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The estimation of the hazard function of earthquakes in aceh province with likelihood approach

Ikhsan Maulidi[1,4], Fanny Novika[2], Mahmudi[1], Vina Apriliani[3,4*], Muhamad Syazali[5]

[1] Universitas Syiah Kuala, Indonesia
[2] Sekolah Tinggi Manajemen Asuransi Trisakti, Indonesia
[3] Universitas Islam Negeri Ar-Raniry, Indonesia
[4] Kanazawa University, Japan
[5] Universitas Pertahanan, Indonesia

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*Correspondence: E-mail:
vina_april17@yahoo.com

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ABSTRACT

In this article, we propose a novel application of the single decrement method with a likelihood approach to estimate the hazard function of earthquake events in Aceh province. While this method has traditionally been used in actuarial sciences for mortality table estimation, its application in seismic hazard estimation represents a new perspective in the field of earthquake risk analysis. To enhance the accuracy of the model, we applied the Box-Cox transformation to normalize the data and used simple regression to formulate the hazard function. Our results demonstrate that a cubic equation provides a more accurate model compared to linear and quadratic equations, as evidenced by the lower Mean Square Error (MSE). This study offers a new approach to hazard rate estimation that surpasses conventional methods by providing more informative and interpretable results for earthquake risk assessment.

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INTRODUCTION

An earthquake is an activity of releasing energy from the earth suddenly, quickly, and creeping in all directions as a seismic wave (Jena, Pradhan, & Beydoun, 2020). According to Jena, Pradhan, Beydoun, Nizamuddin, et al. (2020), earthquakes have very strong destructive power and wide reach. In general, there are three categories of sources of earthquake events, namely collapse, volcanic activity, and tectonics (Jena, Pradhan, Beydoun, Nizamuddin, et al., 2020). In Indonesia, tectonic earthquakes are the most frequent (Liu et al., 2023; Tokuda & Nagao, 2023).

Geographically, the Indonesian archipelago is located at the confluence of three earth crust plates, namely Eurasia, the Pacific, and Indo-Australia (Koshimura, Oie, Yanagisawa, & Imamura, 2009). Geologically, Indonesia is located at the meeting point of two major earthquake lines, namely the Pacific Rim and the Transasiatic Alps. Therefore, Indonesia has quite high earthquake activity (Putra, Kiyono, Ono, & Parajuli, 2012; Triyoso, 2023). In recent years, natural disasters due to earthquakes have often occurred and caused loss of life and property. In Aceh, the last largest earthquake occurred on December 26, 2004, centered off the west coast of Aceh Province with Ms = 8.9. The earthquake triggered a tsunami that affected 11 Asian countries with a death toll of more than 80,000 people. Moreover, the earthquake that occurred in Pidie Jaya claimed many lives in 2016.

The level of danger has a significant influence on the theory of the possibility of an earthquake occurring (Jena, Pradhan, Beydoun, Al-Amri, & Sofyan, 2020). If the hazard level is known, then the joint function distribution for the realization of earthquake event data in (0, T) can be identified (Danciu, Kale, & Akkar, 2018). Therefore, an accurate parametric model is needed to estimate the level of danger (Banyunegoro, Alatas, Jihad, Eridawati, & Muksin, 2019). In general, the level of earthquake hazard is calculated using a single decrement method with a likelihood approach (Nurtiti, Kresna, Islamiyati, & Raupong, 2013), which is adapted from the actuarial science method for estimating mortality tables (Price, Drovandi, Lee, & Nott, 2018). This approach contrasts with the traditional point process method introduced by Vere-Jones in 1995 (Daley, 2006), which relies on nonlinear equations that are difficult to solve analytically and therefore require numerical solutions. Our study shows that the single decrement hazard rate method provides more informative results compared to the point process likelihood method, which offers a new perspective on earthquake hazard estimation (Nurtiti et al., 2013). Other related articles looking at the level of danger can be seen in Anagnos & Kiremidjian (1988); Corral (2005); Kameshwar & Padgett (2014); Kijko & Sellevoll (1989); and Youngs & Coppersmith (1986).

The results of this study show that the single decrement hazard rate method provides more informative results than the point process hazard rate likelihood method. Although this method originates from actuarial science, where it is used to estimate mortality rates, it is adaptable to earthquake hazard estimation because both phenomena deal with probabilistic events over time (Novika, Maulidi, Marsanto, & Amalina, 2022). Earthquakes, like mortality, can be modeled as a series of stochastic events that occur within a specific timeframe, making the single decrement method a suitable alternative (Cipta et al., 2021).

Moreover, the single decrement method allows for a more nuanced understanding of the hazard function by incorporating variables and assumptions relevant to seismic activity, such as regional seismicity rates and fault activity. Several studies in hazard analysis have explored similar probabilistic approaches (Anagnos & Kiremidjian, 1988; Cipta et al., 2021; Corral, 2005; Kijko & Sellevoll, 1989), though none have directly applied the actuarial-based decrement method.

Our study builds on this foundation, demonstrating that this actuarial approach offers a clearer, more interpretable measure of hazard rates compared to traditional point process models, which often require complex numerical solutions. The estimation of hazard rates using the single decrement method consists of two sub-methods: the likelihood approach and the moment approach (Socquet, Hollingsworth, Pathier, & Bouchon, 2019). Using the likelihood approach, exit time information is required, that is, the information on the number of earthquake events after t_0. As usual in actuarial theory, in this article the hazard rate at the point t_0 is symbolized by μ_{t_0} (Zhao, Khosa, Ahmad, Aslam, & Afify, 2020).

METHOD

The method used to estimate the hazard rate values in this article is a single decrement method, which is a method in the actuarial study of developing a mortality table (Lee, Ha, & Lee, 2021). The data used in the analysis is data that is sourced from BMKG. The data are the earthquake data in Aceh during the 1980-2013 period, which had a strength of more than or equal to 5 SR. Following are the steps to formulate earthquake hazard rate, which:

1. Estimate the value of earthquake hazard rate from the data in 1980-2012 with consideration of waiting time for linear and exponential distribution.
2. Determine the best parametric equation to determine the hazard rate by using the assumption of linear, quadratic, and cubic equations.

Suppose that $X(t_0) = T - t_0$ expresses a waiting time of the forward earthquake event if t_0 is the time of the first earthquake event that has been known and T is a time of the next earthquake. For example, if the first earthquake occurred in 2013 and the next earthquake occurred in 2017, then $X(t) = 2017 - 2013 = 4$ years.

Let μ, S, and f in order be a hazard rate, a survival function, and a probability density function. The hazard rate at time t_0 can be defined as

$$\mu_{t_0} = \lim_{\Delta t_0 \to 0} \frac{P(t_0 \le T \le t_0 + \Delta t_0 \mid T > t_0)}{\Delta t_0} \approx \frac{f(t_0)}{S(t_0)} \quad (1)$$

Suppose that $y = t_0$, by integrating both sides of Equation (1) we obtain

$$S(y) = \exp\left[-\int_{t_0}^{t_0+\Delta t_0} \mu_y \, dy\right].$$

If $t_0 = 0$, is a short moment after the earthquake, then

$$\begin{aligned} {}_{\Delta t_0}p_{t_0} = S(t_0) &= P(T > t_0 + \Delta t_0 \mid T > t_0) \\ &= \exp\left[-\int_0^{\Delta t_0} \mu(t_0 + s)\, ds\right] \end{aligned}$$

is a survival function.

The estimation of the hazard rate by using a single decrement with the Likelihood approach requires exit time information, which is a time on the earthquake event. Let d_{t_0} be a number of earthquakes that occurred in the interval $[t_0, t_0 + 1]$ and $n_{t_0} - d_{t_0}$ be a number of earthquakes that occurred after $t_0 + 1$. Likelihood L for the i-th earthquake in the interval $(t_i, t_i + 1]$ is given as follows, assuming there is no event until t_0.

$$L_i = f(t_0(i) \mid T > t_0(i)) = \frac{f(t_0(i))}{S(t_0)} = \frac{S(t_0(i))\, \mu(t_0(i))}{S(t_0(i))} \quad (2)$$

is the contribution of the i-th earthquake on L. Since $y_i = t_0(i) + t_0$ is a time of the i-th earthquake that occurred in the interval $(_0, t_0 + 1]$ with $0 < y_i \le 1$, then

$$L_i = \frac{S(t_0 + y_i)\, \mu(t_0 + y_i)}{S(t_0)} = {}_{y_i}p_{t_0} \mu_{t_0 + y_i} \quad (3)$$

The contribution of a number of earthquake d_{t0} on L is $\prod_{i=1}^{d} {}_{y_i}p_{t_0}\mu_{t_0+y_i}$. The $n_{t_0} - d_{t_0}$, a number of earthquakes after $t_0 + 1$, is $(p_{t_0})^{n_{t_0} - d_{t_0}}$. In this case, n_{t_0} represents a number of earthquakes that occur on and after t_0. Therefore, the total of the likelihood is

$$L = (1 - q_{t_0})^{n_{t_0} - d_{t_0}} \prod_{i=1}^{d_{t_0}} {}_{y_i}p_{t_0}\mu_{t_0+y_i} \quad (4)$$

Solving Equation (4), it is required to make an assumption that the distribution ${}_{y_i}p_{t_0}\mu_{t_0+y_i}$ as a function of q_{t_0}. Here, it has been assumed that l_{t_0+y}, which is a number of earthquakes after $t_0 + y$ as a linear function and an exponential function. By using the linear assumption for $l_{t_0+y} = a + by$, it can be obtained $\widehat{q_{t_0}} = \frac{d_{t_0}}{n_{t_0}}$ which is the maximum likelihood estimation for q_{t_0}. Furthermore, the hazard rate values can be obtained by using the equation

$$\widehat{\mu_{t_0}} = \frac{\widehat{q_{t_0}}}{1-\widehat{q_{t_0}}} \quad (5)$$

And after the hazard rate has been known for each point, the parametric equation can be estimated by using the regression method.

Suppose that l_{t_0+y} is an exponential function, $l_y = ab^y$, then the estimation of the hazard rate is

$$\hat{\mu} = \frac{d_{t_0}}{(n_{t_0}-d_{t_0})+\sum_{i=0}^{d_{t_0}} y_i}, \quad (6)$$

which is the maximum likelihood estimation for μ.

RESULTS AND DISCUSSION

The estimation of hazard rates of the earthquakes

Here we show the plot of the magnitude to the time and the location of the earthquake area study.

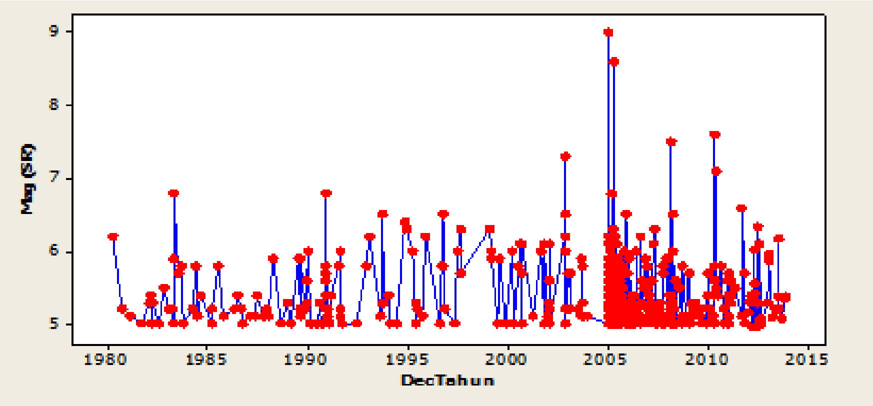

Figure 1. Plot of the Magnitude with Respect to the Time for the Area Study.

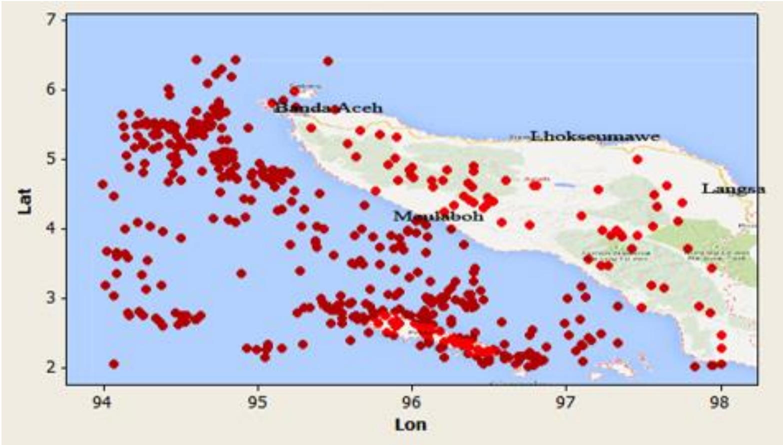

Figure 2. Plot of the Earthquake Event Location for the Area Study.

The estimation of hazard rate using the single decrement method with the likelihood approach can be obtained by using Equations (5) and (6). This following table is the result of the hazard rate estimation by using Equation (5).

Table 1. The Result of the Hazard Rate with the Likelihood Approach for the Waiting Time Assumed as a Linear Function when There is No Earthquake until t_0.

Interval	Tahun	d_{t_0}	n_{t_0}	q_{t_0}	μ_{t_0}
(0,1]	1980	2	490	0.0041	0.0041
(1,2]	1981	2	488	0.0041	0.0041
(2,3]	1982	6	486	0.0123	0.0125
(3,4]	1983	8	480	0.0167	0.0169
(4,5]	1984	4	472	0.0085	0.0085
(4,6]	1985	4	468	0.0085	0.0086
(6,7]	1986	4	464	0.0086	0.0087
(7,8]	1987	6	460	0.0130	0.0132
(8,9]	1988	4	454	0.0088	0.0089
(9,10]	1989	8	450	0.0178	0.0181
(10,11]	1990	15	442	0.0339	0.0351
(11,12]	1991	9	427	0.0211	0.0215
(12,13]	1992	2	418	0.0048	0.0048
(13,14]	1993	5	416	0.0120	0.0122
(14,15]	1994	5	411	0.0122	0.0123
(15,16]	1995	6	406	0.0148	0.0150
(16,17]	1996	5	400	0.0125	0.0127
(17,18]	1997	4	395	0.0101	0.0102
(18,19]	1998	0	391	0.0000	0.0000
(19,20]	1999	6	391	0.0153	0.0156
(20,21]	2000	7	385	0.0182	0.0185
(21,22]	2001	5	378	0.0132	0.0134
(22,23]	2002	15	373	0.0402	0.0419
(23,24]	2003	9	358	0.0251	0.0258
(24,25]	2004	46	349	0.1318	0.1518
(25,26]	2005	142	303	0.4686	0.8820
(26,27]	2006	37	161	0.2298	0.2984
(27,28]	2007	28	124	0.2258	0.2917
(28,29]	2008	25	96	0.2604	0.3521
(29,30]	2009	13	71	0.1831	0.2241
(30,31]	2010	1	58	0.3103	0.4500
(31,32]	2011	13	40	0.3250	0.4815
(32,33]	2012	16	27	0.5926	1.4545

Table 2. The Result of the Hazard Rate with the Likelihood Approach for the Waiting Time Assumed as an Exponential Function when There is No Earthquake until t_0.

Interval	Tahun	$\sum_{i=1}^{n} y_i$	$n_{t_0} - d_{t_0}$	d_{t_0}	q_{t_0}	μ_{t_0}
(0,1]	1980	0.9657	488	2	0.0041	0.0041
(1,2]	1981	0.8068	486	2	0.0041	0.0041
(2,3]	1982	2.1123	480	6	0.0123	0.0124
(3,4]	1983	3.0767	472	8	0.0166	0.0168
(4,5]	1984	1.7630	468	4	0.0084	0.0085
(4,6]	1985	1.7383	464	4	0.0085	0.0086
(6,7]	1986	2.1780	460	4	0.0086	0.0087
(7,8]	1987	3.5945	454	6	0.0129	0.0131
(8,9]	1988	1.9109	450	4	0.0088	0.0089
(9,10]	1989	6.2876	442	8	0.0175	0.0178
(10,11]	1990	9.2794	427	15	0.0332	0.0344
(11,12]	1991	3.3616	418	9	0.0209	0.0214
(12,13]	1992	1.3273	416	2	0.0048	0.0048
(13,14]	1993	2.7808	411	5	0.0119	0.0121
(14,15]	1994	2.3136	406	5	0.0121	0.0122
(15,16]	1995	3.1849	400	6	0.0147	0.0149
(16,17]	1996	3.5643	395	5	0.0124	0.0125
(17,18]	1997	2.0684	391	4	0.0101	0.0102
(18,19]	1998	0	391	0	0	0
(19,20]	1999	2.3068	385	6	0.0153	0.0155
(20,21]	2000	3.6753	378	7	0.0180	0.0183
(21,22]	2001	3.4219	373	5	0.0131	0.0133
(22,23]	2002	8.1972	358	15	0.0393	0.0410
(23,24]	2003	5.0767	349	9	0.0248	0.0254
(24,25]	2004	45.5945	303	46	0.1166	0.1320
(25,26]	2005	50.4287	161	142	0.4018	0.6716
(26,27]	2006	16.2095	124	37	0.2088	0.2639
(27,28]	2007	15.3630	96	28	0.2009	0.2514
(28,29]	2008	8.3890	71	25	0.2395	0.3149
(29,30]	2009	5.9109	58	13	0.1690	0.2034
(30,31]	2010	8.9068	40	18	0.2690	0.3680
(31,32]	2011	3.6904	27	13	0.2975	0.4236
(32,33]	2012	6.2	11	16	0.4819	0.9302

The estimation of the parametric hazard function

The results in Table 1 and Table 2 consist of the value of the hazard rates with the waiting time linear and exponential function at time t_0 denoted by the symbol μ_{t_0}. To estimate the regression equation of the hazard rates, it is needed that the property that the hazard rate must be normally distributed (Lu, Wang, Huang, & Chen, 2023). However, according to the QQ-plot results of the hazard rate in Table 1 and Table 2, the hazard rates are not normally distributed, so it is necessary to normalize this hazard value by first removing the value of the hazard rate, which is an outlier.

By using Box-Cox Transformation for the hazard rate values from Table 1 and Table 2, we obtained the value of λ, respectively, as -0.5 and -0.5. It means that the transformation that should be applied is $(\mu_{t_0})^{-0.5}$ for both of the assumptions. So, it is needed to inverse transform to obtain μ_{t_0}. The inverse transformation is

$$\mu_{t_0} = (\mu^*_{t_0})^{-2} = \frac{1}{{\mu^*_{t_0}}^2} \quad (7)$$

The process to determine the parametric model has been done by the regression method for the hazard rate using the linear model $\mu^*_{t_0} = \beta_0 + \beta_1 t_0 + \varepsilon$, the quadratic model $\mu^*_{t_0} = \beta_0 + \beta_1 t_0 + \beta_2 t_0^2 + \varepsilon$, and the cubic model $\mu^*_{t_0} = \beta_0 + \beta_1 t_0 + \beta_2 t_0^2 + \beta_3 t_0^3 + \varepsilon$.

Based on the results of the transformation of the hazard rate values in Table 1, it has been obtained that the linear and cubic models are the exact models by the test of the significance of the regression coefficients, while the quadratic model is not exact. Moreover, the cubic model is the best model because the Mean Square Error (MSE) of the cubic model is smaller than the linear model.

By using the inverse transformation, Equation (7), the equation of the hazard rate in Table 1 can be expressed as

$$\widehat{\mu_{t_0}} = \frac{1}{(13.567 - 0.919t_0 + 0.061t_0^2 - 0.002t_0^3)^2} \quad (8)$$

If Equation (8) is assumed to be able to predict the hazard rate in the future, then this model can be used. For example, the hazard rate in Aceh in 2020, given the information that the last event occurred in 2018, is

$$\widehat{\mu_2} = \frac{1}{(13.567 - 0.919(2) + 0.061(2)^2 - 0.002(2)^3)^2} = 0.00699$$

Table 3. The Model Estimation of the Hazard Rate in Table 1.

Model	MSE	Regression Test ($\alpha = 0.1$)
$\widehat{\mu^*_{t_0}} = 12.838 - 0.349t_0$	287.346	Exact
$\widehat{\mu^*_{t_0}} = 11.762 - 0.123t_0 - 0.008t_0^2$	147.970	No Exact
$\widehat{\mu^*_{t_0}} = 13.567 - 0.919t_0 + 0.061t_0^2 - 0.002t_0^3$	105.768	Exact

Based on the results of the transformation of the hazard rate values in Table 2, it has been obtained that the linear and cubic models are the exact models by the test of the significance of the regression coefficients, while the quadratic model is not exact. Moreover, the cubic model is the best model because the Mean Square Error (MSE) of the cubic model is smaller than the linear model.

By using the inverse transformation, as Equation (7), the equation of the hazard rate in Table 2 can be expressed as

$$\widehat{\mu_{t_0}} = \frac{1}{(13.574 + 0.917t_0 - 0.060t_0^2 - 0.002t_0^3)^2}. \quad (9)$$

If Equation (9) is assumed to be able to predict the hazard rate in the future, then this model can be used. For example, the hazard rate in Aceh in 2020, given the information that the last event occurred in 2018, is

$$\widehat{\mu_2} = \frac{1}{(13.574 + 0.917(2) - 0.060(2)^2 - 0.002(2)^3)^2} = 0.00436$$

Table 4. The Model Estimation of the Hazard Rate in Table 2.

Model	MSE	Regression Test ($\alpha = 0.1$)
$\widehat{\mu^*_{t0}} = 12.824 - 0.344t_0$	280.494	Exact
$\widehat{\mu^*_{t0}} = 11.778 - 0.125t_0 - 0.007t_0^2$	144.312	No Exact
$\widehat{\mu^*_{t0}} = 13.574 + 0.917t_0 - 0.060t_0^2 - 0.002t_0^3$	103.253	Exact

CONCLUSIONS AND SUGGESTIONS

Based on the analysis that has been done, it is obtained that the estimation of the hazard rate with likelihood approached for the number of the events, which assumed linear and exponential. The results are given in Table 1 and Table 2. Furthermore, the parametric model to estimate these hazard rates has been modeled as a linear, a quadratic, and a cubic equation. The cubic equation is the best model to predict the hazard rate accurately.

Future research could involve applying temporal point process models to analyze the timing patterns of earthquakes, integrating probabilistic approaches in seismic risk analysis to account for uncertainties in hazard estimation. Additionally, exploring multi-hazard approaches that consider the risks of earthquakes alongside other natural disasters would be beneficial.

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