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Odd Harmonious Labeling on Edge Amalgamation from Double Quadrilateral Graphs

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ABSTRACT

A graph that has odd harmonious labeling properties is called an odd harmonious graph. The purpose of this research is to obtain a new class graphs construction which is a family of odd harmonious graphs. The research method used consisted of several stages, namely research preparation, research investigation and verification of research results. The results of this study, we will give a edge amalgamation construction of $\boldsymbol{n}$ double quadrilateral graphs $\boldsymbol{DQ}$, denoted by $\circledast\,\boldsymbol{DQ}(\boldsymbol{n})$ and graph obtained by connecting between two graphs $\circledast\,\boldsymbol{DQ}(\boldsymbol{n})$ with line graph $\boldsymbol{L_2}$, denoted by $\circledast\left(\boldsymbol{DQ}(\boldsymbol{n}), \boldsymbol{L_2}, \boldsymbol{DQ}(\boldsymbol{n})\right)$. It has further been proven that graph $\circledast\,\boldsymbol{DQ}(\boldsymbol{n})$ and graph $\circledast\left(\boldsymbol{DQ}(\boldsymbol{n}), \boldsymbol{L_2}, \boldsymbol{DQ}(\boldsymbol{n})\right)$ have odd harmonious labeling properties, such that all of them are odd harmonious graphs.

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INTRODUCTION

Graph labeling has become a combinatoric field that has developed very rapidly in recent years both in theory and application research on labeling graphs is very important because in several journals submitted by Gallian in 2019, it is evident that graph labeling can be applied to various scientific fields between others are coding theory, data base management, secret sharing schemes and cryptography. Graph labeling is the mapping of a set of points, a set of lines or a combination of both to positive numbers with certain properties (Gallian, 2019).

One new type of labeling of graphs is the labeling of odd harmonious graphs discovered by Liang and Bai in 2009. Previously in 1980 Graham and Sloane introduced the definition of harmonious graphs. Harmonious graph $G(p, q)$ is a graph that fulfills the properties of the harmonious labeling function, namely that there is an injective vertex labeling function $f: V(\mathrm{G}) \rightarrow Z_q$ in such a way as to induce a function of labeling the edge that is wise $f^*: E(\mathrm{G}) \rightarrow Z_q$ with $f^*(xy) = \left(f(x) + f(y)\right) \bmod q$ is a bijective function and Z_q is a set of modulo integers q (Liang & Bai, 2009).

Liang and Bai extended the definition of harmonious graph to the odd harmonious graph definition. Odd harmonious graphs are graphs $G(p,q)$ which have the properties of the odd harmonious labeling function, there is an injection function $f: V(\mathrm{G}) \rightarrow \{0,1,2,3,\ldots,2q-1\}$ such that the induced function $f^{*}: E(\mathrm{G}) \rightarrow \{1,3,5,7,\ldots,2q-1\}$ defined by $f^{*}(xy) = f(x) + f(y)$ is an bijection (Liang & Bai, 2009).

Relevant research results can be seen in some of the published journals and proceedings, among others. The generalized prism graph is an odd harmonious graph (Saputri, Sugeng, & Froncek, 2013), union of snake graphs and pleated of snake graphs are odd harmonious graph (Firmansah, 2016), the Dutch windmill graph is an odd harmonious graph (Firmansah & Yuwono, 2017a), Cartesian product operation results graphs are odd harmonious graph (Firmansah & Yuwono, 2017b), the variation of the double quadrilateral windmill graphs are odd harmonious graph (Firmansah, 2017), amalgamation of the Dutch windmill graphs are odd harmonious graph (Firmansah & Wahid Syaifuddin, 2018), grid graph is an odd harmonious graphs (Jeyanthi, Philo, & Youssef, 2019), super subdevision graphs are odd harmonious graphs (Jeyanthi, Philo, & Siddiqui, 2019).

Double quadrilateral windmill graphs $DQ^{(n)}$ and amalgamation of double quadrilateral windmill graphs $DQ^{(n)} * L_2 * DQ^{(n)}$ are odd harmonious graph (Firmansah & Syaifuddin, 2018). In this study, the authors are interested in constructing a new graph $\circledast DQ(n)$. The fundamental difference between graphs $DQ^{(n)}$ with graphs $\circledast DQ(n)$ lies in the amalgamation operation used. Graph $DQ^{(n)}$ formed from n graph double quadrilateral DQ with vertex amalgamation operation, while graphs $\circledast DQ(n)$ formed from n double quadrilateral graph DQ with edge amalgamation operation, denoted by $\circledast DQ(n) = DQ * DQ * DQ * \ldots * DQ$.

The author also developed a graph $\circledast DQ(n)$ will become graph $\circledast \left(DQ(n), L_2, DQ(n)\right) = DQ * DQ * \ldots * DQ * L_2 * DQ * DQ * \ldots * DQ$ that is, the graph obtained by connecting between two graphs $\circledast DQ(n)$ with line graphs L_2. Furthermore, graph $\circledast DQ(n)$ and graph $\circledast \left(DQ(n), L_2, DQ(n)\right)$ both of them will be proven have odd harmonious labeling function such that it is an odd harmonious graph.

The results of this study proved to be able to add scientific insights in the field of combinatorics especially for the development of graph labeling theory. More specifically the development of the theory of odd harmonious graph labeling. Namely by getting a new graph class which is a family of odd harmonious graphs.

METHOD

The research method used in this study consisted of several stages. The first stage is the preparation of research which is to construct the construction of a new graph to get a vertex set, edge set, size and order of a graph. The second stage is a research investigation which is to provide a vertex label on the graph which is injective and an edge label that is bijective which meets the properties of the odd harmonious labeling function. The third stage is the verification of the results of the research in the form of theorem construction accompanied by mathematical proof. The final stage is to give an example of label construction on a simple graph to facilitate understanding.

RESULTS AND DISCUSSION

The results and discussion in this study are stated in the form of the construction of definitions and theorems and their proofs.

Edge amalgamation operation from graph G and graph H denoted by $G * H$ is a graph operation that combines edges $ab \in$

G and $cd \in H$ into one edge $uv \in G * H$. Symbol $\circledast\, G(n) = G * G * G * \ldots * G$ express the edge amalgamation of as many graphs G.

Definition 1. Graph $\circledast\, DQ(n) = DQ * DQ * DQ * \ldots * DQ$ is a graph obtained from n double quadrilateral graphs DQ with edge amalgamation operation.

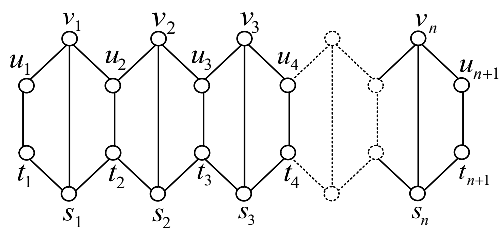

Figure 1. Construction and vertex notation of a graph $\circledast\, DQ(n)$

Based on Figure 1 obtained
$V(\circledast\, DQ(n)) = \{s_i | 1 \leq i \leq n\} \cup \{t_i | 1 \leq i \leq n+1\} \cup \{u_i | 1 \leq i \leq n+1\} \cup \{v_i | 1 \leq i \leq n\}$ and
$E(\circledast\, DQ(n)) = \{s_i t_i | 1 \leq i \leq n\} \cup \{s_i t_{i+1} | 1 \leq i \leq n\} \cup \{s_i v_i | 1 \leq i \leq n\} \cup \{t_i u_i | 1 \leq i \leq n+1\} \cup \{u_i v_i | 1 \leq i \leq n\} \cup \{u_{i+1} v_i | 1 \leq i \leq n\}$
then obtained order $p = 4n + 2$ and size $q = 6n + 1$.

Theorem 1. Graph $\circledast\, DQ(n)$ is an odd harmonious graph.
Proof:

Defined the vertex labeling function $f: V(\circledast\, DQ(n)) \to \{0,1,2,3, \ldots, 12n+1\}$ as follows:

$f(s_i) = 2i - 2, 1 \leq i \leq n$ (1.1)
$f(t_i) = 4i - 3, 1 \leq i \leq n+1$ (1.2)
$f(u_i) = 6n + 2i - 2, 1 \leq i \leq n+1$ (1.3)
$f(v_i) = 4i - 1, 1 \leq i \leq n$ (1.4)

It will be shown that the function f injective. Based on the equation (1.1), (1.2), (1.3), and (1.4) obtained.

(a) The value of the label for each vertex is
$0,2,4,6, \ldots, 2n-2 \equiv 2 (\text{mod } 2)$,
$1,5,9,13, \ldots, 4n+1 \equiv 1 (\text{mod } 4)$,
$6n, 6n+2, 6n+4, 6n+8 \ldots, 8n \equiv 6n (\text{mod } 2)$, and
$3,7,11,15, \ldots, 4n-1 \equiv 3 (\text{mod } 4)$.
so each vertex has a different label.

(b) Vertex set after labeling
$f(V(\circledast\, DQ(n))) = \{0,2,4,6, \ldots, 2n-2\} \cup \{1,5,9,13, \ldots, 4n+1\} \cup \{6n, 6n+2, 6n+4, 6n+8, \ldots, 8n\} \cup \{3,7,11,15, \ldots, 4n-1\}$
$= \{0,1,2,3,4,5,6,7,8,9, \ldots, 2n-2, 4n-1, 4n+1, 6n, 6n+2, 6n+4, 6n+8, \ldots, 8n\} = \{0,1,2,3, \ldots, 8n\}$
$\subseteq \{0,1,2,3, \ldots, 12n+1\}$

Then based on (a) and (b) it is proven that the function f injective.

Next is defined edge labeling function $f^*: E(\circledast\, DQ(n)) \to \{1,3,5,7, \ldots, 12n+1\}$ as follows:

$f^*(s_i t_i) = 6i - 5, 1 \leq i \leq n$ (1.5)
$f^*(s_i t_{i+1}) = 6i - 1, 1 \leq i \leq n$ (1.6)
$f^*(s_i v_i) = 6i - 3, 1 \leq i \leq n$ (1.7)
$f^*(t_i u_i) = 6n + 6i - 5, 1 \leq i \leq n+1$ (1.8)
$f^*(u_i v_i) = 6n + 6i - 3, 1 \leq i \leq n$ (1.9)
$f^*(u_{i+1} v_i) = 6n + 6i - 1, 1 \leq i \leq n$ (1.10)

It will be shown that the function f^* bijective. Based on the equation (1.5), (1.6), (1.7), (1.8), (1.9) and (1.10) obtained.

(c) The label value of each edge is
$1,7,13,19, \ldots, 6n-5 \equiv 1 (\text{mod } 6)$,
$5,11,17,23, \ldots, 6n-1 \equiv 5 (\text{mod } 6)$,
$3,9,15,21, \ldots, 6n-3 \equiv 3 (\text{mod } 6)$,
$6n+1, 6n+7, 6n+13, 6n+19, \ldots, 12n+1 \equiv 1 (\text{mod } 6)$,
$6n+3, 6n+9, 6n+15, 6n+21, \ldots, 12n-3 \equiv 3 (\text{mod } 6)$, and
$6n+5, 6n+11, 6n+17, 6n+23, \ldots, 12n-1 \equiv 5 (\text{mod } 6)$
so that each edge has a different label.

(d) Edge set after labeling
$f^*(E(\circledast\, DQ(n))) =$
$\{1,7,13,19, \ldots, 6n-5\} \cup \{5,11,17,23, \ldots, 6n-1\} \cup \{3,9,15,21, \ldots, 6n-3\} \cup \{6n+1, 6n+7, 6n+13, 6n+19, \ldots, 12n+1\} \cup \{6n+3, 6n+9, 6n+15, 6n+21, \ldots, 12n-3\} \cup \{6n+5, 6n+11, 6n+17, 6n+23, \ldots, 12n-1\}$
$= \{1,3,5,7,9,11,13,15,17,19, \ldots, 6n-5, 6n-3, 6n-1, 6n+1, 6n+3, 6n+$

$$5, 6n+7, 6n+9, 6n+11, 6n+13, 6n+15, 6n+17, 6n+19, \ldots, 12n-3, 12n-1, 12n+1\}$$
$$= \{1,3,5,7,\ldots,12n+1\}$$

Then based on (c) and (d) it is proven that the function f^* bijective.

Furthermore, because function f injective such that the induced function f^* bijective then graph $\circledast\, DQ(n)$ is an odd harmonious graph. ■

Example 1. Here are given odd harmonious labeling on the graph $\circledast\, DQ(6)$ (Figure 2) and $\circledast\, DQ(7)$ (Figure 3).

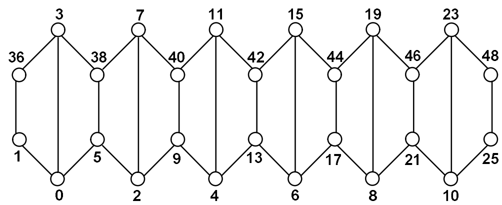

Figure 2. Odd harmonious labeling on a graph $\circledast\, DQ(6)$

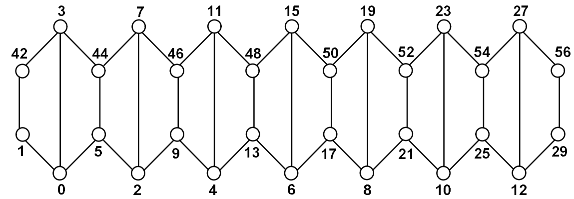

Figure 3. Odd harmonious labeling on a graph $\circledast\, DQ(7)$

Definition 2. Graph $\circledast\big(DQ(n), L_2, DQ(n)\big) = DQ * DQ * \ldots * DQ * L_2 * DQ * DQ * \ldots * DQ$ is a graph obtained by connecting between two graphs $\circledast\, DQ(n)$ with line graph L_2.

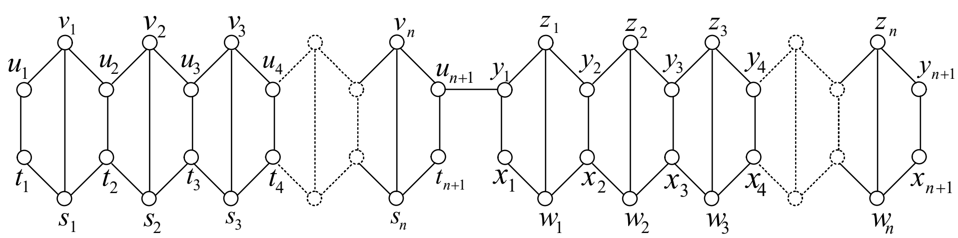

Figure 4. Construction and vertex notation of a graph $\circledast\big(DQ(n), L_2, DQ(n)\big)$.

Based on Figure 4 obtained

$V\Big(\circledast\big(DQ(n), L_2, DQ(n)\big)\Big) = \{s_i|1 \le i \le n\} \cup \{t_i|1 \le i \le n+1\} \cup \{u_i|1 \le i \le n+1\} \cup \{v_i|1 \le i \le n\} \cup \{w_i|1 \le i \le n\} \cup \{x_i|1 \le i \le n+1\} \cup \{y_i|1 \le i \le n+1\} \cup \{z_i|1 \le i \le n\}$ and $E\Big(\circledast\big(DQ(n), L_2, DQ(n)\big)\Big) = \{s_it_i|1 \le i \le n\} \cup \{s_it_{i+1}|1 \le i \le n\} \cup \{s_iv_i|1 \le i \le n\} \cup \{t_iu_i|1 \le i \le n+1\} \cup \{u_iv_i|1 \le i \le n\} \cup \{u_{i+1}v_i|1 \le i \le n\} \cup \{u_{n+1}y_1\} \cup \{w_ix_i|1 \le i \le n\} \cup \{w_ix_{i+1}|1 \le i \le n\} \cup \{w_iz_i|1 \le i \le n\} \cup \{x_iy_i|1 \le i \le n+1\} \cup \{y_iz_i|1 \le i \le n\} \cup \{y_{i+1}z_i|1 \le i \le n\}$

an order is obtained $p = 8n+4$ and size $q = 12n+3$.

Theorem 2. Graph $\circledast\big(DQ(n), L_2, DQ(n)\big)$ is an odd harmonious graph.

Proof:

Defined the vertex labeling function $f: V\Big(\circledast\big(DQ(n), L_2, DQ(n)\big)\Big) \to \{0,1,2,3,\ldots,24n+5\}$ as follows :

$f(s_i) = 2i-2, 1 \le i \le n$ (2.1)

$f(t_i) = 4i-3, 1 \le i \le n+1$ (2.2)

$f(u_i) = 6n+2i-2, 1 \le i \le n+1$ (2.3)

$f(v_i) = 4i-1, 1 \le i \le n$ (2.4)

$f(w_i) = 4n+4i+1, 1 \le i \le n$ (2.5)

$f(x_i) = 14n+2i, 1 \le i \le n+1$ (2.6)

$f(y_i) = 4n+4i-1, 1 \le i \le n+1$ (2.7)

$f(z_i) = 8n+2i, 1 \le i \le n$ (2.8)

It will be shown that the function f injective. Based on the equation (2.1), (2.2), (2.3), (2.4), (2.5), (2.6), (2.7) and (2.8) obtained

(a) The value of the label for each vertex is
$0,2,4,6,\dots,2n-2 \equiv 2(\text{mod } 2)$,
$1,5,9,13,\dots,4n+1 \equiv 1(\text{mod } 4)$,
$6n, 6n+2, 6n+4, 6n+8 \dots, 8n \equiv 6n(\text{mod } 2)$,
$3,7,11,15,\dots,4n-1 \equiv 3(\text{mod } 4)$,
$4n+5, 4n+9, 4n+13, 4n+17,\dots,8n+1 \equiv 1(\text{mod } 4)$,
$14n+2, 14n+4, 14n+6, 14n+8,\dots,16n+2 \equiv 14n(\text{mod } 2)$,
$4n+3, 4n+7, 4n+11, 4n+15,\dots,8n+3 \equiv 3(\text{mod } 4)$, and
$8n+2, 8n+4, 8n+6, 8n+8,\dots,10n \equiv 8n(\text{mod } 2)$,
so each vertex has a different label.

(b) Vertex set after labeling
$f\left(V\left(\circledast\left(DQ(n), L_2, DQ(n)\right)\right)\right) = \{0,2,4,6,\dots,2n-2\} \cup \{1,5,9,13,\dots,4n+1\} \cup \{6n, 6n+2, 6n+4, 6n+8,\dots,8n\} \cup \{3,7,11,15,\dots,4n-1\} \cup \{4n+5, 4n+9, 4n+13, 4n+17,\dots,8n+1\} \cup \{14n+2, 14n+4, 14n+6, 14n+8,\dots,16n+2\} \cup \{4n+3, 4n+7, 4n+11, 4n+15,\dots,8n+3\} \cup \{8n+2, 8n+4, 8n+6, 8n+8,\dots,10n\}$
$= \{0,1,2,3,4,5,6,7,8,9,\dots,2n-2, 4n-1, 4n+1, 4n+3, 4n+5, 4n+7, 4n+9, 4n+11, 4n+13, 4n+15, 4n+17,\dots,6n, 6n+2, 6n+4, 6n+8,\dots,8n, 8n+1, 8n+2, 8n+3, 8n+4, 8n+6, 8n+8,\dots,10n, 14n+2, 14n+4, 14n+6, 14n+8,\dots,16n+2\}$
$= \{0,1,2,3,\dots,16n+2\}$

Then based on (a) and (b) it is proven that the function f injective.

Next is defined edge labeling function
$f^*: E\left(\circledast\left(DQ(n), L_2, DQ(n)\right)\right) \to \{1,3,5,7,\dots,24n+5\}$ as follows:

$f^*(s_i t_i) = 6i-5, 1 \le i \le n$ (2.9)
$f^*(s_i t_{i+1}) = 6i-1, 1 \le i \le n$ (2.10)
$f^*(s_i v_i) = 6i-3, 1 \le i \le n$ (2.11)
$f^*(t_i u_i) = 6n+6i-5, 1 \le i \le n+1$ (2.12)
$f^*(u_i v_i) = 6n+6i-3, 1 \le i \le n$ (2.13)
$f^*(u_{i+1} v_i) = 6n+6i-1, 1 \le i \le n$ (2.14)
$f^*(u_{n+1} y_1) = 12n+3$ (2.15)
$f^*(w_i x_i) = 18n+6i+1, 1 \le i \le n$ (2.16)
$f^*(w_i x_{i+1}) = 18n+6i+3, 1 \le i \le n$ (2.17)
$f^*(w_i z_i) = 12n+6i+1, 1 \le i \le n$ (2.18)
$f^*(x_i y_i) = 18n+6i-1, 1 \le i \le n+1$ (2.19)
$f^*(y_i z_i) = 12n+6i-1, 1 \le i \le n$ (2.20)
$f^*(y_{i+1} z_i) = 12n+6i+3, 1 \le i \le n$ (2.21)

It will be shown that the function f^* bijective. Based on the equation (2.9), (2.10), (2.11), (2.12), (2.13), (2.14), (2.15), (2.16), (2.17), (2.18), (2.19), (2.20), and (2.21) obtained

(c) The label value of each edge is
$1,7,13,19,\dots,6n-5 \equiv 1(\text{mod } 6)$,
$5,11,17,23,\dots,6n-1 \equiv 5(\text{mod } 6)$,
$3,9,15,21,\dots,6n-3 \equiv 3(\text{mod } 6)$,
$6n+1, 6n+7, 6n+13, 6n+19,\dots,12n+1 \equiv 1(\text{mod } 6)$,
$6n+3, 6n+9, 6n+15, 6n+21,\dots,12n-3 \equiv 3(\text{mod } 6)$,
$6n+5, 6n+11, 6n+17, 6n+23,\dots,12n-1 \equiv 5(\text{mod } 6)$,
$12n+3 \equiv 3(\text{mod } 6)$,
$18n+7, 18n+13, 18n+19, 18n+25,\dots,24n+1 \equiv 1(\text{mod } 6)$,
$18n+9, 18n+15, 18n+21, 18n+27,\dots,24n+3 \equiv 3(\text{mod } 6)$,
$12n+7, 12n+13, 12n+19, 12n+25,\dots,18n+1 \equiv 1(\text{mod } 6)$,
$18n+5, 18n+11, 18n+17, 18n+23,\dots,24n+5 \equiv 5(\text{mod } 6)$,
$12n+5, 12n+11, 12n+17, 12n+23,\dots,18n-1 \equiv 5(\text{mod } 6)$, and
$12n+9, 12n+15, 12n+21, 12n+27,\dots,18n+3 \equiv 3(\text{mod } 6)$
so that each edge has a different label.

(d) Edge set after labeling
$f^*\left(E\left(\circledast\left(DQ(n), L_2, DQ(n)\right)\right)\right) = \{1,7,13,19,\dots,6n-5\} \cup \{5,11,17,23,\dots,6n-1\} \cup \{3,9,15,21,\dots,6n-3\} \cup \{6n+1, 6n+7, 6n+13, 6n+19,\dots,12n+1\} \cup \{6n+3, 6n+9, 6n+15, 6n+21,\dots,12n-3\} \cup \{6n+5, 6n+11, 6n+17, 6n+23,\dots,12n-1\} \cup \{12n+3\} \cup \{18n+7, 18n+$

$13,18n+19,18n+25,\dots,24n+1\} \cup$
$\{18n+9,18n+15,18n+21,18n+$
$27,\dots,24n+3\} \cup \{12n+7,12n+$
$13,12n+19,12n+25,\dots,18n+1\} \cup$
$\{18n+5,18n+11,18n+17,18n+$
$23,\dots,24n+5\} \cup \{12n+5,12n+$
$11,12n+17,12n+23,\dots,18n-1\} \cup$
$\{12n+9,12n+15,12n+21,12n+$
$27,\dots,18n+3\}$
$=\{1,3,5,7,9,11,13,15,17,19,\dots,6n-$
$5,6n-3,6n-1,6n+1,6n+3,6n+$
$5,6n+7,6n+9,6n+11,6n+$
$13,6n+15,6n+17,6n+$
$19,\dots,12n-3,12n-1,12n+$
$1,12n+3,12n+5,12n+7,12n+$
$9,12n+11,12n+13,12n+15,12n+$
$17,12n+19,12n+21,12n+$
$23,\dots,18n-1,18n+1,18n+$
$3,18n+5,18n+7,18n+9,18n+$
$11,18n+13,18n+15,18n+$
$17,18n+19,18n+21,18n+$
$23,18n+25,18n+27,18n+$
$29,\dots,24n+1,24n+3,24n+5\}$
$=\{1,3,5,7,\dots,24n+5\}$

Then based on (c) and (d) it is proven that the function f^* bijective.

Furthermore, because function f injective such that the induced function f^* bijective then graph $\circledast \left(DQ(n), L_2, DQ(n)\right)$ is an odd harmonious graph. ■

Example 2. Here are given odd harmonious labeling on a graph $\circledast \left(DQ(5), L_2, DQ(5)\right)$ (Figure 5) and $\circledast \left(DQ(6), L_2, DQ(6)\right)$ (Figure 6).

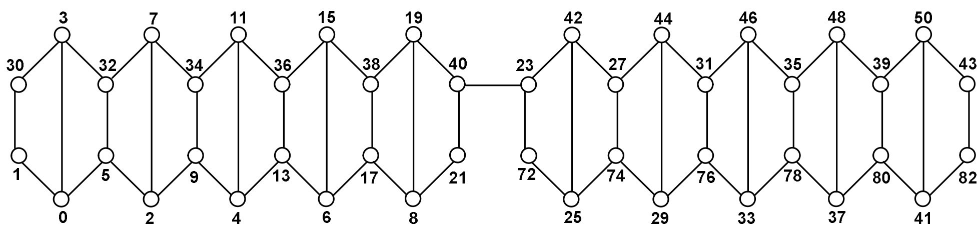

Figure 5. Odd harmonious labeling on a graph $\circledast \left(DQ(5), L_2, DQ(5)\right)$

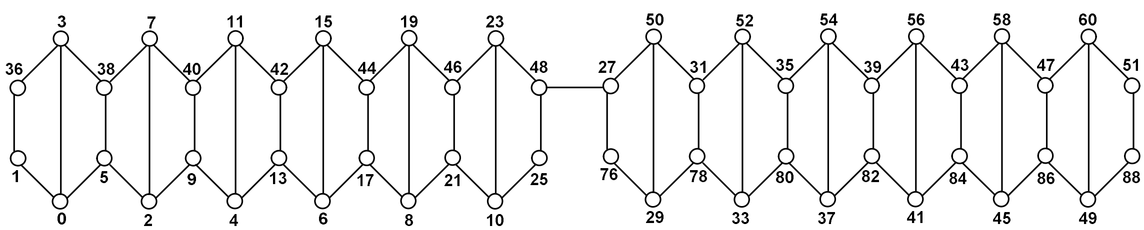

Figure 6. Odd harmonious labeling on a graph $\circledast \left(DQ(6), L_2, DQ(6)\right)$

When comparing the results of this study with previous relevant research, similarities and differences are obtained. The similarity lies in the results of this study, which have been proven that graphs $\circledast DQ(n)$ and graphs $\circledast \left(DQ(n), L_2, DQ(n)\right)$ is part of the odd harmonious family of graphs, the proof of which can be seen in Theorem 1 and Theorem 2. While the difference lies in different research objects. The graph class observed is a new graph class resulting from the development of edge amalgamation operations from the double quadrilteral graph DQ, can be seen in Definitions 1 and Definition 2.

CONCLUSIONS AND SUGGESTIONS

Based on the results and discussion, it can be concluded that graph $\circledast DQ(n)$ that is, the graph obtained from the results of n double quadrilateral DQ with edge amalgamation operations is an odd harmonious graph. The other side is graph

$\circledast\left(DQ(n), L_2, DQ(n)\right)$ that is, the graph obtained by connecting between two graphs $\circledast DQ(n)$ with line graphs L_2 also an odd harmonious graph.

Furthermore, this research can be developed by finding the odd harmonious labeling construction of graphs $\circledast \left(DQ(n), L_2, DQ(n), \ldots, L_2, DQ(n)\right)$ that is, the graph obtained by connecting between m graph $\circledast DQ(n)$ with $m-1$ line graph L_2.

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