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Rainbow vertex connection number and strong rainbow vertex connection number on slinky graph (Sl_nC_4)

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ABSTRACT

A graph is said rainbow connected if no path has more than one vertices of the same color inside. The minimum number of colors required to make a graph to be rainbow vertex-connected is called rainbow vertex connection-number and denoted by $rvc(G)$. Meanwhile, the minimum number of colors required to make a graph to be strongly rainbow vertex-connected is called strong rainbow vertex connection-number and denoted by $srvc(G)$. Suppose there is a simple, limited, and finite graph G. Thus, $G = (V(G), E(G))$ with the determination of k-coloring $c: V(G) \rightarrow \{1,2,\dots,k\}$. The research aims at determining rainbow vertex connection-number and strong rainbow vertex connection-number on slinky graphs (Sl_nC_4). Moreover, the research method applies a literature study with the following procedures; drawing slinky graphs (Sl_nC_4), looking for patterns of rainbow vertex connection-number, and strong rainbow vertex connection-number on slinky graphs (Sl_nC_4), then proving the theorems obtained from the previous pattern. It is obtained $rvc(Sl_nC_4) = 2n - 1$, $srvc(Sl_2C_4) = 4$, and $srvc(Sl_nC_4) = 3n - 3$ for $n \geq 3$.

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INTRODUCTION

The problem of four colors is one of the cases of graph coloring which uses regional coloring that first appeared when trying to color maps in the United States of America (Balakrishnan & Ranganathan, 2012). As mentioned by Coxeter (1971) that map of the United States will be colored to distinguish neighboring countries with at most five or six colors to be used. To color states in the United States what is the minimum number of colors used if every two states that share frontiers are required to be colored differently. After conducting research on this problem, it was found that indeed four colors are needed to color all states in the United States (Chartrand & Zhang, 2012).

As the topic of graph coloring developed, one specific topic appeared in graph coloring, the Rainbow Connection Number. Rainbow connection number was first introduced by Chartrand (2008). Let G is a non-trivial connected graph consisting of a set of (V, E) with a set of vertices G called $V(G)$ and a set of edges G (which may be empty) is called $E(G)$ (Budayasa, 2007). The $u - v$ path is said to be a rainbow path if there are no two edges with the same color. It says rainbow k-coloring if each pair of u and v in G has a rainbow path with u and v as its end vertices (Chartrand et al., 2008).

Rainbow connection number have several types, one of which is rainbow vertex connection number. Rainbow vertex connection number was introduced by Krivelevich & Yuster (2010). Let G graph be non-trivial connected. Path P with k-vertex coloring is called rainbow vertex path. Next, the rainbow vertex connection number was developed by Li et al. (2014) and found the strong rainbow vertex connection number.

Many researchers have developed rainbow vertex connection number on various types of graphs. Among them, research conducted by Bustan (2019) in her thesis that discusses rainbow vertex connection number on star wheel graphs. Then, Bustan (2016) that discusses rainbow vertex connection number on cycle star graphs. Bustan & Salman (2018) examined the rainbow vertex connection number of star fan graphs. Furthermore, Dafik et al. (2018) that discusses strong rainbow and vertex connection of graphs resulting from Edge-Comb products. Moreover, Chen et al. (2018) that discusses strong rainbow connection of graphs.

Furthermore, based on previous research, we took research on slinky graphs to find the rainbow vertex connection number and strong rainbow vertex connection number. Slinky graph is a graph that we have developed using cycle graphs. The slinky graph with the notation $\boldsymbol{Sl_nC_4}$ is the multiplication of n cycle graphs by attaching a copy of the cycle graph right next to it. The cycle graph used in this article is the $\boldsymbol{C_4}$ cycle graph (Vasudev, 2006). So, $\boldsymbol{Sl_nC_4}$ is a slinky graph with $\boldsymbol{n \geq 2}$.

METHOD

This study uses a literature study research method (*library research*). In this research, a study was conducted on books, textbook, journals, and scientific articles about numbers connected to the rainbow points.

The steps taken in this research are formulate the problems discussed and studying various references of coloring the rainbow vertices on graphs. Describe the problem to analyze the problems that have been obtained. Prove patterns and the theorems. We draw a slinky graph one by one with $\boldsymbol{n}$ from 2 to 8. Then, looking for the pattern of rainbow vertex connection number $\boldsymbol{rvc}\ (\boldsymbol{G})$ of $\boldsymbol{G}$ and the strong rainbow vertex connection umber $\boldsymbol{srvc}\ (\boldsymbol{G})$ of $\boldsymbol{G}$. Prove the theorem obtained from the previous pattern. Formulate conclusions based on the proven theorem analysis results.

RESULTS AND DISCUSSION

Definition 1. *Suppose* n *is a positive integer where* $n \geq 2$ *and* $\mathrm{Sl_nC_4}$ *is slinky graph. The slinky graph (*$\mathrm{Sl_nC_4}$*) is a multiplication of cycle graph* $\mathrm{C_4}$ *of* n *and has a diameter or is denoted as* diam(G) *of* 2n.

The slinky graph (Sl_nC_4) is formed by a set of vertices and edges which are respectively defined by:

$V(G) = \{u_i | i \in [1,2n-2]\} \cup v_i | i \in [1,4n]$

$E(G) = \{v_i v_{i+1}, v_{4n+1} = v_1 | i \in [1,4n]\} \cup$

$v_i u_{i-1}, v_i u_i | i \in [1,2n-2], i \text{ even} \cup$

$\{v_{4n-i}u_i, v_{4n-i}u_{i-1} | i \in [1, 2n - 2], i \text{ even}\}$

The following figures are slinky graph Sl_2C_4 and Sl_5C_4 using Definition 1, which are shown in Figure 1 (Ummah, 2013).

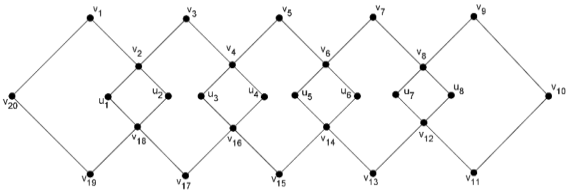

Figure 1. Slinky Graph Sl_5C_4

Rainbow Vertex Connection on Slinky Graphs (Sl_nC_4)

Theorem 1. *Suppose n is a positive integer where* $\mathrm{n} \geq 2$ *and* $\mathrm{Sl_nC_4}$ *are slinky graphs, then*

$$\mathrm{rvc(G)} = 2\mathrm{n} - 1$$

Proof:

Since $diam(Sl_nC_4) = 2n$, it will be shown that $rvc(G) \leq 2n - 1$. For this reason, the coloring defined presented in Definition 2 as follows.

Definition 2. *Suppose* n *is a positive integer where* $\mathrm{n} \geq 2$ *and* $\mathrm{Sl_nC_4}$ *are slinky graphs, then defined* c *coloring* $\mathrm{c}: \mathrm{V(G)} \to \{1,2,3, \dots, 2\mathrm{n} - 1\}$ *as follows*

$\mathrm{u_i} = 1, \quad \mathrm{i} \in [1, 2\mathrm{n} - 2]$

$\mathrm{v_i} = 2\mathrm{n} - 1, \quad \mathrm{i} = 2\mathrm{n} - 1 \wedge \mathrm{i} = 4\mathrm{n} - 2$

$\mathrm{v_i} = \mathrm{i} \bmod (2\mathrm{n} - 1), \ \mathrm{i} \in [1, 4\mathrm{n}], \mathrm{i} \neq 2\mathrm{n} - 1 \wedge \mathrm{i} \neq 4\mathrm{n} - 2$

So, based on Definition 2, it is obtained the rainbow vertex coloring of slinky graph Sl_6C_4 shown in Figure 2.

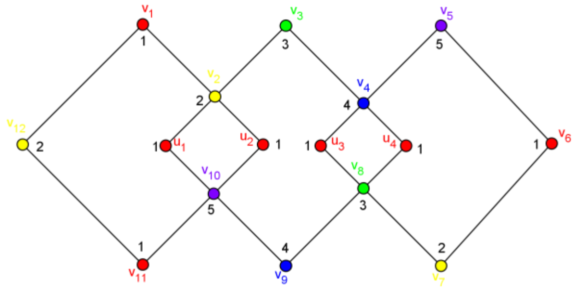

Figure 2. Rainbow Vertex Coloring Sl_3C_4

Furthermore, for each pair of neighboring x and y vertices there is clearly a rainbow path. This is shown in Table 1, where for each pair of vertices $x, y \in V(G)$ there is a rainbow path with c-coloring (Wibisono, 2008).

Table 1. Rainbow Path Sl_nC_4

Case	x	y	Condition	Rainbow Path
1.	v_i	v_j	$i, j \in [1, 2n+1], i < j$	$v_i, v_{i+1}, \dots, v_j$
			$i \in [2n+1, 4n], i < j$	$v_i, v_{i+1}, \dots, v_j$
			$j \in [\, 2n+1, 4n+1 \,] \| v_{4n+1} = v_1$	
			$i, j \in [2n-1, 4n-1], i < j$	$v_i, v_{i+1}, \dots, v_j$
			$i, j \in [2n, 4n], i < j$	$v_i, v_{i+1}, \dots, v_j$
2.	u_i	u_j	$i, j \in [1, 2n-2], i, j \ odd$	$u_i, v_{i+1}, \dots, v_{j+1}, u_j$
			$i, j \in [1, 2n-2], i, j \ even$	$u_i, v_i, \dots, v_j, u_j$
			$i, j \in [1, 2n-2], i < j$ $i \ odd, j \ even$	$u_i, v_{i+1}, \dots, v_j, u_j$
			$i, j \in [1, 2n-2], i < j,$	$u_i, v_i, \dots, v_{j+1}, u_j$

Case	x	y	Condition	Rainbow Path
			$i\ even, j\ odd$	
3.	v_{4n}	u_i	$i \in [1,2n-2], i$ odd	$v_{4n}, v_1, v_2, \dots, v_{i+1}, u_i$
			$i \in [1,2n-2]$, i even	$v_{4n}, v_1, v_2, \dots, v_i, u_i$
4.	u_i	v_{2n}	$i \in [1,2n-2]$, i odd	$u_i, v_{i+1}, v_{i+2}, \dots, v_{2n}$
			$i \in [1,2n-2]$, i even	$u_i, v_i, v_{i+1}, \dots, v_{2n}$
5.	v_i	v_{4n-i}	$i \in [1,2n-2]$, i odd	$v_i, v_{i+1}, u_i, v_{4n-i-1}, v_{4n-i}$
6.	v_i	v_k	$i \in [3,n], i\ odd\ , j\ even$	$v_i, v_{i+1}, \dots, v_j, u_j, v_{4n-j},$
			$j \in [2n+1-i, 2n-2],$	$\dots, v_k$
			$k \in [4n-j, 2n+1]$	
			$i \in [n+1,2n-1], i\ odd$	$v_i, v_{i+1}, u_{i+1}, v_{4n-i-1}, \dots, v_k$
			$k \in [2n+1,3n-1]$	
			$i \in [1,n+1], i\ odd$	$v_i, v_{i+1}, u_{i+1}, v_{4n-i-1}, \dots, v_k$
			$k \in [3n-1,4n-1]$	
			$i \in [n+1,2n-1], i\ odd$	$v_i, v_{i+1}, u_{i+1}, v_{4n-i-1}, \dots, v_k$
			$k \in [3n+1,2n+i-2]$	
			$i \in [1,2n-2], i\ even$	$v_i, u_i, v_{4n-i}, \dots, v_k$
			$k \in [2n+1,4n-2]$	
			$i \in [3,n-2], i\ odd$	$v_i, v_{i+1}, u_{i+1}, v_{4n-i-1}, v_{4n-i-2},$
			$k \in [2n+i, 3n-a]$	$\dots, v_k$
			$a = \begin{cases} 2, \text{odd} \\ 3, \ even \end{cases}$	
7.	v_i	v_{4n-j+1}	$i \in [n+2,2n-1], i\ odd$	$v_i, v_{i-1}, v_{i-2}, \dots, v_j, u_j, v_{4n-j},$
			$j \in [2,2n-i+1], j\ even$	v_{4n-j+1}

Strong Rainbow Vertex Connection on Slinky Graphs(Sl_nC_4)

Theorem 2. *Suppose n is a positive integer where* n ≥ 2 *and* $\mathrm{Sl_nC_4}$ *are slinky graphs, then*

$$\mathrm{srvc(G)} = \begin{cases} 4, & \text{for n} = 2 \\ 3\mathrm{n} - 3, & \text{for n} \geq 3 \end{cases}$$

Proof:

Proof of theorem 2.2 is divided into 2 cases, namely:

Case 1. If $n = 2$.

Because $diam(Sl_{2C_4}) = 4$, so $srvc(Sl_2C_4) \geq 3$. Furthermore, it will be shown that $srvc(Sl_2C_4) \geq 4$. Let $srvc(Sl_2C_4) \leq 3$, then there is c^* a 3-strong rainbow vertex coloring on Sl_2C_4 defined by $c^*: V(G) \to \{1,2,3$. Without loss of generality, let a coloring defined as follows:

$$v_i = i, i \in [1,3]$$

Note that v_6, u_1 and u_2 can't be colored 2. Let the color is 2, then there is a path that is not rainbow, namely v_1, v_2, u_1, v_6, v_5. So, let the 1 color to color the vertices u_1 and u_2, the color 3 to color the vertex v_3 Vertices v_5 and v_7 can't be colored 3. If given a color of 3, then there is a path that is not rainbow, namely v_4, v_5, v_6, v_7, v_8. So, let's say that v_5 is colored2 and v_7 colored 1. The vertex v_4 can only be colored 1. Let the vertex v_4 is colored 2 or 3, there is a rainbow path that is not a rainbow that is v_1, v_2, v_3, v_4, v_5. Note that the vertex v_8 can't be colored 1 or 2. If given a color of 1 or 2, there will be a path that is not rainbow, namely v_7, v_8, v_1, v_2, v_3. The vertex v_8 can't be colored 3. If given a color of 3, there will be a path that is not rainbow, namely v_1, v_8, v_7, v_6, v_5.

Because slinky graph Sl_2C_4 is not a 3-strong rainbow vertex coloring, we obtained that $srvc(Sl_2C_4) \geq 4$.

Furthermore, it will be proven $srvc \leq 4$. For this reason, the coloring defined presented in Definition 3 as follows:

Definition 3. *Suppose Sl_2C_4 is slinky graph, then defined coloring $c: V(G) \rightarrow \{1,2,3,4\}$ as follows*

$$v_i = i, i \in [1,3]$$

$$v_4 = v_8 = u_i = 4, i \in [1,2]$$

$$v_5 = 2,$$
$$v_6 = 3,$$
$$v_7 = 1,$$

So, based on Definition 3, it is obtained the strong rainbow vertex coloring of slinky graph Sl_2C_4 shown in Figure 3.

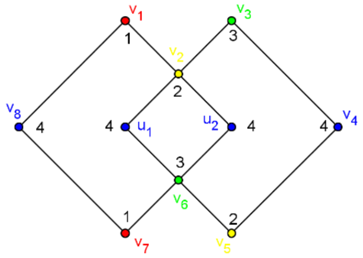

Figure 3. Strong Rainbow Vertex Coloring Sl_2C_4

Furthermore, for each pair of neighboring x and y vertices there is clearly a rainbow path. This is shown in Table 2, where for each pair of vertices $x, y \in V(G)$ there is a strong rainbow path with c-coloring (Wibisono, 2008).

Table 2. Strong Rainbow Path Sl_2C_4

Cases	x	y	Condition	Rainbow Path
1.	v_i	v_j	$i,j \in [1,5], i < j$	$v_i, v_{i+1}, \dots, v_j$
			$i \in [5,8], i < j$	$v_i, v_{i+1}, \dots, v_j$
			$j \in [\,5,9] \vert v_9 = v_1$	
			$i,j \in [3,7], i < j$	$v_i, v_{i+1}, \dots, v_j$
			$i,j \in [4,8], i < j$	$v_i, v_{i+1}, \dots, v_j$
2.	v_7	v_i	$i \in [1,3]$	$v_7, v_8, v_1, \dots, v_i$
3.	v_8	v_i	$i \in [1,4]$	$v_8, v_1, \dots, v_i$
4.	u_1	u_2		u_1, v_2, u_2
				u_1, v_6, u_2
5.	v_8	u_1		v_8, v_1, v_2, u_1
				v_8, v_7, v_6, u_1
6.	v_8	u_2		v_8, v_1, v_2, u_2
				v_8, v_7, v_6, u_2
7.	u_1	v_4		u_1, v_2, v_3, v_4
				u_1, v_6, v_5, v_4
8.	u_2	v_4		u_2, v_2, v_3, v_4
				u_2, v_6, v_5, v_4
9.	v_1	v_5		$\mathrm{v}_1, \mathrm{v}_2, \mathrm{u}_1, \mathrm{v}_6, \mathrm{v}_5$
				v_1, v_2, u_2, v_6, v_5

Cases	x	y	Condition	Rainbow Path
10.	v_3	v_7		v_3, v_2, u_1, v_6, v_7
				v_3, v_2, u_2, v_6, v_7

Case 2. If $n \geq 3$.

Let $3n-3=s$. Because $diam(Sl_nC_4)=2n$, so $srvc(Sl_nC_4) \geq 2n-1$. Furthermore, it will be shown that $srvc(Sl_nC_4) \geq s$ for $n \geq 3$. Let $srvc(Sl_nC_4) \leq s-1 = 3n-4$, then there is c^* a $s-1$-strong rainbow vertex coloring on Sl_{nC_4} defined by $c^*: V(G) \rightarrow \{1,2,3,\dots,s-1$. Without loss of generality, let a coloring defined as follows:

$$v_i = i, i \in [1,2n-1]$$

Note that vertices u_i, u_{i-1} and v_{4n-i} for $i \in [1,2n-2] \forall i$ even can't be colored $c \in [2,2n-2]$. Vertices v_j for $j \in [2n+1,4n-1]$, vertices u_i and u_{i-1} for $i \in [1,2n-2] \forall i$ even can't be colored with the same color. Suppose the vertices are colored so, there will be a path that is not rainbow like in the Table 3.

Table 3. Non-Rainbow Path

Case	Condition	Non-Rainbow Path
1.	$i,j \in [1,2n-1], i<j, j\ odd$	$v_i, \dots, v_{j+1}, u_j, v_{4n-j-1}, \dots, v_k$
	$k \in [2n+1,4n-j-1]$	
2.	$i,j \in [1,2n-1], i<j, j\ even$	$v_i, \dots, v_j, u_j, v_{4n-j}, \dots, v_k$
	$k \in [2n+1,4n-j]$	
3	$i,j \in [1,2n-1], i>j, j\ odd$	$v_i, \dots, v_{j+1}, u_j, v_{4n-j-1}, \dots, v_k$
	$k \in [4n-j-1,4n-1]$	

Note that vertices v_{4n-i} for $i \in [1,2n-2] \forall i$ even can only be colored with 1 or $2n-1$ color and can't be colored with the same color, so there will only be 2 vertices v_{4n-i} for $i \in [1,2n-2] \forall i$ even can be colored. Color 2 or $2n-2$ can only be used to color vertices v_{4n-1} and v_{2n+1}. Suppose the color 2 or $2n-2$ is used to color the vertices v_i for $i \in (2n+2,4n-2)$, then there will be a path that is not rainbow like in Table 3 for the case of 1,2,3, and 4. The odd can only be colored using the same color. Suppose the vertices v_i and v_{4n-i} for $i \in [3,2n-3] \forall i$ odd colored with a different color, then there will be a path that is not rainbow as in Table 3 for cases 1,2,3, and 4.

It is known that the path $v_{2n+1}, v_{2n+2}, \dots, v_{4n-1}$ must have a different vertices color, so it takes $(n-3)$-more colors to color each vertices v_{4n-i} for $i \in [1,2n-2] \forall i$ even remains. The vertices v_{4n} can't be colored $c \in [1,2n-2]$. Suppose that the colors are $c \in [1,2n-2]$, then there is a path that is not a rainbow that is $v_{4n-1}, v_{4n}, v_1, v_2, \dots, v_{2n-1}$. The vertex v_{4n} cannot be colored $c \in [1,3n-4]$. Suppose that the color is $c \in [1,3n-4]$, then there is a path that is not rainbow, namely $v_1, v_{4n}, v_{4n-1}, \dots, v_{2n+1}$. Furthermore, v_{2n} can't be colored $c \in [2,2n-1]$. Suppose that the color is $c \in [2,2n-1]$, then there is a path that is not rainbow, namely $v_1, v_2, \dots, v_{2n+1}$. The vertex v_{2n} can't be colored $c \in [1,3n-4]$. Suppose that the colors are $c \in [1,3n-4]$, then there is a path that is not rainbow which is $v_{2n-1}, v_{2n}, v_{2n+1}, \dots, v_{4n-1}$. Vertices u_i for $i \in [1,2n-2]$ cannot be colored $c \in [1,3n-4]$. If colored $c \in [1,3n-4]$, then there is a path that is not rainbow as in Table 3 for the case of 1, and 4.

Because slinky graph Sl_nC_4 for $n \geq 3$ are not a $(s-1)$-strong rainbow vertex coloring,we obtained thath $srvc(Sl_nC_4) \geq s = 3n-3$.

Furthermore, it will be proven $srvc(Sl_nC_4) \leq s$ for $n \geq 3$. or this reason, the coloring defined presented in Definition 4 as follows:

Definition 4. *Suppose n is a positive integer where $n \geq 3$ and Sl_nC_4 are slinky graphs, then defined coloring $c: V(G) \to \{1,2,3,\dots,s\}$ as follows*

$$v_i = i, i \in [1,2n-1]$$

$$v_{2n} = v_{4n} = u_i = 3n-3, i \in [1,2n-2]$$

$$v_{2n+1} = 2n-2$$

$$v_{4n+i-3} = i, i \in [1,2]$$

$$v_{4n-i} = i, i \in [3,2n-3], i \text{ odd}$$

$$v_{2n+2i} = 2n+i-2, i \in [1,n-2]$$

So, based on Definition 4, it is obtained the strong rainbow vertex coloring of slinky graph Sl_3C_4 and Sl_7C_4 shown in Figure 4.

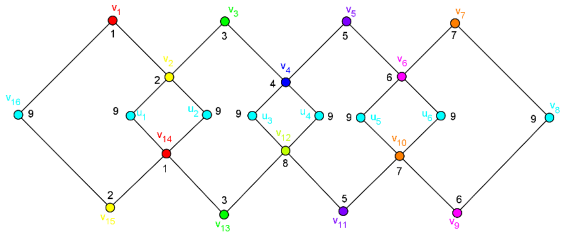

Figure 4. Strong Rainbow Vertex Coloring Sl_4C_4

Furthermore, for each pair of neighboring x and y vertices there is clearly a rainbow path. This is shown in Table 4, where for each pair of vertices $x, y \in V(G)$ there is a strong rainbow path with c-coloring.

Table 4. Strong Rainbow Path Sl_nC_4 for $n \geq 3$

Case	x	y	Condition	Rainbow Path
1.	v_i	v_j	$i,j \in [1,2n+1], i<j$	$v_i, v_{i+1}, \dots, v_j$
			$i \in [2n+1,4n], i<j$	$v_i, v_{i+1}, \dots, v_j$
			$j \in [2n+1,4n+1] \mid v_{4n+1} = v_1$	
			$i,j \in [2n-1,4n-1], i<j$	$v_i, v_{i+1}, \dots, v_j$
			$i,j \in [2n,4n], i<j$	$v_i, v_{i+1}, \dots, v_j$
2.	v_{4n-1}	v_i	$i \in [1,2n-1]$	$v_{4n-1}, v_{4n}, v_1, \dots, v_i$
3.	v_{4n}	v_i	$i \in [1,2n]$	$v_{4n}, v_1, \dots, v_i$
4.	u_i	u_j	$i,j \in [1,2n-2],\ i,j\ odd$	$u_i, v_{i+1}, \dots, v_{j+1}, u_k$
			$i,j \in [1,2n-2], i,j\ even$	$u_i, v_i, \dots, v_j, u_j$
			$i,j \in [1,2n-2], i<j,$ $i\ odd, j\ even$	$u_i, v_{i+1}, \dots, v_j, u_j$
			$i,j \in [1,2n-2], i<j$ $i\ even, j\ odd$	$u_i, v_i, \dots, v_{j+1}, u_j$
5.	u_i	u_k	$i,k \in [1,2n-2], i<k,$ $i,k\ odd$	$u_i, v_{4n-i-1}, v_{4n-i-2} \dots, v_{4n-k-1}, u_k$
			$i,k \in [1,2n-2], i<k$ $i\ odd, k\ even$	$u_i, v_{4n-i-1}, v_{4n-i-2} \dots, v_{4n-k}, u_k$

Case	x	y	Condition	Rainbow Path
			$i,k \in [1,2n-2], i < k$ i $even, k$ odd	$u_i, v_{4n-i}, v_{4n-i-1} \dots, v_{4n-k-1}, u_k$
			$i,k \in [1,2n-2], i < k,$ i,k $even$	$u_i, v_{4n-i}, v_{4n-i-1} \dots, v_{4n-j}, u_j$
6.	v_{4n}	u_i	$i \in [1,2n-2], i$ odd	$v_{4n}, v_1, v_2, \dots, v_{i+1}, u_i$
			$i \in [1,2n-2], i$ $even$	$v_{4n}, v_1, v_2, \dots, v_i, u_i$
7.	u_i	v_{2n}	$i \in [1,2n-2], i$ odd	$u_i, v_{i+1}, v_{i+2}, \dots, v_{2n}$
			$i \in [1,2n-2], i$ $even$	$u_i, v_i, v_{i+1}, \dots, v_{2n}$
8.	u_i	v_j	$i \in [1,2n-2], i$ odd $j \in [2n,4n]$	$u_i, v_{4n-i-1}, \dots, v_j$
			$i \in [1,2n-2], i$ $even$ $j \in [2n,4n]$	$u_i, v_{4n-i}, \dots, v_j$
9.	v_i	v_{4n-i}	$i \in [1,2n-2], i$ odd	$v_i, v_{i+1}, u_i, v_{4n-i-1}, v_{4n-i}$
			$i \in [1,2n-2], i$ odd	$v_i, v_{i+1}, u_{i+1}, v_{4n-i-1}, v_{4n-i}$
			$i \in [3,2n], i$ odd	$v_i, v_{i-1}, u_{i-2}, v_{4n-i+1}, v_{4n-i}$
			$i \in [3,2n], i$ odd	$v_i, v_{i-1}, u_{i-1}, v_{4n-i+1}, v_{4n-i}$
10.	v_i	v_k	$i,j \in [1,2n-1], i < j,$ j odd $k \in [2n+1,4n-j-1]$	$v_i, \dots, v_{j+1}, u_j, v_{4n-j-1}, \dots, v_k$
			$i,j \in [1,2n-1], i < j,$ j $even$ $k \in [2n+1,4n-j]$	$v_i, \dots, v_j, u_j, v_{4n-j}, \dots, v_k$
			$i,j \in [1,2n-1], i > j,$ j odd $k \in [4n-j-1,4n-1]$	$v_i, \dots, v_{j+1}, u_j, v_{4n-j-1}, \dots, v_k$
			$i,j \in [1,2n-1], i > j,$ j $even$ $k \in [4n-j,4n-1]$	$v_i, \dots, v_j, u_j, v_{4n-j}, \dots, v_k$

CONCLUSIONS AND SUGGESTIONS

Based on the main results, to determine the rainbow vertex-connection number on slinky graphs can use Theorem 1, for example, on a slinky graph $\boldsymbol{Sl_4C_4}$, we got $\boldsymbol{rvc(G) = 7}$. To determine the strong rainbow vertex-connection number on slinky graph can use Theorem 2, for example, on a slinky graph $\boldsymbol{Sl_2C_4}$, we got $\boldsymbol{srvc(G) = 4}$, and on a slinky graph $\boldsymbol{Sl_5C_4}$, we got $\boldsymbol{srvc(G) = 12}$.

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