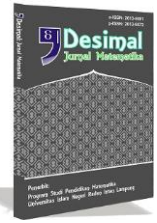




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Optimization of bantuan pangan non tunai (BPNT) distribution using bilevel linear programming in siantar martoba subdistrict pematang siantar

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ABSTRACT

Bantuan Pangan Non Tunai (BPNT) is assistance that is in the second decile; in other words, KPMs who are in the first decile will also get assistance. A limited quota of BPNT recipients will result in people in categories such as the elderly group not getting this assistance. This problem arises because there is a significant increase in the population of the elderly group every year. The research method uses secondary data from data sources, namely the social service office, to develop a bilevel model for the problem of distributing food aid on cash by regularizing the bilevel model so that a linear programming model with a single objective function is obtained. In the regularization stage, the gradient descent method is used to find the optimal value of the penalty parameter. From the calculation results of the regularized model, it is found that the values of the variables $x_1 = 2956$, $x_2 = 583$, and $x_3 = 250$ have a value of $Z = 3804$. This bilevel linear programming model approach provides a strong basis for planning and decision-making related to the distribution of Non-Cash Food Assistance (BPNT) in Siantar Martoba Subdistrict. Therefore, it can be assumed that this bilevel linear programming approach can be used as a guideline for related agencies in allocating resources efficiently.

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INTRODUCTION

Poverty is a complex phenomenon that involves many factors. Poverty can be defined as the inability to fulfill economic, social, and other needs. The causes of poverty can be grouped into two categories: natural factors such as a poor

environment, inadequate knowledge, and natural disasters, and non-natural factors such as failed economic growth, corruption, unstable political conditions, and failure of day-to-day operations (Iskandar, 2012). Poverty can be intergenerational, where if an individual is born into a poor family, the individual

cannot get out of the scope of poverty, thus forming a new poor family (Hadi, Soewarsono, Lan, & Jati, 2019).

In Indonesia, poverty persists as a significant concern, notwithstanding the nation's abundant natural resource potential. Based on data from the Badan Pusat Statistik (BPS) of Indonesia, the poverty rate was recorded at 9.57% in September 2022, indicating a slight increase of 0.03% from March 2022 (Badan Pusat Statistika, 2023). The high poverty rate hampers development and welfare, thus contradicting the goals and aspirations of the country, especially in advancing the welfare of the people as stipulated in the 1945 Constitution.

To address the issue of poverty, the Indonesian government has made various efforts, one of which is through the Bantuan Pangan Non Tunai (BPNT) program. BPNT is a transformation of the Beras Sejahtera (RASTRA) social assistance policy aimed at reducing the financial burden of poor communities when purchasing food items (Nabila, Suharso, & Hartanto, 2021). However, in its implementation, there are still some problems, such as limited quotas of beneficiaries and allocations that have not been optimized. In Siantar Martoba Subdistrict, Pematang Siantar City, BPNT has become an important assistance program for the poor. However, the limited quota of beneficiaries is an obstacle that is often faced, especially for vulnerable groups such as the elderly, who are more in need.

Optimization is a branch of mathematics that optimizes by maximizing or minimizing the objective function by considering the constraints that are taken into account. One commonly used optimization method is linear programming, where the goal is to find the maximum or minimum value of a linear function with respect to a number of linear constraints (Mashayekh, Stadler, Cardoso, & Heleno, 2017). Linear programs require

non-negative values of variables that are maximized or minimized under some linear restrictions, with properties including linearity, proportionality, additivity, divisibility, and certainty, where proportionality implies that a variable's contribution to the objective or restriction is proportional to the level of the variable's value, additivity implies that there is no cross-multiplication between activities, and certainty implies that all model parameters are constants (Lumbantoruan, 2020). A linear programming model has three basic components: decision variables to be set, objectives to be optimized (maximize or minimize), an objective function (Erfianti & Muhajir, 2019), and fundamental concepts of linear programming, including the introduction of the fundamental elements of linear programming problems. To optimize the distribution of Bantuan Pangan Non Tunai (BPNT) in Siantar Martoba Subdistrict, this research will use a bilevel linear programming approach. This study uses a bilevel linear programming approach because ordinary linear programming cannot model the optimization of two levels of interdependent decisions. With two levels of decisions, namely the upper level for the total allocation of subsidy funds and the lower level for the allocation of subsidy funds to specific aid categories, a bilevel approach is needed to ensure effective and sustainable resource allocation in the Bantuan Pangan Non Tunai (BPNT) program in Siantar Martoba Subdistrict. Bilevel linear programming is an optimization method that contains two interrelated levels (Sinha, Soun, & Deb, 2019). These processes are integrated into a two-level structure that makes the upper-level problem sequentially dependent on the optimization problem of the lower-level problem (Abbassi, Chaabani, Said, & Absi, 2020). In this context, bilevel linear programming is a nested optimization problem where the

upper-level feasible region is determined by the solution set of the lower level (Zhang & Li, 2018). A bilevel linear programming system has a hierarchical structure that involves two decisions, the upper and lower levels (Dempe, 2020). In the first stage, the leader will make a decision to allocate the quota between three categories, namely LINJAMSOS (Perlindungan dan Jaminan Sosial), DAYASOS (Pemberdayaan Sosial), and REHSOS (Rehabilitasi Sosial), to maximize the quota of the three categories. However, in the second stage, the decision should also consider optimizing the quota for the REHSOS category.

This research will take examples from previous studies that have successfully applied Bi-level Linear Programming in different contexts. For example, research by Ziliaskopoulos & Papalamprou (2022) regarding the development of subsidy policies for agriculture with the aim of minimizing environmental impacts. In this study, we will analyze the implementation of Bantuan Pangan Non Tunai (BPNT) distribution in Siantar Martoba Subdistrict, Pematang Siantar City, using a bilevel linear programming approach. This approach is an element of novelty introduced in this research. Therefore, this study aims to optimize the distribution of BPNT by considering the limited quota of beneficiaries and maximizing the allocation for the REHSOS (Rehabilitasi Sosial) category, in the hope that it can be done more efficiently and on target.

METHOD

The research took place using secondary data from the Pematang Siantar City Social Service Office, namely data on

the number of Keluarga Penerima Manfaat Bantuan Pangan Non Tunai (BPNT) recipients in the period July–August 2023 in Siantar Martoba Subdistrict, with data on the number of BPNT in Siantar Martoba Subdistrict as many as 3900 families, and data on the balance of subsidies per KPM, which is Rp. 200,000.00, from the LINJAMSOS (Perlindungan dan Jaminan Sosial), DAYASOS (Pemberdayaan Sosial), and REHSOS (Rehabilitasi Sosial) categories.

Table 1. Number of KPM Receiving BPNT July–August 2023

KPM Type	Variable	Quantit y	Proporti on
LINJAMSOS	x_1	2456	0.758
DAYASOS	x_2	576	0.178
REHSOS	x_3	209	0.064

The variables x_1 , x_2 , and x_3 were chosen to represent the LINJAMSOS (Perlindungan dan Jaminan Sosial), DAYASOS (Pemberdayaan Sosial), and REHSOS (Rehabilitasi Sosial) categories. The top-level objective function is to maximize the quota of Bantuan Pangan Non Tunai (BPNT) recipients in Siantar Martoba Subdistrict, while the lower-level objective function is to maximize the REHSOS category by considering the proportion of previous distribution data. The bilevel linear programming model is then modified into an ordinary linear programming model by adding a penalty parameter to the lower-level objective function for regularization. Furthermore, the objective function and constraints are converted into a standard linear program form or canonical form to facilitate optimization with the simplex method. Details can be seen in Figure 1.

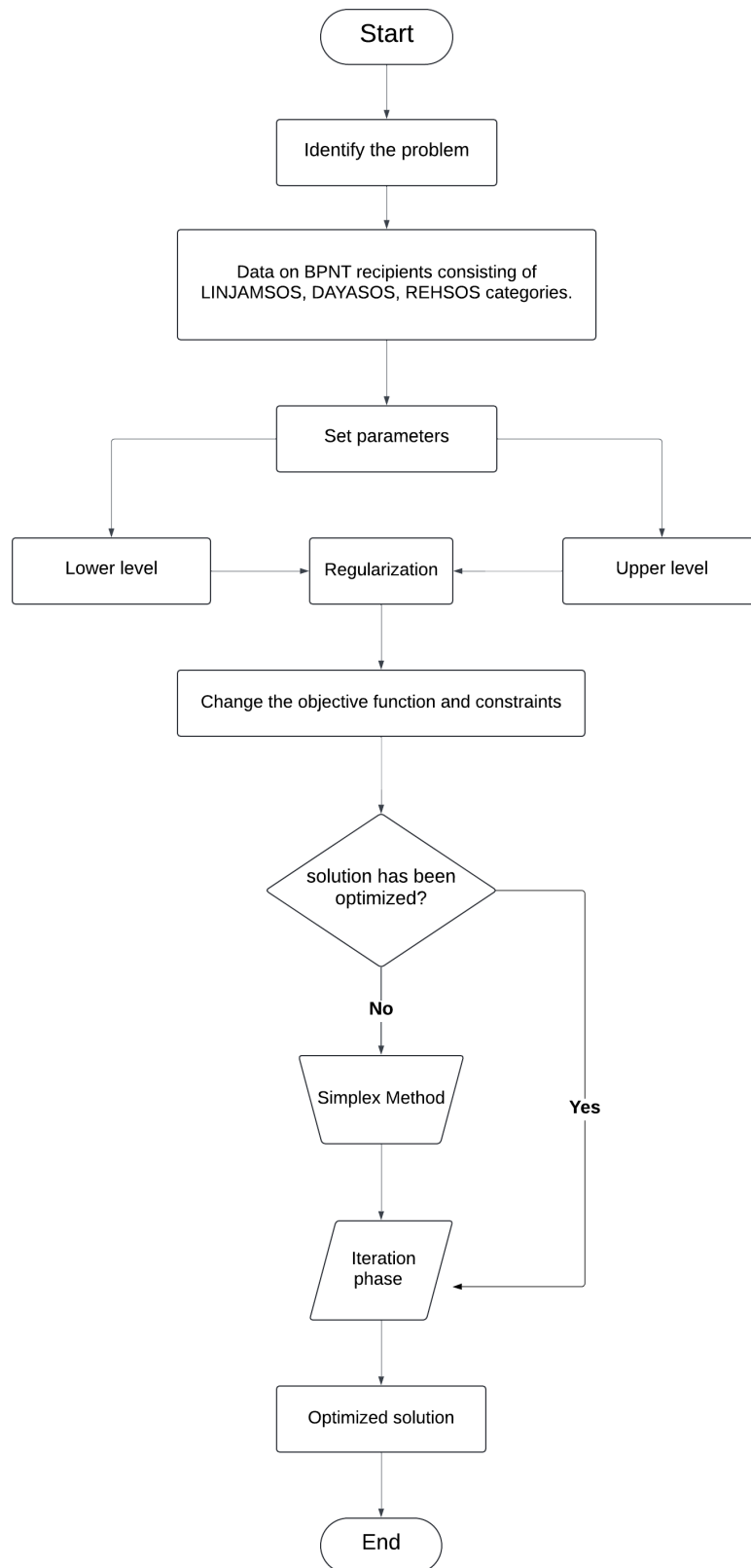


Figure 1. Research flowchart diagram

RESULTS AND DISCUSSION

The quota optimization process in this study consists of three main stages,

namely: first, case modeling to identify the variables and parameters required in the problem formulation; second, model

regularization to transform the Bilevel Linear Programming Model into an ordinary linear programming model by adding a penalty parameter to the objective function; and third, regularization model optimization using the simplex method to find the optimal solution that satisfies all predefined constraints and requirements.

Case Modeling

The Bilevel Linear Programming model in this study develops three main variables, namely LINJAMSOS (Perlindungan dan Jaminan Sosial), DAYASOS (Pemberdayaan Sosial), and REHSOS (Rehabilitasi Sosial), to enable optimal resource allocation and sustainability in the distribution of Bantuan Pangan Non Tunai (BPNT). At the top level, the model maximizes the quota of BPNT recipients in Siantar Martoba Subdistrict, while at the bottom level, the focus is on maximizing the REHSOS category by considering the proportion of previous distribution data. The constraint function in this model involves the number of Keluarga Penerima Manfaat (KPM), the amount of funds disbursed, as well as limitations on the number of quotas for each category of assistance, including budget restrictions to maintain the continuity of the BPNT program. By ensuring the proper allocation of resources, the model aims to avoid wasting resources, ensure program effectiveness, and provide the greatest impact on groups in need. From the above explanation, the determined bilevel linear programming model can be written as follows:

$$\max x_1 + x_2 + x_3$$

$$\max 0.064x_3$$

$$\text{s.t. } x_1 + x_2 + x_3 \leq 3900$$

$$2x_1 + 2x_2 + 2x_3 \leq 7578$$

$$x_1 \leq 2956.2$$

$$x_2 \leq 694.2$$

$$x_3 \leq 249.2$$

$$x_1, x_2, x_3 \geq 0$$

Where:

x_1 : Variable of LINJAMSOS category

x_2 : Variable of DAYASOS category

x_3 : Variable of REHSOS category

Model Regularization

Regularization is a method typically used to solve linear programming optimization problems and involves augmenting the precision of the data with a penalty parameter (Ehrhardt, Gazzola, & Scott, 2023). The purpose of model regularization is to avoid overfitting, where the model overfits the data, by reducing the complexity of the model and keeping the objective function in balance. One method that can be used to find the optimal value of the penalty parameter is the gradient descent method. The gradient descent method is an optimization algorithm used to find the optimal value of an objective function by varying the parameters iteratively according to the direction of the function gradient. This objective function has a constraint function. If the solution is constrained, a penalty parameter is usually required to handle the problem of constraint violation (Shen & Chen, 2023). The penalty parameter is used as a control against overfitting when training the model, and applying regularization to the bilevel model with a penalty parameter can reduce the risk of overfitting. The penalty parameter affects the learning process of the model to create an optimal balance between flexibility and generalization. The gradient descent method is used to find the optimal value of the penalty parameter in this study.

$$\max (x_1 + x_2) + \lambda(x_3)$$

$$\text{s.t. } x_1 + x_2 + x_3 \leq 3900$$

$$2x_1 + 2x_2 + 2x_3 \leq 7578$$

$$x_1 \leq 2956.2$$

$$x_2 \leq 694.2$$

$$x_3 \leq 249.2$$

$$x_1, x_2, x_3 \geq 0$$

The gradient descent iteration process to search for the optimal value of the penalty parameter will have the opportunity to be multiplied in the iteration process, and the search may quickly encounter feasible regions. As a result, an optimal solution of low quality may be obtained. To obtain optimal parameter values, it is important to set appropriate tolerance limits during the optimization process. The tolerance limit serves as a convergence criterion, which determines how small a change in the objective function value or gradient norm is desirable before the process is stopped iterating (Jamhuri & Subiono, 2021). Setting an appropriate tolerance limit ensures that the results are reliable and relevant to the problem at hand. In this study, a threshold value of less than or equal to 0.01 was set ($\epsilon \leq 0.01$). Using gradient descent, we can update the parameter values to get a more optimal penalty parameter. Parameters can be updated using the gradient descent formula. Initialize the starting values $x_1 = -1$, $x_2 = -1$, $x_3 = 1$ with values of $\lambda = 1$ and $\alpha = 0.01$. λ is the penalty parameter, and α is the learning rate. The results of iterations in updating the value of the penalty parameter using the Python programming language. The results of iteration in updating the value of the penalty parameter can be seen in Table 2

Table 2. Results of 1st Iteration to 73rd Iteration

Iterati on	lamda (λ)	Function Result	Relative Changes (ϵ)
1	1	-0.9360	-
2	0.9994	-0.9674	0.0335059
3	0.9987	-0.9987	0.0324178
4	0.9981	-1.0301	0.0313982

Iterati on	lamda (λ)	Function Result	Relative Changes (ϵ)
5	0.9975	-1.0614	0.0304407
6	0.9969	-1.0928	0.0295399
7	0.9963	-1.1241	0.0286908
8	0.9957	-1.1555	0.0278891
9	0.9951	-1.1868	0.0271310
10	0.9945	-1.2182	0.0264130
11	0.9939	-1.2495	0.0257320
12	0.9933	-1.2809	0.0250852
13	0.9928	-1.3122	0.0244702
14	0.9922	-1.3436	0.0238845
15	0.9917	-1.3749	0.0233262
16	0.9911	-1.4062	0.0227934
17	0.9906	-1.4376	0.0222844
18	0.9900	-1.4689	0.0217976
19	0.9895	-1.5003	0.0213317
20	0.9890	-1.5316	0.0208852
21	0.9885	-1.5629	0.0204570
22	0.9880	-1.5943	0.0200460
23	0.9875	-1.6256	0.0196512
24	0.9870	-1.6569	0.0192717
25	0.9865	-1.6882	0.0189065
26	0.9860	-1.7196	0.0185549
27	0.9856	-1.7509	0.0182161
28	0.9851	-1.7822	0.0178895
29	0.9847	-1.8135	0.0175744
30	0.9842	-1.8448	0.0172702
31	0.9838	-1.8762	0.0169763
32	0.9833	-1.9075	0.0166923
33	0.9829	-1.9388	0.0164176
34	0.9825	-1.9701	0.0161519
35	0.9821	-2.0014	0.0158945
36	0.9817	-2.0327	0.0156453
37	0.9812	-2.0641	0.0154038
38	0.9809	-2.0954	0.0151696
39	0.9805	-2.1267	0.0149424
40	0.9801	-2.1580	0.0147219
41	0.9797	-2.1893	0.0145078
42	0.9793	-2.2206	0.0142999
43	0.9790	-2.2519	0.0140978
44	0.9786	-2.2832	0.0139014
45	0.9783	-2.3145	0.0137104
46	0.9779	-2.3458	0.0135246
47	0.9776	-2.3771	0.0133437
48	0.9773	-2.4084	0.0131676
49	0.9770	-2.4397	0.0129961
50	0.9766	-2.4710	0.0128290
51	0.9763	-2.5023	0.0126662
52	0.9760	-2.5336	0.0125074
53	0.9757	-2.5649	0.0123526
54	0.9755	-2.5962	0.0122016
55	0.9752	-2.6275	0.0120542
56	0.9749	-2.6588	0.0119103
57	0.9746	-2.6901	0.0117699
58	0.9744	-2.7214	0.0116327

Iteration	lamda (λ)	Function Result	Relative Changes (ϵ)
59	0.9741	-2.7527	0.0114987
60	0.9739	-2.7840	0.0113677
61	0.9736	-2.8153	0.0112397
62	0.9734	-2.8465	0.0111146
63	0.9732	-2.8778	0.0109922
64	0.9730	-2.9091	0.0108724
65	0.9728	-2.9404	0.0107553
66	0.9726	-2.9717	0.0106407
67	0.9724	-3.0030	0.0105284
68	0.9722	-3.0343	0.0104186
69	0.9720	-3.0656	0.0103110
70	0.9718	-3.0968	0.0102056
71	0.9716	-3.1281	0.0101023
72	0.9715	-3.1594	0.0100012
73	0.9713	-3.1907	0.0099020

In the 73rd iteration, the relative change value of the objective function reaches 0.0099020, which is already below the threshold value set at 0.01. The relative change in the gradient descent method can be used to show changes in the optimal value of the objective function. The relative change is used to measure the convergence of the gradient descent algorithm, whether the parameters have reached the optimum or not (Wungguli, Silalahi, & Guritman, 2015). The convergence criterion has been met, and the iteration can be stopped. The value of the penalty parameter is 0.9713. Furthermore, the penalty parameter value that has been found is substituted into the regularization model. The regularized model with a penalty parameter of 0.9713 can be written as follows::

$$\max x_1 + x_2 + 1.062x_3$$

$$\text{s.t. } x_1 + x_2 + x_3 \leq 3900$$

$$2x_1 + 2x_2 + 2x_3 \leq 7578$$

$$x_1 \leq 2956.2$$

$$x_2 \leq 694.2$$

$$x_3 \leq 249.2$$

$$x_1, x_2, x_3 \geq 0$$

Regularization Model Optimization

The application of the model to the case study uses the simplex method in solving the optimization of the previously regularized model to obtain optimal results. In this context, the variable Z is the objective function of the optimization.

Objective function:

$$\text{Max } Z = x_1 + x_2 + 1.062x_3$$

Constraint functions:

$$x_1 + x_2 + x_3 \leq 3900$$

$$2x_1 + 2x_2 + 2x_3 \leq 7578$$

$$x_1 \leq 2956.2$$

$$x_2 \leq 694.2$$

$$x_3 \leq 249.2$$

Transforming functions into canonical form,

$$Z - x_1 - x_2 - 1.062x_3 - 0S_1 - 0S_2 - 0S_3 - 0S_4 - 0S_5$$

$$x_1 + x_2 + x_3 + 1S_1 + 0S_2 + 0S_3 + 0S_4 = 3900$$

$$2x_1 + 2x_2 + 2x_3 + 0S_1 + 1S_2 + 0S_3 + 0S_4 = 7578$$

$$x_1 + 0x_2 + 0x_3 + 0S_1 + 0S_2 + 1S_3 + 0S_4 + 0S_5 = 2456$$

$$0x_1 + x_2 + 0x_3 + 0S_1 + 0S_2 + 0S_3 + 1S_4 + 0S_5 = 576$$

$$0x_1 + 0x_2 + x_3 + 0S_1 + 0S_2 + 0S_3 + 0S_4 + 1S_5 = 209$$

By using the simplex method, optimal results are obtained that illustrate the correct quota allocation for each aid category, as shown in table 3 below.

Table 3. Simplex table of iteration results

V D	Z	x_1	x_2	S_1	S_2	S_3	S_4	S_5	NK
S_1	0	0	0	0	1	-	0	0	111
						0.5			
x_2	0	0	1	0	0	0.5	-	0	583.2
							1		
x_1	0	1	0	0	0	0	1	0	2956.2
S_4	0	0	0	0	0	-	1	1	111
						0.5			
x_3	0	0	0	1	0	0	0	0	249.6
Z	1	0	0	0	0	0.5	0	0	0.062

In the table above, the base variable (VD) is the variable whose value is equal to the right-hand side of the equation. The right value (NK) is the value on the right side of the equation, which is the value after the equals sign (Ariyanti & Azizah, 2019). The result found is the value of $x_1 = 2956.2 \approx 2956$, $x_2 = 583.2 \approx 583$, and $x_3 = 249.6 \approx 250$. After finding the values of x_1 , x_2 , and x_3 , the next step is to replace the values of x_1 , x_2 , and x_3 into the objective function, which is $\max Z = x_1 + x_2 + 1.062x_3$ to calculate the maximum quota of the three categories. Value of $Z = 2956.2 + 583.2 + 1.0659 (249.6) = 3804.475 \approx 3804$ families in Siantar Martoba Subdistrict, Pematang Siantar City. Quota x_1 is the LINJAMSOS (Perlindungan dan Jaminan Sosial) category with as many as 2956 families in Siantar Martoba Subdistrict, Pematang Siantar City; x_2 is the DAYASOS (Pemberdayaan Sosial) category with as many as 583 families in Siantar Martoba Subdistrict, Pematang Siantar City; and x_3 is the REHSOS (Rehabilitasi Sosial) category with as many as 250 families in Siantar Martoba Subdistrict, Pematang Siantar City.

In a previous study (Ziliaskopoulos & Papalamprou, 2022), the approach used was to use a bilevel linear programming model to combine the objective functions into a single function by using the complementary excess slack condition (or Kuhn-Tucker condition). On the other hand, this study adopts the bilevel linear programming approach by regularizing

the objective function to make it a single function. The regulation process is performed by adding a penalty parameter at the lower level, which is then iterated using the gradient descent method to find the optimal value of the penalty parameter. After the value of the penalty parameter is found, it is then substituted into the regularization model and optimized using the simplex method. The findings of the previous study emphasized the development of a basic bilevel linear programming model to minimize the environmental impact of the agricultural sector and its relationship with government subsidy policies. On the other hand, this research makes a new contribution to designing Bantuan Pangan Non Tunai (BPNT) distribution policies by finding the optimal quota for BPNT distribution in Siantar Martoba Subdistrict, Pematang Siantar City..

CONCLUSIONS AND SUGGESTIONS

The conclusion obtained from the results of research and discussion is that this bilevel linear programming approach presents a strong basis for planning and decision-making related to the distribution of BPNT in Siantar Martoba Subdistrict, Pematang Siantar City, because it is able to find the optimal quota for BPNT distribution in Siantar Martoba Subdistrict, Pematang Siantar City. The analysis is based on the objective function in the bilevel model, which prioritizes the optimal quota of the LINJAMSOS (Perlindungan dan Jaminan Sosial), DAYASOS (Pemberdayaan Sosial), and REHSOS (Rehabilitasi Sosial) categories, giving priority to the lower-level social rehabilitation to be maximized. The results found are that quota x_1 is the LINJAMSOS category with as many as 2956 families in Siantar Martoba Subdistrict, Pematang Siantar City; quota x_2 is the DAYASOS category with as many as 583 families in Siantar Martoba Subdistrict, Pematang Siantar City; and

quota x_3 is the REHSOS category with as many as 250 families in Siantar Martoba Subdistrict, Pematang Siantar City, with a maximum quota of 3804 families in Siantar Martoba Subdistrict, Pematang Siantar City.

Although we have obtained optimal quota results with the approach that has been taken, there is a weakness in this research: the value of the penalty parameter is not investigated in depth to determine whether the parameter value is the global or local optimal value. The global optimum value is considered better than the local optimum value because it reflects the overall best possible solution in the context of the problem at hand. The local optimal value is only the best solution that can be found within a certain area of the solution space, while the global optimal value includes the best solution in the entire possible solution space. Therefore, future research is recommended to conduct further searches to find the global optimal value of the penalty parameter.

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