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Ideal distribution route: An optimization approximation by using random search method

Abdu Afin Arsy, Faris Alaudin Shalih, Fulkan Kafilah Al Husein, Achmad Abdurrazzaq*

The Republic of Indonesia Defense University, Indonesia

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*Correspondence: E-mail:
razzaq.ganesha@gmail.com

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ABSTRACT

Distribution is the process of transferring goods from producers to consumers. In this process, both producers and consumers always expect a more efficient distribution system. One way to create an efficient distribution channel is by determining the ideal point for vital objects. By determining the ideal point, an optimal solution can be obtained in minimizing costs and improving the efficiency of the distribution system. This research discusses the determination of the ideal point using numerical optimization methods. Analytical and numerical approaches are used through the Modified Random Search method to formulate and analyze a mathematical model that can provide an optimal solution to the distribution problem. The proposed algorithm will be implemented to solve the energy distribution problem in the real environment. The aim is to test the effectiveness and accuracy of the proposed algorithm based on the solutions obtained. Based on the experimental results, distribution problems in general and energy distribution in particular are addressed better, and the distribution process is more efficient.

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INTRODUCTION

Distribution is a process of transferring goods from producers to consumers (Bumblauskas et al., 2020). Distribution is a crucial factor in running various businesses. A company's decision in determining the distribution channel can determine the profit for the company itself if done correctly (Takata, 2019). The distribution channel is the route that must be passed through the flow of goods from the producer to the distributor or intermediary, or from the wholesaler to the consumer (Ayu, 2021; Hidayah, 2021). In distributing goods, selecting the right distribution channel becomes important to minimize costs and maximize efficiency. One of the factors that affects the selection of the distribution channel is the distance between vital objects and all objects connected to the vital objects (Pratama, 2019). The closer the traveled distance, the more optimal results can be achieved. This problem is important to solve because it can affect the security and sustainability of the system. For example, in the manufacturing industry, optimizing the location of critical machines or components can increase efficiency and productivity while reducing production downtime and machine maintenance costs (Xue et al., 2020). This will result in significant benefits to the company. In addition, in the field of transportation, optimizing the location of roads or parking lots can reduce traffic congestion and travel time, as well as improve driver safety (Rao et al., 2023). However, on the other hand, if the layout is not optimal, it can cause a very detrimental effect on important objects or components. For example, in building design, suboptimal location of sprinkler systems or fire extinguishers can result in delays in fire control and increase the risk of fire escalation (Yao et al., 2019). Likewise in the oil and gas industry, suboptimal drilling locations can cause leaks and serious environmental hazards (Lu et al., 2020). To achieve effective and efficient conditions, optimization methods need to be performed and models need to be evaluated so that measures of effectiveness and efficiency can be measured.

The solution to the problem above is to optimize the distance of the distribution system (Chandra & Setiawan, 2018), (Ridha Permana et al., 2020). The types will be discussed first as well as their advantages and disadvantages. Optimization is the act of achieving the best result in a given situation by minimizing the required effort or maximizing the expected result (Moehle et al., 2019). Optimization methods can be approached from an analytical or numerical perspective (Hanafi et al., 2010). In terms of analysis, optimization is carried out using differential and integral methods, while in the numerical part, optimization is carried out using more complex and difficult methods such as the Random Search Optimization (RSO) method. This method is used to find the best solution to an optimization problem by generating a large number of random solutions from the entire possible search space, then selecting the best solution from the solution set. The RSO method is suitable for problems that have a very wide and complex search space for solutions (Battiti & Brunato, n.d.). This method does not require prior information of the problem to find the solution, and each test iteration uses a random solution so that the method can avoid local traps and give the possibility to find the best solution in the entire search space. The RSO method has been used in various optimization problems and is increasingly used in the field of machine learning and optimization models. In a study conducted by Bergstra and Bengio (2012), the RSO technique succeeded in producing better solutions than more sophisticated optimization techniques such as grid search or sequential model-based optimization. This proves that the RSO method can be used to improve the hyperparameter optimization process in machine learning models. Another advantage of this method is its ability to avoid local traps and provide the possibility to find the best solution in the entire search space. However, this method also has disadvantages because it is less efficient in finding solutions in a narrower search space and has a high time complexity.

Optimization methods have a long history in the development of mathematics and computer science (Martins, 2021). The Golden Section Method was first discovered by the ancient Greek mathematician, Euclid, in the 3rd century BC (Nugraha, 2019). This method was used to solve the problem of extremal functions in certain intervals and is still used today. Meanwhile, the quadratic interpolation method and Newton's method were developed in the 17th century by two famous scientists, John Wallis and Sir Isaac Newton (Rasheed, 2021; Rebentrost, 2021). The quadratic interpolation method uses polynomial interpolation to approximate the function, while the Newton method uses derivatives to approximate extreme points (Rasheed, 2021). The Hooke-Jeeves method, which was first discovered in 1961 by British scientists, Robert Hooke and T.A. Jeeves, uses trial-and-error approximation to find extreme points in the search space (Xiong, 2022). Meanwhile, steepest ascent/descent methods and Random Search methods were developed in the 1950s and 1960s by scientists focusing on optimization problems, including George Dantzig and Nelder-Mead. The steepest ascent/descent method follows a stepwise gradient function, while the Random Search method uses a random approach to find the best solution to the optimization problem (García-Nava, 2022; Li, 2020). Then, in 1996, British scientists David Wolpert and William Macready developed a new optimization method known as the No Free Lunch Theorem (Adam, 2019). This theorem states that there is no single optimization method that can optimize all optimization problems. Scientists must choose a method that fits the problem they want to solve.

Each optimization method has its advantages and disadvantages. For example, linear optimization methods can produce solutions quickly and accurately but are limited to problems with linear objective functions and linear constraints (Akbari, 2019). Meanwhile, the non-linear optimization method is wider in its application because it can handle non-linear objective functions and non-linear constraints but requires longer calculation time and can get stuck in the local solution. The RSO method has the advantage of avoiding traps locally and finding the best solution across the search space (Yang, 2020). However, since this method relies on random solution generation, it sometimes requires many iterations to find a good solution. Additionally, there is a risk that some search points will not be explored at all in a random search, which may result in sub-optimal solutions. Therefore, it is necessary to carefully set the parameters in the RSO process to ensure that the search finds an effective and efficient solution.

In this article, an optimization method to determine the flow energy distribution will be proposed by implementing algorithm (in this case, RSO) and analyzing (in this case, Extreme Point Study). Furthermore, Chapter 2 will discuss about the proposed algorithm (Modified Random Search). Afterwards, the experimental results will be displayed in Chapter 3 in case of algorithms implementation in real environment. Finally, chapter 4 will explain the conclusion of this research.

METHOD

A Modified Random Search program will be created to evaluate the points on the objective function in a rectangular area to find the smallest point and then divide the area one-fourth smaller than the initial area that has been determined. It runs continuously until the error value converges towards the predetermined value. However, before making the program, the characteristics of the objective function have been studied first so that the value of the convergence error can always be reached and certainly no damage can occur in the program. In addition, the Modified Random Search program is also flexible which means the user can specify their area to evaluate the number of points to evaluate, the desired error value, and the partition to use. Certainly, by making the program flexible, each user with different tool capabilities can still run this program on each computer and get effective and efficient results. This is expected to improve the performance of the energy distribution system and in general topics later. Next, the working algorithm of the Modified Random Search program that has been developed is presented as follows:

First. Initialize the search region by specifying the four outermost points of the expected kernel so that the search region is formed in the shape of a square.

Second. Determine the region with the smallest total distance based on the search region that has been set. therefore, the search region can be reduced according to the smallest distance sum.

Third. This procedure is carried out until the difference between the smallest distances produced is equal to the smaller than α.

Fourth. After obtaining the smallest sum of distances that meet this requirement, the optimal point has been obtained.

In the process of determining the optimal point, careful and systematic steps are essential to ensure accurate and reliable results. Random search belongs to the optimization field of Stochastics and Global Optimization. Random search is a direct search method because it doesn't require an instance to search for a continuous domain. This basic approach is related to techniques that provide small improvement such as Directed Random Search and Adaptive Random Search (Reza Ahmadzadeh, 2023). This function implements algorithm minimization, based on random searches repeated. At each iteration, the function scrambles the vector in the search area and finds the one that minimizes the target function. Then, a smaller search area is defined around this minimizer. The process repeats, shrinking the search region to convergence. The algorithm formulated above has been implemented in MATLAB code and saved as an M-file. The working details of the code is presented in the flowchart on Figure 1.

The difference between the proposed method (Modified Random Search) and other random search methods is due to the limitation of the observed area is limited and closed. The density of points can be selected individually to suit the computer strength of each user. The method does not require derivatives in the process of finding the optimal point. It can also converge to the global optimal point. Even if the function has multiple local maxima, the algorithm is able in determining the optimal point in the objective function stated above. The assumption used in determining the optimal point is that each point has no weight, meaning that each point is considered equally important. In addition, in the process of determining the distance between points, the direction is not taken into account, so the distance calculated is only based on the Euclidean distance or the straight distance between the points. Although there are several assumptions in determining the optimal point, systematic and accurate steps are still required to ensure optimal and reliable results. The objective function used is the sum of the shortest distances from each point to the point as:

$$\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}) = \sum_{i=1}^{n} \sqrt{(x - x_i)^2 + \left(y - y_i\right)^2} \qquad (1)$$

where $\boldsymbol{x}$ and $\boldsymbol{y}$ denotes the location of objective point and $\boldsymbol{x_i}$ and $\boldsymbol{y_i}$ is the location of the considered point.

RESULTS AND DISCUSSION

Before starting to define the objective function, the analysis will be carried out on the characteristics of the intended objective function. Analysis is performed by taking derivatives with respect to the x-axis and y-axis. This analysis is done to show that the mathematical analysis approach in this problem is not feasible and the numerical approach is more viable.

Mathematical Analysis Approach

Distance is a numerical measurement that indicates how far apart the position of an object is from another object. In everyday terms, distance can refer to the physical length between two positions or an estimate based on certain criteria (e.g., the distance between Building A and Building B). Distance is a scalar quantity expressed as a value or number without any direction (Gramedia.com, accessed July 2023). According to Abdur (2017), distance is divided into several types based on the coordinates of points in space, including Euclidean Distance.

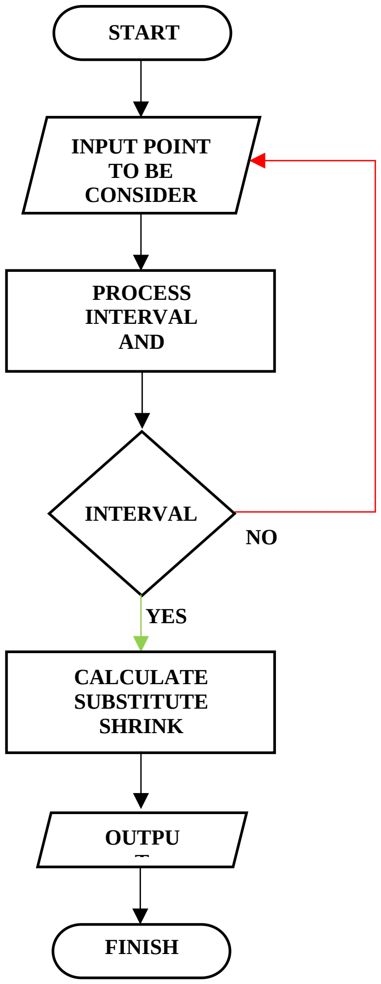

Figure 1. Flowchart of the proposed method

The distance between two points in Euclidean space of n dimensions is calculated using the Pythagorean theorem. On Manhattan Distance, the distance between two points in space is calculated by summing the horizontal and vertical displacements required to reach the target point. On Chebyshev Distance, the distance between two points in space is calculated by measuring the maximum displacement between the coordinates of those points. On Minkowski Distance, the distance between two points in space is a generalization of Euclidean distance and Manhattan distance.

In the context of energy distribution, the author chooses Euclidean distance for the n dimensional mapping model because of its advantages, including easy calculation of distances, accurate geometric representation, and consistency with intuition about distance in everyday life, where the distance between two points is determined based on the straight-line length connecting them.

Function to calculate the sum of distances in n dimension:

$$f(x_a, x_b, \dots, x_n) = \sum_{i=1}^{n} \sqrt{\left(x_a - x_{i_a}\right)^2 + \left(x_b - x_{i_b}\right)^2 + \cdots + \left(x_n - x_{i_n}\right)^2} \qquad (2)$$

The Euclidean formula can be used generally in more than 2 dimensions (Kiki, 2018). However, in the context of optimizing distribution, the Euclidean formula will be applied to 2-dimensional fields. The shortest distance will be sought using the Euclidean formula in 2 dimensions so as to minimize the distance, which means minimizing costs and optimizing work (Harahap & Khairina, 2017). For that, in this article it's used 2-dimensional function such as:

$$\boldsymbol{f(x,y)=\sum_{i=1}^{n}\sqrt{(x-x_i)^2+(y-y_i)^2}}\;;\quad \{(\mathbf{x},\mathbf{y})|\mathbf{x},\mathbf{y}\in\mathbb{R}\} \tag{3}$$

The Euclidean distance function in 2 dimensions is a function that measures the distance between two points in a two-dimensional space using the Euclidean method. The characteristics of the Euclidean distance function in 2 dimensions can be examined in several aspects such as domain, range, convexity, and uniqueness of its solutions.

For the domain of the function f(x, y) mentioned above, its domain is determined as the set of all pairs of points that are members of real numbers in a two-dimensional space. As for its range, f(x, y) yields non-negative values, which is consistent with its implementation, namely the distance between two points. Regarding its convexity, the Euclidean distance function in 2 dimensions is considered a convex function (Dattorro, 2009). A function is said to be convex if the line connecting two points on the graph of the function lies above the graph of the function. In this case, the line connecting two points on the graph of the Euclidean distance function in 2 dimensions always lies above or on the graph of the function. Next, the uniqueness of derivative solutions will be analyzed. This analysis is important because the existence of derivative values can serve as the basis for optimizing a function as well. In this case, first, the function is differentiated to examine the existing extreme points in the function. The derivative solution point can be searched by searching the extreme point analysis. According to Purcell (1995), the extreme point occurs in 3 situations:

i) The singular point, when $x = x_i$ and $y = y_i$
ii) Endpoint hose. In this case, there is no endpoint hose because the hose stretched from $(-\infty, \infty)$
iii) The stationary point, a point that satisfies $\frac{\partial F}{\partial x} = 0$ and $\frac{\partial F}{\partial y} = 0$

So that, the derivative of the function above shows as below:

$$\boldsymbol{\frac{\partial F}{\partial x}=\sum_{i=1}^{n}\frac{x-x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}}}$$

$$\boldsymbol{0=\sum_{i=1}^{n}\frac{x-x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}}}$$

$$\boldsymbol{0=\sum_{i=1}^{n}\frac{x}{\sqrt{(x-x_i)^2+(y-y_i)^2}}-\sum_{i=1}^{n}\frac{x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}}}$$

$$\boldsymbol{x\left(\sum_{i=1}^{n}\frac{1}{\sqrt{(x-x_i)^2+(y-y_i)^2}}\right)=\sum_{i=1}^{n}\frac{x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}}}$$

$$\bar{x} = x = \frac{\sum_{i=1}^{n} \frac{x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}}}{\sum_{i=1}^{n} \frac{1}{\sqrt{(x-x_i)^2+(y-y_i)^2}}} \quad (4)$$

in the same way,

$$\bar{y} = y = \frac{\sum_{i=1}^{n} \frac{y_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}}}{\sum_{i=1}^{n} \frac{1}{\sqrt{(x-x_i)^2+(y-y_i)^2}}} \quad (5)$$

Solution (4) and (5) is still implicit and can be written in pairs (x, y) that fulfill:

$$\sum_{i=1}^{n} \frac{(x - x_i) + (y - y_i)}{\sqrt{(x - x_i)^2 + (y - y_i)^2}} = 0, x \neq x_i, y \neq y_i \quad (6)$$

The equation shows that the solution is from the extreme points are infinite, so that the solution is also not unique. Next, the author will implement modified random search algorithm. Generally, this method will work by randomly trying points in an environment with an algorithm that can automatically look for approaches to the smallest value (global minimum).

Algorithm Implementation

As an implementation, we will take a case study on finding the optimal point for energy distribution in the Sentul Campus of the Republic of Indonesia Defense University. The buildings will be represented in the form of x and y coordinates in Figure 2.

n	X	Y
1	3	16
2	33	58
3	36	89
4	47	25
5	56	8
6	88	38
7	34	−21
8	16	−32
9	−33	−8
10	−3	−32
11	-51	32
12	-21	72

Figure 2. UNHAN 2D map image with Cartesian Coordinates

Next, in using the Modified Random Search Method to find the optimal point, the objective function will be defined in equation (7) based on the mentioned data above.

$$f(x,y) = \sqrt{(x-3)^2+(y-16)^2} + \sqrt{(x-33)^2+(y-58)^2} + \sqrt{(x-36)^2+(y-89)^2} + \sqrt{(x-47)^2+(y-25)^2} + \sqrt{(x-56)^2+(y-8)^2} + \sqrt{(x-88)^2+(y-38)^2} + \sqrt{(x-(-21))^2+(y-72)^2} + \sqrt{(x-(-51))^2+(y-32)^2} + \sqrt{(x-(-33))^2+(y-(-8))^2} + \sqrt{(x-(-3))^2+(y-(-32))^2} + \sqrt{(x-34)^2+(y-(-21))^2} + \sqrt{(x-16)^2+(y-(-32))^2} \quad (7)$$

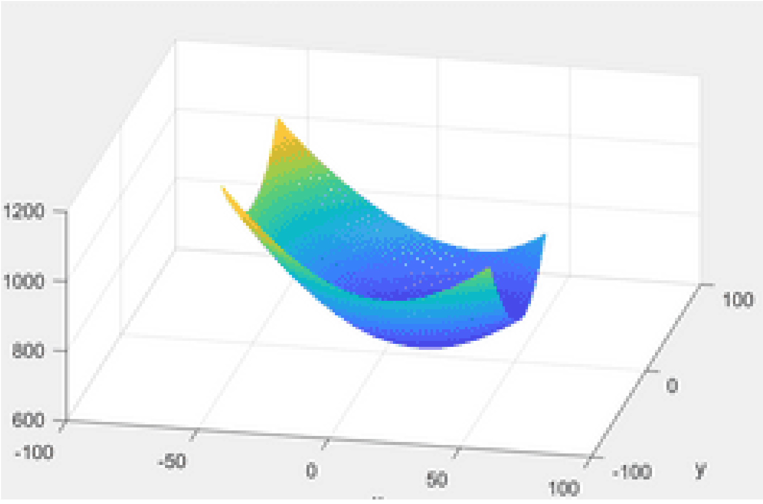

Figure 3. Mesh grid plot of the objective function

After obtaining the objective function, the singular point with the smallest value of the objective function is first checked. Then, the value is stored for subsequent comparison with the value generated from the modified random search algorithm to obtain the smallest value. The singular point that yields the smallest value in this process is (3.16), where the total value is 655.2127.

The modified random search algorithm will commence the search for the smallest value. To illustrate the process, the mesh grid of the objective function is displayed. In the first iteration, the function is randomly evaluated within the bounds defined by the outermost points considered above.

Next, the algorithm will divide the 4 areas and in each of the 4 partitioned parts, it will randomly select a minimum value to compare with each other. In the first iteration, 656.7781 is the obtained value. Then, the area bounded by the outermost x and y points is changed by the algorithm to one of the 4 partitions. Next, second iteration will be carried out in the same way as the first iteration. This second iteration is done to search for more detailed values and compare them with the results of the first iteration. From the second iteration, value 633.49 is obtained. To consider whether the last iteration is enough or another iteration is needed, the difference of values gained from the first and second iteration should be calculated. The author determines that 0.05 is the maximum difference value to accept the coordinates gained from the last iteration as the result. Because the difference between both values is more than 0.05, third iteration should be performed. From the third iteration, value 633.4471 is gained.

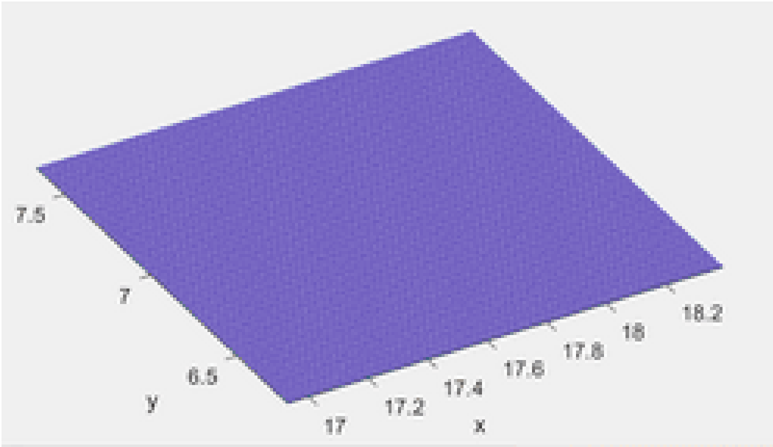

Figure 4. Range of solution points as an evaluation of the algorithm's goodness

As the difference between the values obtained from the second and third iterations is below 0.05, which was set as the maximum difference value from the beginning, it is clear that the algorithm has found the lowest point value. Therefore, the algorithm finishes working and determines the coordinates with the lowest point value to be the ideal point. The ideal point is (18,5.8) with the value of 633.4471, which is lower than the lowest singular point value of 655.2127. On a 2D map, the ideal final point is shown at Figure 5.

Figure 5. Ideal point on the 2D map

After identifying the ideal point, a direct field review is conducted to determine whether the intended location is truly feasible for installing energy installations. It turns out that the spot is not possible to be built, as the intended location covers several access roads and is be demolished. Because of that issue, the ideal final point is irrelevant to reality. Therefore, another point is determined that is relevant to reality, called the relevant point. Through the image below, two relevant points will be shown, which have values closest to the ideal point located near the infrastructure that can't.

Figure 6. Solution point in the actual map

Relevant Point 1 is defined at coordinate (15,40) and Relevant point 2 is defined at coordinate (28,30) on the map. Relevant point values are gained by calculating the result of putting both relevant point coordinates into equation 2. The value of Relevant Point 1 is 728.0995 and the value of Relevant Point 2 is 683.8911.

To find the best relevant points, we need to analyze the error between the ideal point value and each relevant point values. The relevant point which has the smallest error percentage will be chosen. The following formula will be used to calculate the percentage difference between the relevant point and the ideal point.

$$\frac{\textbf{Relevant Point Value} - \textbf{Ideal Point Value}}{\textbf{Relevant Point Value}} \times 100\% \quad (8)$$

Next, the percentage error between the ideal point and Relevant Point 1 will be calculated below.

$$\frac{\mathbf{728.0995 - 633.4471}}{\mathbf{728.0995}} \times 100\% = 13\%$$

Lastly, the percentage error between the ideal point and Relevant Point 2 will be calculated below.

$$\frac{\mathbf{683.8911 - 633.4471}}{\mathbf{683.8911}} \times 100\% = 7.38\%$$

Turns out that the error percentage of Relevant Point 2 is smaller than the error percentage of Relevant Point 1. Based on that result, coordinates (28,31) are chosen as the spot of vital object installation.

CONCLUSIONS AND SUGGESTIONS

Overall, the modified random search method has been successfully implemented either its algorithm or in its application in choosing the best location to install vital object. The developed model has been proven to be quite representative based on the test results. Furthermore, the model represents a suitable approach for implementation in other cases.

Some considerations are needed while implementing this method. The result gained by using this method should be confirmed at the real spot to see whether the coordinates are relevant to the real situations of the spot or not. Besides that, certain device specifications are needed in using this algorithm. Considering these aspects will contribute to a comprehensive evaluation of the Modified Random Search Method and guide future research directions.

REFERENCES

Abdur, R. A., Tohir, M., Valentino, E., Imron, Z., & Taufiq, I. (2017). *Buku guru matematika*. Kementerian Pendidikan dan Kebudayaan. (ISBN 978-602-282-085-7).

Adam, S. P., Alexandropoulos, S. A. N., Pardalos, P. M., & Vrahatis, M. N. (2019). No free lunch theorem: A review. *Approximation and Optimization: Algorithms, Complexity and Applications*, 145, 57-82. https://doi.org/10.1007/978-3-030-12767-1_5

Akbari, M. & Haddadpour, H. (2019). Linear optimization methods for solving multi-objective optimization problems. *Journal of Optimization in Industrial Engineering*, 12(28), 27-34. 10.22094/joie.2019.1870499.1591

Amelia, L. (2013). Model optimasi produksi minyak sawit dan inti sawit menggunakan pendekatan hibrid sistem pakar kabur dan random direct search. *Jurnal Inovisi,* 9(2), 79–87.

Ayu Muchlisa, N., & Surianto, M. A. (2021). analisis saluran distribusi pada PT. Panahmas Dwitama Distrindo Jember. *Jurnal Indonesia Sosial Sains*, 2(12), 2059–2068. https://doi.org/10.59141/jiss.v2i12.480

Xue, B., Sun, C., Chu, H., Meng, Q., & Jiao, S. (2020). Method of electronic component location, grasping and inserting based on machine vision. *2020 International Wireless Communications and Mobile Computing (IWCMC), Limassol, Cyprus*, 2020, pp. 1968-1971, doi: 10.1109/IWCMC48107.2020.9148099.

Battiti, R., & Brunato, M. (2019). *Reactive search optimization: learning while optimizing*. Springer. doi: https://doi.org/10.1007/978-3-319-91086-4_15

Bumblauskas, D., Mann, A., Dugan, B., & Rittmer, J. (2020). A blockchain use case in food distribution: Do you know where your food has been?. *International Journal of Information Management*, 52, 102008. https://doi.org/10.1016/J.IJINFOMGT.2019.09.004

Rao, C. S. V. P., Pandian, A., Reddy, C. R., Bajaj, M., Jurado, F., and Kamel, S. (2023). Optimal Location of EV Parking Lot by MAOWHO technique in Distribution System. *2023 5th Global Power, Energy and Communication Conference (GPECOM), Nevsehir, Turkiye*, 2023, pp. 103-107, doi: 10.1109/GPECOM58364.2023.10175745.

Carrizosa, E., Conde, E., Pascual, A., & Romero-Morales, D. (1997). Closest solutions in ideal-point methods. *Advances in Multiple Objective and Goal Programming*, 1, 274–281. doi: https://doi.org/10.1007/978-3-642-46854-4_30

Chandra, A., & Setiawan, B. (2018). Optimasi jalur distribusi dengan Metode Vehicle Routing Problem (VRP) optimizing the distribution routes using Vehicle Routing Problem (VRP) Method. *Jurnal Manajemen Transportasi & Logistik*, 5(2), 105-116. http://dx.doi.org/10.54324/j.mtl.v5i2.233

Trahan, D. H., & Wagon, S. (2018). Elementary differential equations and boundary value problems. *The American Mathematical Monthly,* 86:10, 599-612, DOI: 10.1080/00029890.1979.11994869

Dattorro, J. (2009). *Convex optimization & euclidean distance geometry*. California: Mεβoo Publishing.

Fitiyani, F., Away, Y., & A.Gani, T. (2018). Pengaruh inisialisasi populasi random search pada algoritma berevolusi dalam optimasi Travelling Salesman Problem (Tsp). *Jurnal Nasional Komputasi dan Teknologi Informasi (JNKTI),* 1(2). doi: 10.32672/jnkti.v1i2.770

García-Nava, P. E., Rodríguez-Picón, L. A., Méndez-González, L. C., & Pérez-Olguín, I. J. C. (2022). A study of stopping rules in the steepest ascent methodology for the optimization of a simulated process. *Axioms*, 11(10), 514. doi: https://doi.org/10.3390/axioms11100514

Hanafi, L., Rohmati, E., & Puspita, G. M. (2010). Penyelesaian numerik untuk menentukan nilai optimal pada american option dengan metode beda hingga Fully Implisit dan Crank-Nicolson. Limits: Journal Mathematics and Its Applications, 7(2). doi: http://dx.doi.org/10.12962/j1829605X.v7i2.1433

Harahap, M. K., & Khairina, N. (2017). Pencarian jalur terpendek dengan algoritma dijkstra. *SinkrOn*, 2(2), 18. doi: https://doi.org/10.33395/sinkron.v2i2.61

Hasmi, R. A. (2018). Optimasi perencanaan produksi dengan menggunakan metode linear programming pada CV. Aceh Bakery. *Jurnal Optimalisasi,* 1(1), 43–56. DOI:10.35308/jopt.v1i1.168

Hidayah, R. (2021). *Strategi pemilihan lokasi terhadap kesuksesan usaha berbasis syariah*. Skripsi. Jambi: Universitas Islam Negeri Sulthan Thaha Saifuddin.

James B. Riggs. (1988). *An introduction to numerical methods for chemical engineers*. Texas: Texas Tech University Press, Chapter 6

Kamal, N. Gramedia.com. Diakses tanggal 12 juli 2023. https://www.gramedia.com/literasi/pengertian-referensi/#Sumber_Referensi.

Kamus Besar Bahasa Indonesia

Kiki Setiawan, Dkk., (2018). *Menghitung rute terpendek menggunakan algoritma A* dengan fungsi euclidean distance*. Seminar Nasional Teknologi Informasi dan Komunikasi 2018, (ISSN: 2089-9815), 70–79.

Li, L. & Talwalkar, A. (2020). Random search and reproducibility for neural architecture search. *PMLR: Proceedings of Machine Learning Research,* 115:367-377, 2020.

Ranran, L. & Reynolds, A. (2020). Joint optimization of well locations, types, drilling order, and controls given a set of potential drilling paths. *SPE Journal*. 25. 10.2118/193885-PA.

Lubis, A. N. (2004). Peranan saluran distribusi dalam pemasaran produk dan jasa. Academia, 1–14. https://www.academia.edu/download/56449929/manajemen-arlina_lbs4.pdf

Martins, J. R., & Ning, A. (2021). *Engineering design optimization*. Cambridge University Press.

Moehle, N., Shen, X., Luo, Z. Q., & Boyd, S. (2019). A distributed method for optimal capacity reservation. *Journal* *of Optimization Theory and Applications*, 182(3), 1130–1149. https://doi.org/10.1007/s10957-019-01528-5

Nugraha, D. A., & Lian, K. L. (2019). A novel MPPT method based on cuckoo search algorithm and golden section search algorithm for partially shaded PV system. *Canadian Journal of Electrical and Computer Engineering*, 42(3), 173-182. doi: https://doi.org/10.1109/CJECE.2019.2914723

Pratama, M. A. (2019). *Analisis pendistribusian yang efektif guna penjualan susu frisian flag pada CV. Sumber Makmur Metro ditinjau dari etika bisnis islam*. Skripsi. Lampung: Institut Agama Islam Negeri (IAIN) Metro.

Purcell, E. J. & Varberg, D. (1995). *Kalkulus dan geometri analitis jilid 1*. Alih Bahasa, I Nyoman Susila.; Bana Kartasasmita (Edisi 8 Jilid 1). Jakarta: Erlangga.

Rao, S. S. (2019). *Engineering optimization: theory and practice*. John Wiley & Sons. doi: 10.1002/9781119515592

Rasheed, M., Shihab, S., Rashid, T., & AL-Farttoosi, O. A. A. (2021). Comparison study between classic chord and inverse quadratic interpolation methods. *Journal of Al-Qadisiyah for Computer Science and Mathematics*, 13(1), Page-184. doi: https://doi.org/10.29304/jqcm.2021.13.1.781

Rebentrost, P., Schuld, M., Wossnig, L., Petruccione, F., & Lloyd, S. (2019). Quantum gradient descent and Newton's method for constrained polynomial optimization. *New Journal of Physics*, 21(7), 073023. doi: 10.1088/1367-2630/ab2a9e

Reza Ahmadzadeh (2023). *Random Search Algorithm (https://www.mathworks.com/matlabcentral/fileexchange/38630-random-search-algorithm)*, MATLAB Central File Exchange. Retrieved February 16, 2023.

Ridha P. E., Marisa M. D., & Hidayati, R. (2020). Menggunakan metode simulated annealing (studi kasus: PD Bumi Jaya Indah Kota Pontianak). *In Coding: Jurnal Komputer dan Aplikasi,* 8(3). doi: https://doi.org/10.26418/coding.v8i3.42409

Soetopo, W. (2004). Teknik simulasi untuk optimasi random search dengan menerapkan relaksasi pada perbaikan fungsi tujuan. *Jurnal Teknik*, 79

Steven C. C. & Raymond P. C. (2003). *Numerical Methods for Engineers: With Software and Programming Applications, 4th edition*. New York: McGraw-Hill Company Inc, Part Four

Sunarya, U. & Haryanti, T. (2022). Perbandingan kinerja algoritma optimasi pada metode random forest untuk deteksi kegagalan jantung. *Jurnal Rekayasa Elektrika*, 18(4), 241–247. doi: 10.17529/jre.v18i4.26981

Takata, H. (2019). Transaction costs and capability factors in dual or indirect distribution channel selection: An empirical analysis of Japanese manufacturers. *Industrial Marketing Management*, 83, 94–103. https://doi.org/10.1016/J.INDMARMAN.2018.11.003

Wang, Y., Zhao, N., Jing, H., Meng, B., & Yin, X. (2016). A novel model of the ideal point method coupled with objective and subjective weighting method for evaluation of surrounding rock stability. *Mathematical Problems in Engineering*, 2016. doi: 10.1155/2016/8935156

Wijaya, F. S. (2019). *Optimalisasi pelaksanaan maintenance armada pt. hidup sejahtera Sentosa.* Doctoral dissertation. Universitas Muhammadiyah Gresik.

Wiyanti, D, T. (2013). Algoritma optimasi untuk penyelesaian travelling salesman problem. *Jurnal Transformatika*, 11(1), 1-6. doi: 10.26623/transformatika.v11i1.76

Xiong, F., Wei, B., & Xu, F. (2022). Identification of arch dam mechanical parameters based on sensitivity analysis and Hooke–Jeeves algorithm optimization. *Structures*, 46, 88-98. doi: https://doi.org/10.1016/j.istruc.2022.10.052

Yang, J., & Kim, Y. (2020). A hybrid random search optimization algorithm for global optimization problems. *Journal of Applied Mathematics and Computing*, 63(1-2), 173-186 doi: 10.1007/s12190-020-01438-5

Yao, J., Zhang, X., & Murray, A. T. (2019). Location optimization of urban fire stations: Access and service coverage. *Computers, Environment and Urban Systems*, 73, 184–190. doi: https://doi.org/10.1016/J.COMPENVURBSYS.2018.10.006

Yoash Levron (2023). *Minimization of a function by random iterative search (https://www.mathworks.com/matlabcentral/fileexchange/40276-minimization-of-a-function-by-random-iterative-search)*, MATLAB Central File Exchange. Retrieved February 16, 2023.