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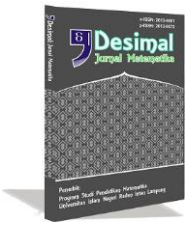

Students' mathematical reasoning abilities are viewed from Van Hiele's theory

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ABSTRACT

Mathematical reasoning is the ability of a person's brain that includes the presentation of statements, making conjectures, performing mathematical manipulations, and presenting conclusions. This study aims to analyze or describe the ability of mathematical reasoning in solving trigonometric problems based on Van Hiele's level of thinking among high school students. This research uses a descriptive qualitative method. Data collection techniques consist of tests and interviews. Data analysis involves data presentation, reduction, and conclusion. The results of this study show that students at level 0 (visualization) of Van Hiele's level of thinking have one indicator of mathematical reasoning ability, which is to present statements. Students at level 1 (analysis) of Van Hiele's level of thinking have two indicators of mathematical reasoning: presenting statements and making conjectures. Students at level 2 (informal cut) of Van Hiele's level of thinking have three indicators of mathematical reasoning: presenting statements, making conjectures, and mathematical manipulations. Students at level 3 (deduction) of Van Hiele's level of thinking have four indicators of mathematical reasoning ability: presenting statements, making conjectures, mathematical manipulations, and drawing conclusions. Therefore, the higher the thinking level of the students at Van Hiele, the higher their mathematical reasoning ability. Whereas, the lower the Van Hiele thinking level of students, the lower their mathematical reasoning ability.

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INTRODUCTION

Mathematics is a subject that makes students who study it discipline in their minds. Mathematics education at school emphasizes structuring reasoning, forming attitudes and skills in applying mathematics. Therefore, if mathematics is taught correctly and can be well received by students, it will improve students' thinking and reasoning skills. Mathematics is the science of organized structures, mathematics discusses facts and relationships, as well as space and shape (Nur'aini et al. 2017). Students are

expected to have a high interest in learning mathematics. Learning mathematics is learning to solve problems (Samo 2017). Problem solving is the main focus of mathematics learning nowadays. By solving mathematical problems, students acquire a way of thinking, get used to, have a high curiosity, and have very high mathematical reasoning skills.

The importance of mathematical reasoning abilities is very influential with the mathematics learning process they follow (Kurnia Putri, Sulianto, and Azizah 2019). This mathematical reasoning ability is the foundation of mathematics (Mahendra, Caswita, and Bharata 2019). Mathematical reasoning is a part of mathematical thinking that includes making formulas and drawing conclusions (Lestari, Subanji, and Irawati 2022). Mathematical reasoning is a brain habit that if developed well and consistently will make it easier to communicate mathematics in writing and orally (Muslimin and Sunardi 2019). The ability to reason is a logical thinking process in gathering facts to draw a conclusion (Pradini and Kesumawati 2022). Mathematical reasoning ability can give a person a logical way of thinking and conclude the learning process (Oktaviana and Aini 2021). In students' mathematical reasoning abilities, it affects their reasoning abilities as quoted by Megawati in (Linola, Marsitin, and Wulandari 2017). Therefore, the ability to solve mathematical problems is an indicator of the achievement of mathematical learning in school that must be mastered by students (Gradini, Yustinaningrum, and Safitri 2022). One of the higher order thinking skills that is important to be developed and mastered by students carefully is reasoning ability (Hidayat et al. 2022). By having good mathematical reasoning skills, students can easily solve daily problems that require reasoning skills (Ariati and Juandi 2022). With this, in learning mathematics, students must have very high reasoning in solving problems, but there are still many different types of students; some have high reasoning, some have medium reasoning, and some have low reasoning. Mathematical reasoning skills can make students better understand and master mathematics learning. Mathematical reasoning skills are very effective when using Van Hiele's theory (Alpian and Anggoro 2020).

Trigonometry is a branch of mathematics that studies the relationship between sides and angles in a triangle (Gusmania and Agustyaningrum 2020). It also discusses triangles and is related to geometry (Hadi and Faradillah 2020). Trigonometry is a topic in mathematics that improves various cognitive skills. It deals with the relationship of side lengths and angles of triangles in real world activities as cited by Sarpe in (Maphutha, Maoto, and Kibirige 2022). Trigonometry is very useful for students to develop their knowledge when entering higher education according to their interests, because trigonometry is not only used in Mathematics, but trigonometry can also be used in other branches of science such as physics, chemistry, geography, engineering, and so on (Insani and Kadarisma 2020). Trigonometry is also one of the earliest mathematical topics that connects algebraic reasoning, geometry and graphics. Trigonometry is used throughout the concept of geometry because every geometric shape that has all straight sides can always be broken down into a set of right triangles (Nofita Telung, Oltje T. Sambuaga, and Derel F. Kaunang 2022). So, trigonometry is fundamental knowledge that is important in solving problems in various fields and can be connected to Van Hiele's theory, which focuses on geometry. students face difficulties in solving geometry problems especially in understanding the questions and determining the strategies used (Miatun, Khusna, and Slamet 2021). In

trigonometry material, students learn about trigonometric ratios in right triangles, trigonometric ratios in various quadrants and also graphs of trigonometric functions and their applications in daily life (Novianti and Riajanto 2021).

According to Van Hiele's theory, to measure the level of student thinking there are 5 levels, namely: level 0 (visualization), level 1 (analysis), level 2 (informal deduction), level 3 (deduction), and level 4 (determination/rigor) which quoted by Alex in (Wulandari and Ishartono 2022). Van Hiele's learning level is divided into five levels of students' geometric thinking, namely; level 1 (visualization), level 2 (analysis), level 3 (informal cut), level 4 (cut), level 5 (rigor) (Umami and Asdarina 2024). The students' level of geometric thinking is divided into 5 levels, namely level 0 (visualization), level 1 (analysis), level 2 (informal cut) and level 4 (rigor) (Astuti, Suryadi, and Turmudi 2019). Van Hiele's level of thinking shows the characteristics of students' thinking process in learning trigonometry and their understanding in the context of trigonometry. Thus, students must pass a certain level maturely before going to the next level. However, students do not need to reach level 4. Each level of Van Hiele's thinking has certain criteria causing students to differ in understanding and solving trigonometric problems. Differences between students in organizing and processing information about trigonometric material can be caused by differences in their mathematical reasoning abilities, leading to differences in their thinking levels.

Based on the background, the researcher is interested in explaining the ability of mathematical reasoning in solving trigonometric problems based on Van Hiele's thinking level. This research is aimed at class X high school students. The results of this study are expected to help students broaden their horizons and learn mathematical reasoning abilities based on Van Hiele's level of thinking.

METHODS

This type of research is a qualitative descriptive study that describes observational data regarding mathematical reasoning abilities in solving trigonometric problems based on Van Hiele's level of thinking among high school students. Here is the qualitative research procedure presented in Figure 1.

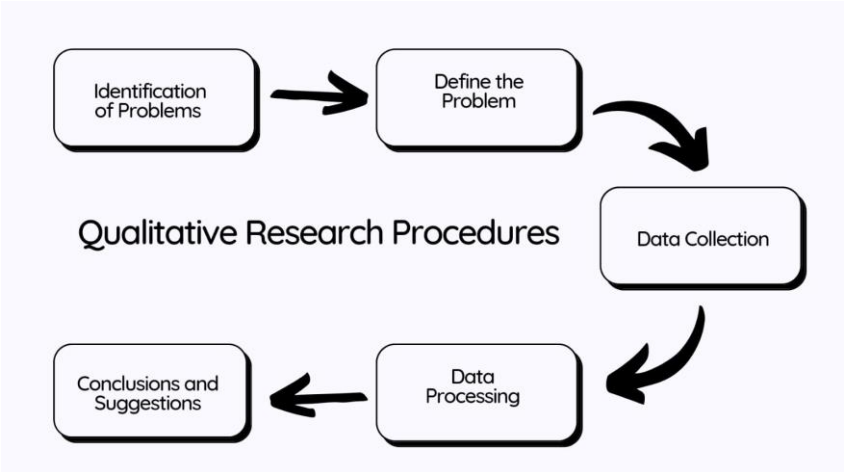

Figure 1. Qualitative Research Procedures

This research was conducted at SMA Negeri 1 Jatinom, with a total of 26 class XA students. Here is the number of students who have a level at Van Hiele's level of thinking presented in Table 1.

Table 1. Number of students who have Van Hiele's level of thinking

No	Van Hiele's Levels of Thinking	Number of Students
1.	Level 0 (Visualization)	1
2.	Level 1 (Analysis)	10
3.	Level 2 (Informal Deduction)	14
4.	Level 3 (Deduction)	1
5.	Level 4 (Rigor)	0

Based on the Table 1, the selected student subjects are a total of 4 students from all levels of Van Hiele's level of thinking, 1 student each. Here are the selected student data, and Van Hiele's thinking level values are presented in Table 2.

Table 2. Name of Subjects with Van Hiele's Levels of Thinking

No	Student	Van Hiele's Levels of Thinking
1.	Student 1 (S1)	Level 0 (Visualization)
2.	Student 2 (S2)	Level 1 (Analysis)
3.	Student 3 (S3)	Level 2 (Informal Deduction)
4.	Student 4 (S4)	Level 3 (Deduction)

In this research, the mathematical indicators used and the aspects studied are quoted by Suendang in (Ramdan and Lessa Roesdiana 2022) presented in Table 3.

Table 3. Indicators of Mathematical Reasoning ability

No.	Indicators of Mathematical Reasoning Ability	Description
1	Presenting a statement	Presenting mathematical statements in writing and drawing
2	Making conjectures	Presenting solution steps
3	Mathematical manipulation	Perform calculations using the solution steps that have been written
4	Draw conclusions	Writing conclusions

In this study, the data collected will be examined using the data triangulation technique to check the validity of the data (Sutama, Sandy, and Fuadi 2017). In this study, the triangulation technique was used to test the validity of the data. The data analysis technique used in this research is an analysis technique that consists of three stages, namely data reduction, data presentation, and drawing conclusions. Before the researcher tests the students, the researcher conducts an expert verification process. A mathematics education lecturer conducts the expert verification process. Until the question instrument is obtained as follows.

Problem number 1

An airplane travels from Airport A to the east for a distance of 12 km and stops at Airport B. Then, the airplane moves again 600 facing east to south for a distance of 15 km and stops at Airport C. After Airport C, the airplane that moved back to Airport D. . Airport D is located between Airports A and C so that Airports B and D are perpendicular to Airports A and C. Determine the distance from Airport B to Airport D!

Problem number 2

Prove if triangle PQR is right-angled at Q. The angle is P = 30^0 and PQ = 18 cm.

a. Length QR = $6\sqrt{3}$ and PR = $12\sqrt{3}$
b. Sin R = $\frac{1}{2}\sqrt{3}$, cos R = $\frac{1}{2}$, tan R = $\sqrt{3}$
c.

Problem number 3

A person whose height is 150 cm is standing near a tree at a distance of $5\sqrt{3}$ m. The angle of elevation of the eye to the top of the tree is 30°. Construct a figure from this situation and find the unknown value in the problem.

RESULTS

Based on the research instrument used. The following is a data analysis of mathematical reasoning abilities in solving Van Hiele problems based on the Van Hiele learning level of high school students.

Data Analysis of Subject S1

The results of S1's answers to questions 1, 2, and 3 are shown below.

Answer to Problem Number 1

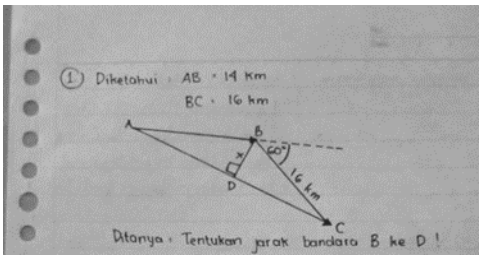

Figure 2. S1's answer to Problem Number 1

Based on the figure in answer number 1, subject S1 can only present a statement. The following interview results reinforce this statement.

P*: "What do you know from problem number 1?"*

S1: "Problem number 1 determines the distance from airport B to airport D."

P: "Then, how do you solve it?"

S1: "First, I write down what is known in the problem, which is AB and BC. Then, I draw it. For the next step, I can't do it."

So, it can be concluded that the S1 subject with level 0 category (visualization) fulfills one indicator of mathematical reasoning ability in question 1. The S1 subject can only fulfill the indicator of mathematical reasoning ability which is presenting mathematical statements in writing and drawing.

Answer to Problem Number 2

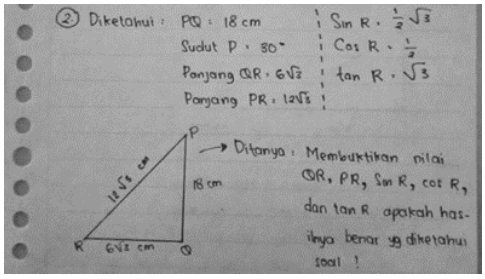

Figure 3. S1's answer to Problem Number 2

Based on the figure in answer number 2, subject S1 can only present a statement. The following interview results reinforce this statement.

P: "What do you know from problem number 2?"

S1: "Proving the statement is true or not."

P: "Then, how do you solve it?"

S1: "What I did first was write down what is known in the problem, namely PQ, angle P, QR, PR, sin R, cos R, and tan R. Then draw it. Then, for the next step, I can't."

So, it can be concluded that the S1 subject with category level 0 (visualization) meets one indicator of mathematical reasoning ability in question number 2. The S1 subject can only meet the indicator of mathematical reasoning ability which is presenting mathematical statements in writing and drawing.

Answer to Problem Number 3

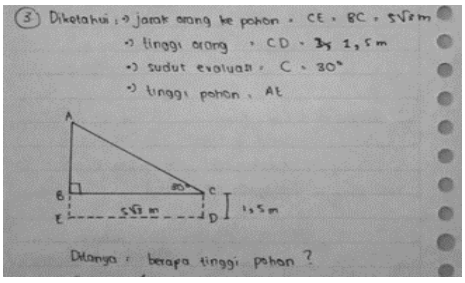

Figure 4. S1's answer to Problem Number 3

Based on the figure in answer number 3, subject S1 can only submit a statement. The following interview results reinforce this statement.

P: "What do you know from question number 3?"
S1: "Drawing and finding the unknown in the question, and the unknown height of the tree"
P: "Then how do you solve it?"
S1: " What I do is first write down what is known in the problem BC, CD, and angle C. Then, I draw it. Then, for the next step, I can't solve it."

So, it can be concluded that subject S1, with category level 0 (visualization), meets one indicator of mathematical reasoning ability in question number 3. However, subject S1 can only meet the indicator of mathematical reasoning ability, which is to deliver mathematical statements in writing. and painting. From questions 1, 2, and 3, it can be concluded that subject S1 has low mathematical reasoning ability.

Data Analysis of Subject S2

The results of S2's answers to questions 1, 2, and 3 are shown below.

Answer to Question Number 1

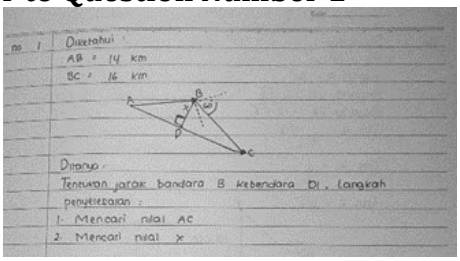

Figure 5. S2's answer to Problem Number 1

Based on the figure in answer number 1, subject S2 can make statements and make assumptions. The following S2 subject interview results reinforce this statement.

P: "What do you know from question number 1?"
S2: "Determining the distance from airport B to airport D."
P: "How do you solve it?"
S2: "I write down what is known in the problem, namely the values of AB and BC, and explain it. Then, I wrote the solution steps for the next step, which is to find the values of AC and x. And then I couldn't answer." And then I couldn't answer."

So, S2 subjects with category level 1 (Analysis) can meet one indicator of mathematical reasoning ability in question number 1. S1 subjects are only able to meet indicators of mathematical reasoning ability, which is to present mathematical statements in writing and drawings. and be able to present the completion steps,

Answer to Problem Number 2

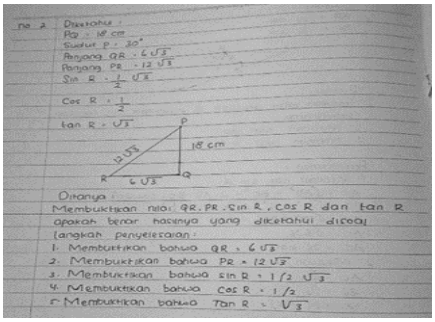

Figure 6. S2's answer to Problem Number 2

Based on the figure in answer number 2, subject S2 can make statements and make assumptions. The following S2 subject interview results reinforce this statement.

P: "What do you know from question number 2?"
S2: "Proving that the statement known in the problem is true or not."
P: "How do you solve it?"
S2: "I write what is known in the problem PQ and angle p. Then draw it. And I also recorded the solution step by proving the value of QR, PR, sin R, cos R and Tan R. Even if I want to answer, I am still confused."

So, subject S2 with category level 1 (Analysis) meets one indicator of mathematical reasoning ability in question number 2. Subject S1 is only able to meet the indicator of mathematical reasoning ability which is to present mathematical statements in writing and drawings and be able to present the steps of improvement.

Answer to Problem Number 3

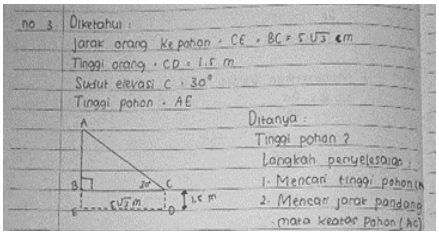

Figure 7. S2's answer to Problem Number 3

Based on the figure in answer number 3, subject S2 can make statements and make assumptions. The following S2 subject interview results reinforce this statement.

P: "What do you know from question number 3?"
S2: "Drawing is the same as finding the unknown in the question, and the unknown height of the tree is then looked for the distance from the eye to the top of the tree."
P: "Then how do you solve it?"
S2: "I write down what is known in the problem i.e. length BC, CD, angle of elevation c. Then draw it. Then write the steps to solve it, which is to find the height of the tree and find the distance from the point to the top of the tree. To answer I am still confused"

So, subject S2, with category level 1 (analysis), fulfills two indicators of mathematical reasoning ability in question number 3. Subject S2 fulfills the indicator of mathematical reasoning ability which is to present mathematical statements in writing and drawings and present the solution steps. From questions 1, 2, and 3, it can be concluded that subject S2 has low mathematical reasoning ability.

Data Analysis of S3 Students

The results of S3's answers to questions 1, 2, and 3 are shown below.

Answer Problem Number 1

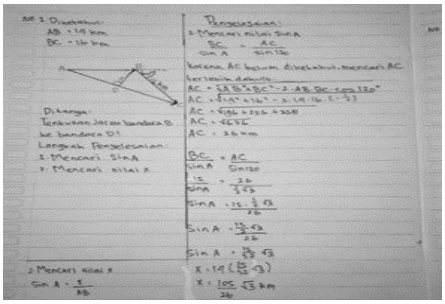

Figure 8. S3's answer to Problem Number 1

Based on the figure in answer number 1, subject S3 can present statements, make conjectures, and perform mathematical manipulations. The results of the following S3 subject interviews reinforce this statement.

P: "What do you know from question number 1?"
S3: "Determining the distance from airport B to airport D."
P: "How do you solve it?"
S3: "I write what is known in the problem which is AB and BC and explain it then for the next step write the solution step which is by finding the value of AC and the value of x. And then I do the calculation so that the value of AC = 26 km and the value of x = 105/26√3 km."

So, S3 subjects with level 2 category (informal cut) can meet the three indicators of mathematical reasoning abilities in question number 1. S3 subjects can meet the indicators of mathematical reasoning abilities that is presenting mathematical statements in writing and images, being able to present. preparation steps, and perform calculations using the solution steps that have been written.

Answer to Problem Number 2

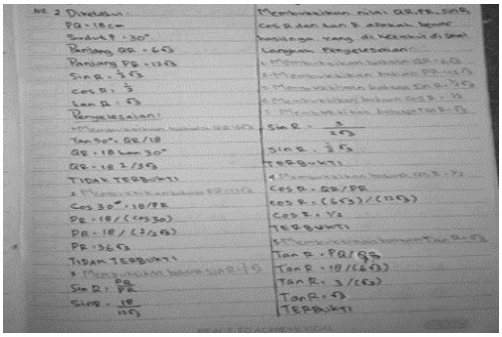

Figure 9. S3's Answer to Problem Number 2

Based on the figure in answer number 2, subject S3 can present statements, make conjectures, and perform mathematical manipulations. The results of the following S3 subject interviews reinforce this statement.

P: "What do you know from question number 2?"
S3: "Proving that the statement known in the problem is not true."
P: "How do you solve it?"
S3: "I write what is known in the problem of length PQ and angle p. Then draw it. And I also wrote the solution steps which is to prove QR, PR, sin R, cos R and tan R. To answer I have, but I don't know if it is correct or not."

So, S3 subjects, with a level 2 category (informal cut), can meet the three indicators of mathematical reasoning ability in question number 2. S3 subjects can meet the indicators of mathematical reasoning ability, which is to present mathematical statements in writing and images, capable. to present the solution steps, and perform calculations using the solution steps that have been written.

Answer to Problem Number 3

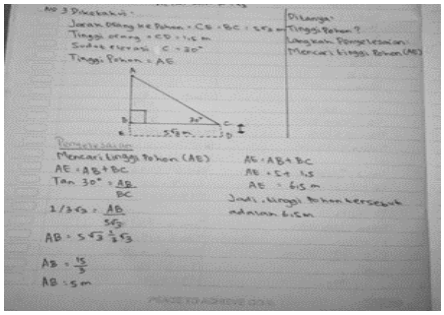

Figure 10. S3's Answer to Problem Number 3

Based on the figure in answer number 3, subject S3 can present statements, make conjectures, perform mathematical manipulations, and draw conclusions. The results of the following S3 subject interviews reinforce this statement.

P: "What do you know from question number 3?"
S3: "Drawing is the same as looking for the unknown in the question, and the unknown height of the tree is then looked for the distance from the eye to the top of the tree."
P: "Then how do you solve it?"
S3: " I write down what is known in the problem which is the length of BC, CD and the elevation angle c. Then draw it. Then write the steps to solve it which is to find the height of the tree. To answer that I have done it, i.e. obtained AE = 6.5 m. And I also wrote the conclusion."

So, S3 subjects with level 2 category (informal cut) are able to meet the four indicators of mathematical reasoning abilities in question number 3. S3 subjects can meet the indicators of mathematical reasoning abilities which are presenting mathematical statements in writing and images, presenting solution steps, perform calculations using the solution steps that have been written, and write conclusions. From questions 1, 2, and 3, it can be concluded that subject S3 has simple mathematical reasoning.

Data Analysis of Subject S4

The results of S4's answers to questions 1, 2, and 3 are shown below.

Answer to Problem Number 1

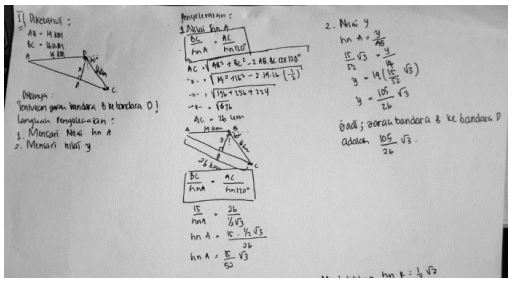

Figure 11. S4's answer to Problem Number 1

Based on the figure in answer number 1, subject S4 can present statements, make conjectures, perform mathematical manipulations, and draw conclusions. The results of the interview with the following S4 subjects reinforce this statement.

P: "What do you know from question number 1?"

S4: "Determine the distance from airport B to airport D."

P: "How do you solve it?"

S4: "I write down what is known in the problem which is AB and BC and explain it, then for the next step I write down the solution step which is to find the value of sin A and the value of y. And then I do the calculation to get the value of sin A = 15/32√3 and the value of y = 105/26√3. Then I also wrote the conclusion that the distance from airport B to airport D is 105/26√3 km."

So, S4 subjects with category level 3 (deduction) can meet the four indicators of mathematical reasoning ability in question number 1. S4 subjects can meet the indicators of mathematical reasoning ability, which is to convey mathematical statements in writing and pictures, present the solution steps, perform calculations using the solution steps that have been written, and write a conclusion.

Answer to Problem Number 2

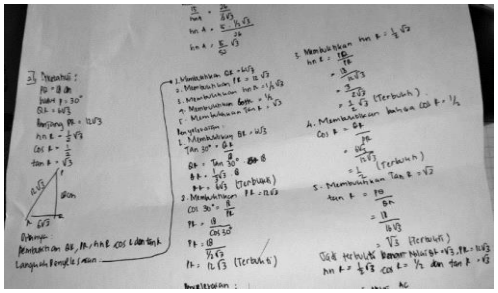

Figure 12. S4's answer to Problem Number 2

Based on the figure in answer number 2, it can be seen that subject S4 can present statements, make guesses, perform mathematical manipulations, and draw conclusions. The results of the interview with the following S4 subjects reinforce this statement.

P: "What do you know from question number 2?"

S6: "Proving that the statement known in the problem is true or not."

P: "How do you solve it?"

S6: "I write what is known in the problem of length PQ and angle p. Then I draw it. And I also wrote down the solution steps which are to prove QR, PR, sin R, cos R and tan R. Then I made calculations that all proved the statement in the problem to be true. Then I also write a conclusion."

So, subject S4, with category level 3 (deduction), meets the four indicators of mathematical reasoning ability in question number 2. Subject S4 can meet the indicator of mathematical reasoning ability, which is to present mathematical statements in writing and pictures, present solution steps, perform calculations using the solution steps that have been written, and write a conclusion.

Answer to Problem Number 3

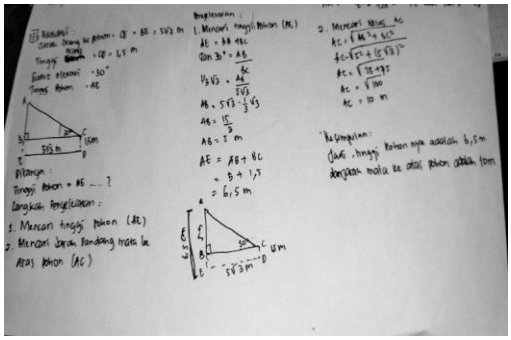

Figure 13. S4's answer to Problem Number 3

Based on the figure in answer number 2, subject S4 can present statements, make conjectures, perform mathematical manipulations, and draw conclusions. The results of the interview with the following S4 subjects reinforce this statement.

P: "What do you know from question number 3?"

S6: "Drawing is the same as finding the unknown in the question, and the unknown height of the tree is then looked for the distance from the eye to the top of the tree."

P: "Then how do you solve it?"

S6: "I write down what is known in the problem which is the length of BC, CD and the elevation angle c. Then draw it. Then write the steps to solve it, which is to find the height of the tree, and the distance of the eye to the tree. To answer, I have done it i.e. obtained AE = 6.5 m and AC = 10 m. And I also wrote the conclusion. And I also wrote the conclusion."

So, S4 subjects with category level 3 (deduction) meet the four indicators of mathematical reasoning abilities in question number 3. S4 subjects can meet the indicators of mathematical reasoning abilities, which is to present mathematical statements in writing and with pictures, able to present. solution steps, perform calculations using the solution steps that have been written, and write a conclusion. From question number 1, 2, and 3, it can be concluded that subject S4 has high mathematical reasoning.

CONCLUSIONS AND SUGGESTIONS

Based on the results of the study described above, it can be concluded that S1 subjects are classified as students who have Van Hiele level 0 thinking (visualization) cannot meet the four indicators of mathematical reasoning ability and can only meet one indicator of mathematical reasoning ability, which is presenting facts. S2 subjects are classified as students who have Van Hiele level 1 thinking (analysis) unable to meet the four indicators of mathematical reasoning ability and can only meet two indicators of mathematical reasoning ability which are presenting statements and making conjectures. S3 subjects are classified as students who have Van Hiele's thinking level 2 (informal cut) unable to meet the four indicators of mathematical reasoning ability and can only meet three indicators of mathematical reasoning which are presenting statements, making conjectures, and mathematical manipulation in form. calculation. S4 subjects are classified as students who have Van Hiele level 3 thinking (deduction) able to meet all four indicators of mathematical reasoning, namely presenting statements, making conjectures, mathematical manipulation in the form of calculations, and conclusions. With this, the lower the mathematical reasoning ability, the lower the level of Van Hiele's thinking. And the higher the mathematical reasoning ability, the higher the level of Van Hiele's thinking.

From the results of the study and conclusions, teachers are expected to be able to prepare mathematical questions that aim to train reasoning abilities and reduce the use of quick methods in learning mathematics. In order to improve students' mathematical reasoning abilities. Future researchers are expected to be able to obtain students who have level 4 (rigor) in Van Hiele's level of thinking and further analyze them into

low, medium or high mathematical abilities.

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