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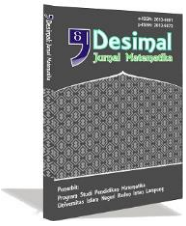

Semicontinuous endowment insurance premium valuation using quadratic fractional age assumptions

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ABSTRACT

The classic assumptions used to calculate fractional ages in valuing insurance premiums with payouts made immediately after death result in discontinuous probabilities of immediate death at integer ages because the assumption only applies to a 1-year time interval, specifically $0 \leq t \leq 1$. The new assumption, namely the quadratic fractional age assumption, has successfully introduced continuity at integer ages. This study discusses the conditions for applying the quadratic fractional age assumption and its influence on the calculation of semi-continuous dual-purpose insurance premiums, where the policyholder's beneficiaries receive the sum assured if the insured individual passes away before the contract ends or receive protection if the policyholder survives until the end of the contract, with premium payments made on a monthly basis. Simulation results indicate that not all mortality tables meet the requirements for the quadratic fractional age assumption, where the value falls between $0 \leq B_x \leq 2d_x$. Only the Commissioners Standard Ordinary Table 1958 meets this criterion. Monthly premiums calculated using the quadratic fractional age assumption yield smaller values compared to premiums calculated using the constant force of mortality assumption and the uniform distribution of death assumption.

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INTRODUCTION

The death benefit of semi-continuous endowment life insurance is paid immediately upon the occurrence of death, and premiums are paid on a monthly, quarterly, annually or semi-annual basis. To calculate the actuarial present value of the life insurance and the reserve for the benefit, the force of mortality at the moment of death is required (Jordan, 1967). The force of mortality can be calculated by assuming that the instantaneous death within the interval $0 \leq t \leq 1$ follows a specific distribution based on fractional age assumptions. The available mortality table only contains the probability of someone's survival and death for whole ages. We need fractional-age assumptions to calculate fractional ages. Fractional age assumptions are needed when policyholder ages are not integers (Brillinger, 1961; Broffitt, 1984; Hoem, 1984; London, 1997). The classic fractional age assumptions used in actuarial science include Linear Interpolation or Uniform Distribution of Death (UDD), Exponential Interpolation or Constant Force of Mortality (CFM), and Harmonic Interpolation or Balducci (Bowers, Gerber, Hickman, Jones, & Nesbitt, 1997). These three assumptions are not dependent on the survival probabilities from the preceding and succeeding years, resulting in a discontinuous force of mortality at whole ages (piecewise continuous function) (Li, Zhao, & Guo, 2013). Willmot (1997) proposed a new fractional age assumption called Fractional Independence (FI). This assumption holds if and only if $\Pr(S \leq s|K = k) = \frac{kPu-k+s^{p}u}{k^{p}u-k+1^{P}u}$. This assumption facilitates the calculation of the actuarial present value (APV). Jones & Mereu (2000) generalized fractional age assumptions with α-power approximations, including UDD, constant rate, and Balducci as special cases when $\alpha_x = 1, -1, 0$. Frostig (2002, 2003) discussed power family approximations of fractional age assumptions and their properties. Further, Jones & Mereu (2002) explored α-power approximations and introduced the Quadratic Survival Function (QSF) and Linear Force of Mortality (LFM). Using QSF, the survival function is assumed to be quadratic with $0 \leq t \leq 1$. The survival function is ${}_tp_x = 1 - \mu_{x+0}t + (\mu_{x+0} - q_x)t^2$ valid only when $0 \leq \mu_{x+0} \leq 2q_x$. Meanwhile, with LFM, the force of mortality is assumed to be linear between two whole ages $\mu_{x+t} = \mu_{x+0} - 2(\log p_x + \mu_{x+0})t$ valid only when $0 \leq \mu_{x+0} \leq 2q_x$. The new fractional age assumption, Quadratic Fractional Age Assumption, successfully eliminates discontinuity in the force of mortality at whole ages, but not all mortality tables can use this quadratic assumption as it only holds when $0 \leq B_x \leq 2d_x$ (Hossain, 2011). Ocktaviani & Effendie (2015) compared the force of mortality values obtained from interpolating the Indonesian Mortality Table (TMI) 2011 using several fractional age assumptions, such as linear, exponential, hyperbolic, LFM, and quadratic. But in that study, there was no application of the fractional age assumption in premium pricing, especially endowment life insurance premiums.

Actuaries commonly use mortality tables, including TMI III, TMI IV for Indonesia, the 1941 Commissioners Standard Ordinary (CSO), the 1958 CSO, and the 1980 CSO Mortality Table for the United States, to calculate the actuarial present value of life insurance premiums. According to Utomo (2021), these mortality tables do not show statistically significant differences in average death rates, as tested using nonparametric Mann-Whitney and Kruskal-Wallis tests. Similarly, research by Narlitasari, Rioke, Setyani, Nurbayti, & Prabowo (2022) indicates that the graph shows results indicating no statistically significant variations in the 2019 TMI mortality chances for both men and women when using the UDD and CFM techniques. Therefore, both approaches can be utilized to determine the likelihood of death at the fractional age of TMI 2019.

From the previous research and the issues we are facing with the determination of life insurance premium rates, where the benefit payments are made immediately after death occurs, using classical fractional age assumptions resulting in a discontinuous force of mortality throughout age, the research objective is to analyze the force of mortality using quadratic fractional age assumptions and compare the premium determination results using quadratic fractional age, UDD, and CFM assumptions.

This research is organized as follows: Section 2 discusses methods. Quadratic fractional age assumptions and monthly premiums for semi-continuous endowment life insurance using quadratic fractional age assumptions and classical assumptions are discussed in this section, as is the research flowchart. Section 3 is the key part of this research. In this section, we discuss the force of mortality using quadratic, UDD, and CFM for males and females using the 1958 CSO mortality table, as well as the calculations of the actuarial present value (net premium) for endowment life insurance with benefits payable at the moment of death and discrete life annuities. Besides, some simulation results are given. In Section 4, we conclude this research and make suggestions.

METHOD

The mortality data used in this research is the CSO 1958 mortality data for both males and females from Society of Actuaries (2012). The CS0 1958 mortality table presents mortality probability data for males and females based on age. The maximum age limit for males is 99 years, and for females, it is 102 years. The insured's age is assumed to be within the age interval of $18 \leq x \leq 58$ years, representing the productive working age. The duration of the insurance contract is considered, with an annual interest rate of 6%, and the death benefit received by the beneficiaries immediately upon the death of the policyholder is assumed to be 1 unit. Someone's death certainly doesn't always occur on their birthday. Someone's death certainly doesn't always occur on their birthday, so we needed to calculate the probability of death at fractional ages using fractional age assumptions.

Quadratic Fractional Age Assumptions

The probability that an individual will live to a certain age after reaching age x can be calculated using mortality statistics and life expectancy tables that reflect the average life expectancy of individuals at that age in a particular population, known as the survival function. The survival function for someone aged x years, assuming quadratic fractional age, is given by:

$$s_x(t) = \frac{l_{x+t}^h}{l_x^h} = \frac{l_x - \left(t - \frac{t^2}{2}\right) B_x - \frac{t^2}{2} B_{x+1}}{l_x} = {}_t p_x^h$$

where $B_x = 2\sum_{i=0}^{\omega-x-1}(-1)^i d_{x+i}$, $B_x + B_{x+1} = 2d_x$, ω is the limiting age in the mortality table used, and d_x indicates the number of people who died at the age of x years.

Let $B_x = Q_x . l_x$ or $Q_x = \frac{B_x}{l_x} = 2\sum_{i=0}^{\omega-x-1}(-1)^i \frac{d_{x+i}}{l_x} = 2\sum_{i=0}^{\omega-x-1}(-1)^i q_{x+i}$ then ${}_t q_x^h = \left(t - \frac{t^2}{2}\right) Q_x + \frac{t^2}{2} p_x Q_{x+1}$.

The force of mortality using quadratic factional age assumptions is

$$\mu(x+t) = \frac{(1-t)Q_x + p_x Q_{x+1} t}{1 - \left[\left(1 - \frac{t^2}{2}\right) Q_x + \frac{t^2}{2} p_x Q_{x+1}\right]}$$

The use of the quadratic fractional age assumption is carried out by ensuring that the mortality table used satisfies the conditions of the quadratic fractional age assumption (Hossain, 2011).

If the CSO 1958 mortality table satisfies condition $0 \leq B_x \leq 2d_x$, then the force of mortality is calculated using the quadratic fractional age assumption. The notation d_x indicates the number of people who died at the age of x years. If the mortality table used does not meet these requirements, then the force of mortality is calculated using UDD, or CFM. Mortality tables with quadratic fractional age assumptions, UDD, and CFM for both males and females are constructed.

Uniform Distribution of Death (UDD) Assumptions

The assumption of the Uniform Distribution of Death (UDD) is the most commonly used because it is simple and easy to understand, but sometimes it can be overly restrictive (Willmot, 1997).

The uniform distribution of death assumptions is employed to estimate fractional ages by utilizing linear interpolation, where ${}_tp_x^U$ is a linear function (Friedler, 1986). According to Goldstein & Lee (2020), the UDD assumption will have t. l_x up to time t years with $0 < t < 1$, where l_x is the number of individuals alive in x years old. This is because if is the number of persons aged x years who died before reaching the age of $x + 1$ year. As a result, the number of individuals alive at all ages, according to the UDD assumption, is

$$l_{x+t}^U = (1 - tq_x)l_x^U$$

The survival function for someone aged x years, assuming UDD fractional age, is given by:

$$s_x(t) = {}_tp_x^U = 1 - tq_x$$

The force of mortality using UDD assumptions is

$$\mu(x + t) = \frac{q_x}{1 - tq_x}$$

Constant Force of Mortality (CFM) Assumptions

Under this assumption, the force of mortality remains constant between integer ages. Consequently, for integer values of x and $0 < t < 1$, we presume that μ_{x+t} does not vary with t, and we denote it as μ_x (Dickson, Hardy, & Waters, 2019).

If l_{x+t} represents the number of individuals aged $x + t$ years, then the survival function of someone from birth to the age of $x + t$ years is given by

$$s_x(t) = {}_tp_x^C = \frac{l_{x+t}^C}{l_x^C}$$

The force of mortality using CFM assumptions is

$$\mu_x^C = -\log p_x$$

The results of constructing mortality tables are used to calculate net single premiums and the present value of term life annuities with payments made at the beginning of the month. The determination of monthly premiums for semi-continuous endowment life insurance uses the principle of premium equivalence.

True Fractional Premiums

The premiums paid more than once within a single period without any adjustment to the death benefit are referred to as true fractional premiums. Based on the principle of premium equivalence, the payment model of true fractional premiums with benefits provided directly upon the occurrence of the insured's death is stated as

$$P^{(m)}(\bar{A}_{x:\overline{n|}}) = \frac{\bar{A}_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}^{(m)}}$$

where m represents the premium payment period. In this research, m = 12 because premium payments are made monthly. $\bar{A}_{x:\overline{n|}}$ represents the actuarial net present value of a death benefit of 1 unit with an insurance protection period of n years. If the insured passes away before reaching age $x + n$, the benefit is paid immediately at that time. However, if the insured survives until age $x + n$, the benefit is paid by the insurance company to the insured at that age. While $\ddot{a}_{x:\overline{n|}}^{(m)}$ represents the present value of life annuity due for an individual aged x years with premium payments made m times per year for n years. The research flowchart is shown in Figure 1.

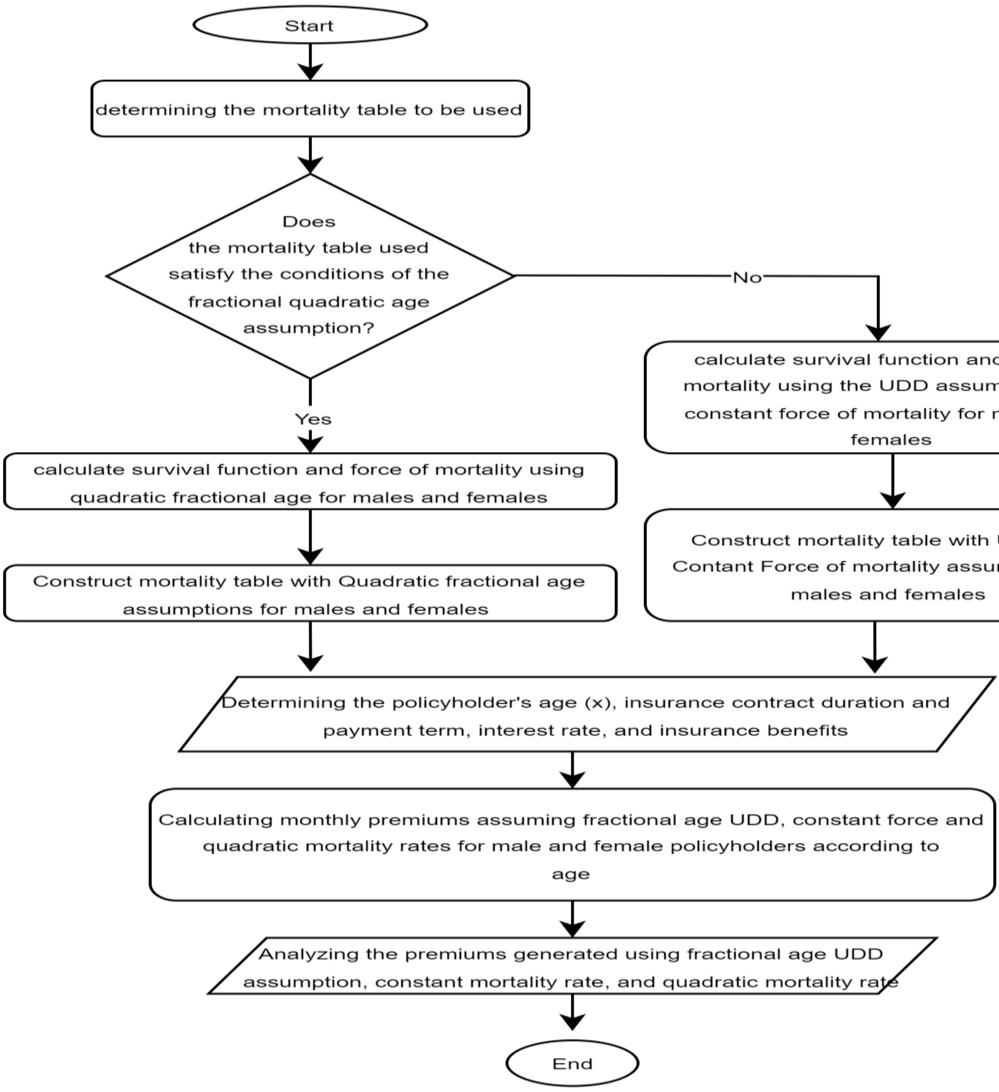

Figure 1. The Research Flowchart

RESULTS AND DISCUSSION

In the CSO 1958 mortality table, the number of deaths for an 18-year-old male is 167, indicating that the value of B_x falls within the interval from 0 to $2d_x$.

$$0 \leq \ 2\sum_{i=0}^{92}(-1)^i d_{18+i} \ \leq \ 2d_{18}$$
$$0 \leq \ 229 \ \leq \ 334$$
$$0 \ \leq \ Bx \ \leq \ 2d_x$$

The force of mortality calculated using UDD, CFM, and Quadratic assumptions for males and females just before reaching the ages of 50 to 55 can be observed in Figures 2 and 3.

Based on Figure 2 and Figure 3, it can be observed that the force of mortality for males with both the UDD and CFM assumptions is discontinuous at all ages. For fractional ages within $0 \leq t \leq 1$, the force of mortality under the UDD assumption always increases, whereas under the constant death rate assumption, it remains constant. Meanwhile, the force of mortality under the quadratic fractional age assumption fluctuates and remains continuous at all ages.

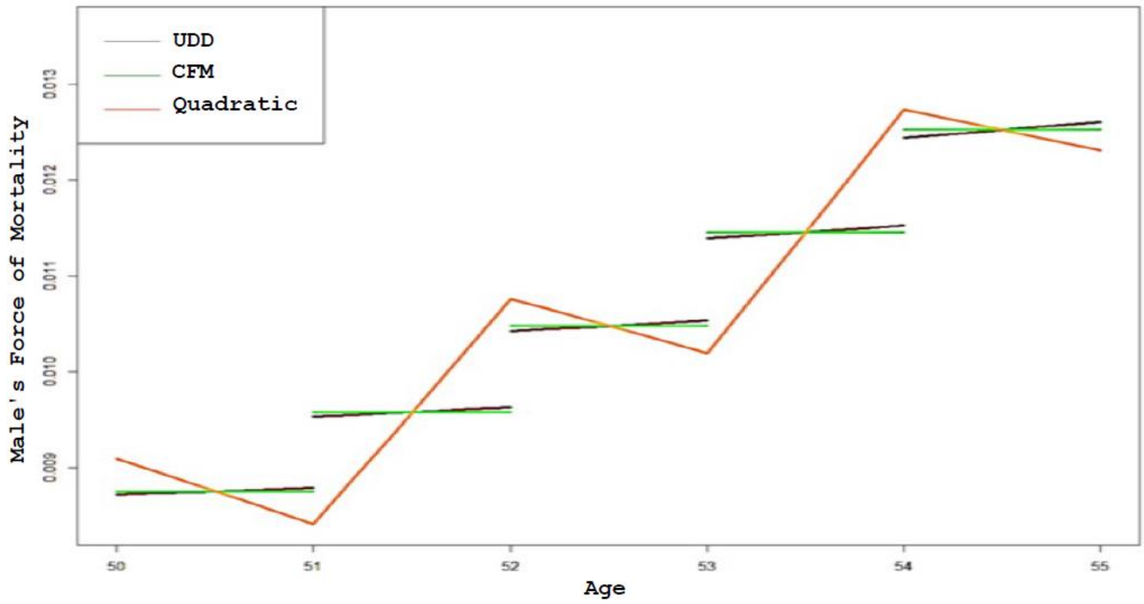

Figure 2. Force of Mortality for Males Using Three Fractional Age Assumptions

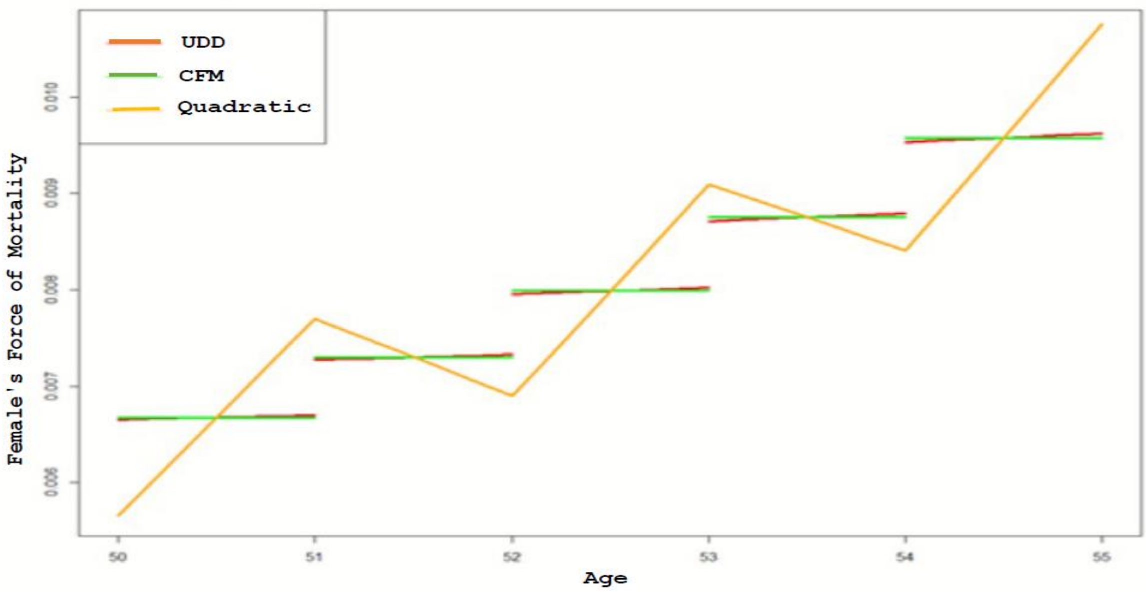

Figure 3. Force of Mortality for Female Using Three Fractional Age Assumptions

The number of newborns, both male and female, is assumed to be 100,000 each, with the probability of death within 1 year denoted as q_0. For males, q_0 is 0.00443, and for females, it is 0.003942. By utilizing the relationship between the values of q_0 and p_0, the probability that male and female infants will survive until the age of 1 year, p_0, is 0.995571 and 0.996084, respectively. The probability of female survival is higher than that of male survival. The information available in the CSO 1958 mortality table is used to find the elements present in each mortality table, using the quadratic fractional age assumption, UDD, and CFM assumptions for both males and females.

Mortality Table Using UDD Assumption

The mortality table using the UDD assumption is used as a reference to identify the elements present in the mortality table with the UDD assumption.

The number of newborn males who reach the age of t years, where $0 \le t \le 1$ is denoted by l_{0+t}, is

$$l_{x+t}^{u} = (1 - tq_x)l_x^u$$
$$l_{0+t}^{U} = (1 - 0.0044295t)100000$$
$$l_{0+t}^{U} = 100000 - 442.95t$$

Meanwhile, for female infants, it is obtained

$$l_{0+t}^{U} = (1 - 0.0039421t)100000$$
$$l_{0+t}^{U} = 100000 - 394.21t$$

Thus, the number of individuals aged $0 + t$ years with $0 \le t \le 1$ for males and females, respectively, is $100000 - 442.95t$ and $100000 - 394.21t$. Assuming $t = 0$, the values of l_0 for males and females are both 100,000. The probability that newborn males will die before reaching the age of t years, denoted by ${}_tq_0$, is

$${}_tq_x^U = t.q_x$$
$${}_tq_0^U = t.q_0$$
$${}_tq_0^U = 0.00443t$$

Meanwhile, for newborn females, it is obtained

$${}_tq_x^U = t.q_x$$
$${}_tq_0^U = t.q_0$$

Thus, the probability that someone who is newborn will die before reaching the age of t years, with $0 \le t \le 1$, for males and females, respectively, is 0.00443t and 0.003942t. Assuming $t = 1$, the values of q_0 for males and females are 0.00443 and 0.003942, respectively. The probability that newborn males will survive until the age of t years is

$${}_tp_x^U = 1 - t.q_x$$
$${}_tp_0^U = 1 - 0.00443t$$

Meanwhile, for females, it is obtained

$${}_tp_0^U = 1 - 0.003942t$$

Thus, the probability that someone who is newborn will survive until the age of t years, with $0 \le t \le 1$, for males and females, respectively, is 1-0.00443t and 1-0.003942t. Assuming $t = 1$, the values of q_0 for males and females are 0.995571 and 0.996058, respectively.

The assumption of the CFM, or exponential distribution of death, is used as a reference to identify the elements present in the mortality table. Thus, the number of individuals aged $0 + t$ years that is denoted by l_{0+t} for males is

$$l_{x+t}^{C} = (p_x^t)l_x^C$$
$$l_{0+t}^{C} = l_0\left(\frac{l_1}{l_0}\right)^t$$
$$l_{0+t}^{C} = 100000(0.995571)^t$$

Meanwhile, for females, it is obtained

$$l_{0+t}^{C} = l_0\left(\frac{l_1}{l_0}\right)^t$$
$$l_{0+t}^{C} = 100000(0.996058)^t$$

Thus, the number of individuals aged $0 +$ t years with $0 \le t \le 1$ for males is $100000(0.995571)^t$, and for females, it is $100000(0.996058)^t$. Assuming $t = 0$, the values of l_0 for males and females are both 100000. The probability that a male individual aged zero will survive until the age of t years, denoted by ${}_tp_0$, is

$${}_tp_x^C = (p_x)^t$$
$${}_tp_0^C = 0.995571^t$$

Meanwhile, for females, it is obtained

$${}_tp_x^C = (p_x)^t$$
$${}_tp_0^C = 0.996058^t$$

Thus, the probability that someone who is newborn will survive until the age of t years, with $0 \le t \le 1$, for males is $(0.995571)^t$, and for females, it is $(0.996058)^t$. Assuming $t = 1$, the values of p_0 for males and females are 0.995571 and 0.996058, respectively. The probability that a male individual aged zero will die within t years, denoted by ${}_tq_0$, is

$${}_tq_x^C = 1 - {}_tp_x^C$$
$${}_tq_0^C = 1 - (0.995571)^t$$

Meanwhile, for females, it is obtained

$${}_tq_0^C = 1 - (0.996058)^t$$

Thus, the probability that someone who is newborn will die before reaching the age of t years, with $0 \le t \le 1$, for males is $1 - (0.995571)^t$, and for females, it is $1 - (0.995571)^t$. Assuming $t = 1$, the values of q_0 for males and females are 0.00443 and 0.003942, respectively.

The values l_x^h, p_x^h, and q_x^h in the mortality table are calculated using the quadratic fractional age assumption by first determining the values of B_x for males and females as shown in Table 1.

Tabel 1. B_x Value for Male and Female

Male		Female	
x	B_x	x	B_x
0	775.0493	0	557.5784
1	110.8507	1	230.8416
2	215.6965	2	75.96419
⋮	⋮	⋮	⋮
98	193.9868	98	277.595
99	0	99	317.1599

The value of Q_x for males aged zero is

$$Q_x = \frac{B_x}{l_x}$$

$$Q_0 = \frac{775.049347222451}{100000} = 0.00775$$

Meanwhile, for female individuals aged zero, it is obtained

$$Q_x = \frac{B_x}{l_x}$$

$$Q_0 = \frac{557.578439030534}{100000} = 0.005576$$

Thus, the obtained values of Q_x for male individuals aged zero are 0.00775, and for females, it is 0.005576. After obtaining the values of B_x and Q_x, the number of individuals at age x can be calculated. The number of male individuals aged $0 + t$ years, denoted by l_x, is

$$l_{x+t}^h = l_x - B_x t + (B_x - d_x)t^2$$

$$l_{0+t}^h = l_0 - B_0 t + (B_0 - d_0)t^2$$

$$l_{0+t}^h = 332{,}099347t^2 - 775{,}049347222451t + 100000$$

Meanwhile, for females, it is obtained

$$l_{x+t}^h = l_x - B_x t + (B_x - d_x)t^2$$

$$l_{0+t}^h = l_0 - B_0 t + (B_0 - d_0)t^2$$

$$l_{0+t}^h = 163{,}368439t^2 - 557{,}578439030534t + 100000$$

Thus, the number of individuals aged $0 + t$ years with $0 \leq t \leq 1$ for males is $332.099347t^2 - 775.049347222451t + 100000$, and for females, it is $163.368439t^2 - 557.578439030534t + 100000$.

Assuming $t = 0$, the values of l_0 for males and females are both 100000. The probability that a newborn male individual will die before reaching the age of t years, ${}_tq_0$, is

$${}_tq_x^h = \left(t - \frac{t^2}{2}\right) Q_x + \frac{t^2}{2} p_x Q_{x+1}$$

$${}_tq_0^h = \left(t - \frac{t^2}{2}\right) Q_0 + \frac{t^2}{2} p_0 Q_1$$

$${}_tq_0^h = 0.007750493t - 0.003320993t^2$$

Meanwhile, for females, it is obtained

$${}_tq_0^h = 0.005575784t - 0.001633684t^2$$

Thus, the probability that someone who is newborn will die before reaching the age of t years, with $0 \leq t \leq 1$, for males is $0.007750493t - 0.003320993t^2$, and for females, it is $0.005575784t - 0001633684t^2$. Assuming $t = 1$, the values of q_0 for males and females are 0.00524 and 0.00266, respectively. The probability that a newborn male individual will survive until t years ahead, ${}_tp_0$, is

$${}_tp_x^h = 1 - \left(\left(t - \frac{t^2}{2}\right) Q_x + \frac{t^2}{2} p_x Q_{x+1}\right)$$

$${}_tp_0^h = 1 - \left(\left(t - \frac{t^2}{2}\right) Q_0 + \frac{t^2}{2} p_0 Q_1\right)$$

$${}_tp_0^h = 1 - 0.00775049t + 0.003320993t^2$$

Meanwhile, for females, it is obtained

$${}_tp_0^h = 1 - 0.005575784t + 0.00163368t^2$$

Thus, the probability that someone who is newborn will survive until the age of t years, with $0 \leq t \leq 1$, for males is $1 - 0{,}00775049t + 0{,}003320993t^2$, and for females, it is $1 - 0{,}005575784t + 0{,}00163368t^2$. Assuming $t = 1$, the values of p_0 for males and females are 0.99557 and 0.996058, respectively.

Application of UDD, CFM, and Quadratic Fractional Age Assumption in Semi-Continuous Endowment Insurance

The simulation of calculating the premium for a semi-continuous endowment life insurance contract with a contract duration of 30 years, a constant interest rate of 6%, and a continuous interest rate of 0.058268908. A benefit of 1 unit is provided if the insured passes away during the contract period, with the probability of death calculated using UDD, CFM, and the quadratic fractional age assumption. If the insured survives until the end of the contract, a payment of 1 unit will be provided by the insurance company. The insured individuals are aged between 18 and 58 years for both males and females.

Monthly Benefits of Endowment Insurance with UDD Assumption

The endowment insurance premium with monthly payments using the UDD assumption involves calculating the net single premium for a 30-year protection period and an initial annuity with monthly premium payments. The net single premium for an 18-year-old male individual, denoted by $\bar{A}_{18:\overline{30|}}$ is

$$\bar{A}_{18:\overline{30|}} = \bar{A}^{1}_{18:\overline{30|}} + A_{18:\overline{30|}}^{\ \ 1}$$
$$= \sum_{k=0}^{29} e^{-k\delta} {}_{k}p_{18}^{U} \int_0^1 e^{-\delta s} q_{18+k}^{U} ds + v^{30}\ {}_{30}p_{18}$$
$$= \sum_{k=0}^{29} e^{-k\delta}\ {}_{k}p_{18}^{U} q_{18+k}^{U}. 0.971423 + v^{30}\ {}_{k}p_{18}$$
$$= 0.1928170424$$

Meanwhile, the initial annuity with monthly premium payments $\ddot{a}^{(12)}_{18:\overline{30|}}$ is

$$\ddot{a}^{(12)}_{18:\overline{30|}} = \frac{id}{i^{(12)}d^{(12)}} \ddot{a}_{18:\overline{30|}} - \frac{i-i^{(12)}}{i^{(12)}d^{(12)}} (1 - {}_{30}E_{18})$$
$$= 13.88778429$$

The endowment insurance premium with monthly payments is

$$\frac{P^{(12)}}{12}(\bar{A}_{18:\overline{30|}}) = \frac{\bar{A}_{18:\overline{30|}}}{12\ddot{a}^{(12)}_{18:\overline{30|}}}$$
$$= 0.00115699427$$

Thus, the obtained endowment insurance premium with monthly payments using the UDD assumption is 0.00115699427 units.

Monthly Premium for Endowment Insurance Using the CFM Assumption

The net single premium for a 30-year protection endowment insurance and the initial annuity with monthly premium payments for an 18-year-old male, denoted by $\bar{A}_{18:\overline{30|}}$ is

$$\bar{A}_{18:\overline{30|}} = \bar{A}^{1}_{18:\overline{30|}} + A_{18:\overline{30|}}^{\ \ 1}$$
$$= \sum_{k=0}^{29} e^{-k\delta}\ {}_{k}p_{18}^{C} \int_0^1 e^{-s\delta} (p_{18+k}^{C})^{s} (-\ln(p_{18+k}^{C}))\, ds + v^{30}\ {}_{30}p_{18}$$
$$= \sum_{k=0}^{29} e^{-k\delta}\ {}_{k}p_{18}^{C} (-\ln(p_{18+k}^{C})) \frac{\left(e^{\ln(p_{18+k}^{C})-\delta} - 1\right)}{\ln(p_{18+k}^{C})-\delta} + v^{30}\ {}_{30}p_{18}$$
$$= 0.19281751$$

Meanwhile, the initial annuity with monthly premium payments, denoted by $\ddot{a}^{(12)}_{18:\overline{30|}}$, is

$$\ddot{a}^{(12)}_{18:\overline{30|}} = \frac{id}{i^{(12)}d^{(12)}} \ddot{a}_{18:\overline{30|}} - \frac{i-i^{(12)}}{i^{(12)}d^{(12)}} (1 - {}_{30}E_{18})$$
$$= 13.88778429$$

Next, the calculation of the endowment insurance premium with monthly payments is

$$\frac{P^{(12)}}{12}(\bar{A}_{18:\overline{30|}}) = \frac{\bar{A}_{18:\overline{30|}}}{12\ddot{a}^{(12)}_{18:\overline{30|}}}$$
$$= 0.00115699708$$

Thus, the obtained endowment insurance premium with monthly payments using the constant force of mortality assumption is 0.00115699708 units.

The Monthly Premium for Endowment Insurance Using the Quadratic Fractional Age Assumption

The net single premium for a 30-year protection endowment insurance and the initial annuity with monthly premium payments for an 18-year-old male, denoted by $\bar{A}_{18:\overline{30|}}$ is

$$\bar{A}_{18:\overline{30|}} = \bar{A}^{1}_{18:\overline{30|}} + A_{18:\overline{30|}}^{\ \ 1}$$
$$= \sum_{k=0}^{29} e^{-k\delta}\ {}_{k}p_{18}^{h} \int_0^1 e^{-\delta s} (1-s) Q_{18+k}^{h} + s\left(p_{18+k}^{h} Q_{18+k}^{h}\right) ds + v^{30}\ {}_{30}p_{18}$$

$$= \sum_{k=0}^{n-1} e^{-k\delta} {}_{k}p_{18}^{h} \left[\frac{\delta Q_{18+k}^{h} + p_{18+k}^{h} Q_{19+k}^{h} - Q_{18+k}^{h}}{\delta^2} - \frac{e^{-\delta}\left(\delta p_{18+k}^{h} Q_{19+k}^{h} + p_{18+k}^{h} Q_{19+k}^{h} - Q_{18+k}^{h}\right)}{\delta^2} \right]$$

$$= 0.192813013$$

Meanwhile, the initial annuity with monthly premium payments, denoted by $\ddot{a}_{18:\overline{30|}}^{(12)}$, is

$$\ddot{a}_{18:\overline{30|}}^{(12)} = \frac{id}{i^{(12)}d^{(12)}} \ddot{a}_{18:\overline{30|}} - \frac{i - i^{(12)}}{i^{(12)}d^{(12)}} (1 - {}_{30}E_{18})$$

$$= 13.88778429$$

Thus, the endowment insurance premium with monthly payments is

$$\frac{P^{(12)}}{12} (\bar{A}_{18:\overline{30|}}) = \frac{\bar{A}_{18:\overline{30|}}}{12\ddot{a}_{18:\overline{30|}}^{(12)}}$$

$$= \ 0.00115697009$$

Thus, the endowment insurance premium with monthly payments using the quadratic fractional age assumption is 0.00115697009 units.

Monthly Premium for Males with UDD, CFM, and Quadratic Fractional Age Assumption

The monthly premiums using the quadratic fractional age assumption, UDD, and CFM for males aged 18 to 58 years can be seen in Table 2.

Table 2. Endowment Life Insurance Premiums with Monthly Payments Using the Quadratic Fractional Age Assumption, UDD, and CFM Assumption for Males

x	UDD	CFM	Quadratic
18	0.001156994	0.001156997	0.001156970
19	0.00116158	0.001161583	0.001161519
20	0.001166565	0.001166568	0.001166532
21	0.001172051	0.001172055	0.001171981
...	...	...	...
...	...	...	...
56	0.003164208	0.003165025	0.003163785
57	0.003344177	0.003345097	0.003343723
58	0.003537584	0.003538619	0.003537211

Based on Table 2, the differences in monthly premium values using the quadratic fractional age assumption, UDD, and CFM for males from ages 18 to 58 years are illustrated in Figure 3.

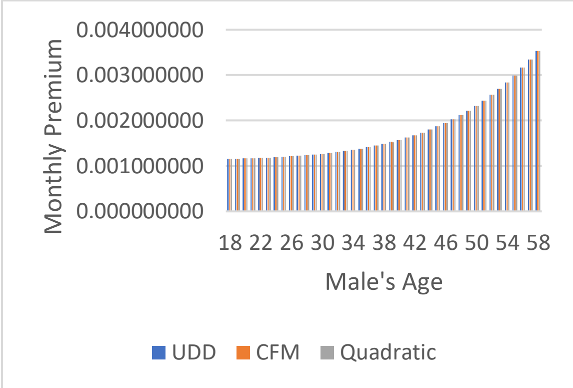

Figure 3. Endowment Insurance Monthly Premiums for Males

The present value of premiums for ages 18 to 58 years with the quadratic fractional age assumption, UDD, and CFM assumption consistently increases with the insured individual's age. As the insured's age increases, the differences in premium amounts between the three assumptions also increase.

Monthly Premiums for Females Using UDD, CFM, and Quadratic Fractional Age Assumption

Table 3. Monthly Endowment Life Insurance Premiums Using the Quadratic Fractional Age Assumption, UDD, and CFM Assumption for Females

x	UDD	CFM	Quadratic
18	0.001144420	0.001144422	0.001144372
19	0.001148517	0.001148519	0.001148498
20	0.001152685	0.001152688	0.001152632
21	0.001156994	0.001156997	0.001156970
...	...	...	...
...	...	...	...
56	0.002697717	0.002698277	0.002697217
57	0.002841688	0.002842326	0.002841240
58	0.002996946	0.002997668	0.002996458

According to Table 3, the differences in monthly premium values using the quadratic fractional age assumption, UDD, and CFM for females are illustrated in Figure 4.

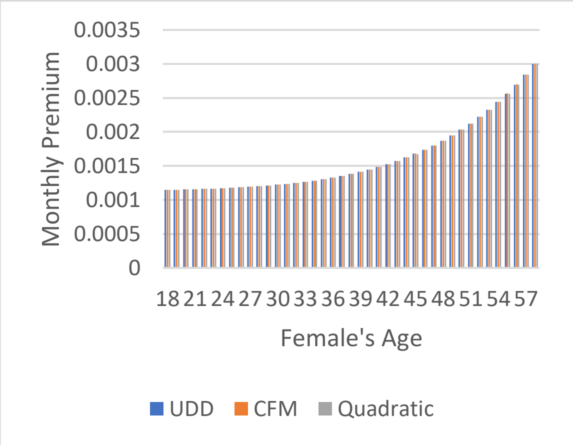

Figure 4. Endowment Insurance Monthly Premiums for Females

Based on Table 3 and Figure 4, it is observed that the premiums for ages 18 to 58 years with the quadratic fractional age assumption, UDD, and CFM assumptions for females consistently increase with the insured individual's age. Furthermore, as the insured's age increases, the differences in premium amounts between the three assumptions also increase.

CONCLUSIONS AND SUGGESTIONS

Based on this research, it can be concluded that among the three fractional age assumptions used, continuity occurs only in calculations using the quadratic fractional age assumption, while for the UDD and CFM assumptions, discontinuity occurs at integer ages. The calculation of semi-continuous endowment insurance premiums with monthly payments determined by the insured individual's age when taking out the insurance indicates that the monthly premiums for male insured individuals are higher compared to those for female insured individuals. The magnitude of monthly premium payments consistently increases with the insured individual's age when taking out the insurance. Based on the assumptions used, the magnitude of premium payments using the quadratic fractional age assumption, UDD, and CFM assumptions is almost the same. As the insured individual's age increases, the differences in premium payments between the three assumptions also increase. When examining the premiums from ages 15 to 58, calculations using the quadratic fractional age assumption result in the lowest premiums, while the CFM assumption leads to the highest premiums.

In further research, the calculation of monthly premiums can be replaced with quarterly or semi-annual calculations using different fractional age assumptions such as Fractional Independence (FI), Linear Force of Mortality (LFM), and various types of insurance such as whole life insurance and term life insurance.

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