Contents lists available at DJM

DESIMAL: JURNAL MATEMATIKA

p-ISSN: 2613-9073 (print), e-ISSN: 2613-9081 (online), DOI 10.24042/djm

http://ejournal.radenintan.ac.id/index.php/desimal/index

The impact of discrete mathematics lectures on students' deductive reasoning: The case of graph theory learning

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ARTICLE INFO

Article History

Received : 24-01-2024
Revised : 15-02-2024
Accepted : 26-02-2024
Published : 30-04-2024

Keywords:
Deductive Reasoning; Graph Theory; Four Color Coloring Theorem; Discrete mathematics.

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Doi: 10.24042/djm.v7i1.21014

ABSTRACT

This research aims to investigate the impact of the discrete mathematics lecture process on deductive reasoning abilities, especially in carrying out proofs in graph theory. This research uses qualitative research based on the results of observations of the learning process in the Discrete Mathematics course. The subjects of this research were three students from the Mathematics Education Study Program who were pre-service teachers and in-service mathematics teachers at a university in Malang. The data collection technique is a description and observation test question sheet. The research results show that the process of learning graph theory in discrete courses is through explanations related to the definition of graph coloring, examples of the application of theorems, practice questions through guided proof with lecturers, and the application of graph coloring theory in everyday life. Overall, the graph theory learning process emphasizes deductive reasoning abilities. Based on the results of the analysis of the four-color theorem test, in general, it succeeded in having a good impact on students in the mathematics education study program.

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INTRODUCTION

Discrete mathematics is one of the courses studied by mathematics education students. The problems in discrete mathematics courses are contextual (Nasution, Lubis, & Firdaus, 2020). The discrete mathematics course contains graph theory material. This course is a mathematical foundation for other courses. In addition, discrete mathematics is related to prerequisite material for subsequent courses. Discrete mathematics exists to train abstract thinking processes, analysis, and logical thinking critically and rationally (Sugiharni, 2018). So, discrete mathematics gives rise to high-level analysis and logical reasoning skills.

The logical process goes through the stages of abstraction, statement sentences, the logical process, and reasoning

(Wahyuni, Roza, & Maimunah, 2019). Logic and Reasoning are two things that are interrelated in presenting arguments: the accuracy of thinking and the process of drawing conclusions. Every individual needs to go through a reasoning process to solve a problem (Kusumaningtyas, Parta, & Susanto, 2021). Mathematical Reasoning is the ability of students to verify arguments with information that has been collected and ends with a conclusion (Saleh, Prahmana, Isa, & Murni, 2018). This follows the discrete mathematics learning process, where the abilities needed to learn graph material are reasoning and logic in solving existing problems.

Mathematical Reasoning is the development of thought patterns and the process of expressing thought patterns in a problem (Sari & Darhim, 2020). Mathematical Reasoning is one of the abilities that students must have in the mathematics learning process (Asdarina & Ridha, 2020). Thus, mathematical reasoning abilities must always be integrated into every mathematics lesson. It can be concluded that Reasoning is a high-level thinking process that requires logic through an abstraction process in problem-solving chess.

Deductive Reasoning is the process of reaching a logical conclusion from several general statements. Deductive Reasoning runs from general things to specific things. Mathematics is deductive Reasoning because mathematics is structured and organized (Afandi & Angkotasan, 2021). The stages in the reasoning process include (1) carrying out a calculation process based on applicable formulas; (2) drawing conclusions based on logical Reasoning; (3) compiling a process of direct, indirect proof, and mathematical induction; and (4) building an analysis and synthesis based on several cases (Fadillah, 2019). In conclusion, deductive Reasoning is a reasoning process to draw conclusions based on agreed-upon rules or theories.

Indicators of deductive Reasoning involves (1) compiling direct and indirect proof using mathematical induction, (2) providing validity of arguments, (3) drawing logical conclusions, (4) providing explanations regarding models, facts, properties, and relationships, (5) compiling valid arguments, (6) providing answers and solution processes, (7) analyzing patterns and relationships in mathematical situations, (8) compiling and studying conjectures, and (9) formulating opponents following the rules of inference. This research only applies three indicators, namely: (1) carrying out a calculation process based on specific formulas and rules; (2) compiling direct or indirect evidence; and (3) drawing logical conclusions.

These three indicators were chosen because they could represent the overall indicators of deductive reasoning. The calculation process based on existing rules includes explanations related to facts, conjecture analysis, and relationship analysis. Then, indicators of direct or indirect evidence include the preparation of arguments and the problem-solving process. Then, indicators for concluding include valid and logical answers from the results of solving problems.

Previous existing research only discussed deductive reasoning with students in schools. The research conducted discusses the deductive reasoning of pre-service and in-service mathematics teachers. This research links mathematical deductive reasoning with learning graph theory in discrete mathematics courses, which has yet to be widely discussed in existing research.

Graph theory is a discrete mathematics topic studied in the observation process. Graph theory is the relationship between discrete objects and their representation (Oktaviana & Abdillah, 2021). Graph theory studies graph elements, types of graphs, graph representation, isomorphic graphs, Eulerian and Hamiltonian paths or circuits, shortest paths, planar graphs, plane graphs, and graph coloring. This research focuses on graph coloring material, namely assigning a color to each point on a graph so that no two neighboring points are given the same color. Especially in the four-color theorem, "the chromatic number in a planar graph is not more than 4" (Jofie, Bahri, & Iqbal Baqi, 2021). This research aims to investigate the impact of the discrete mathematics lecture process on deductive reasoning abilities, especially in carrying out proofs in graph theory.

METHOD

This research uses qualitative research based on the results of observations of the learning process in the Discrete Mathematics course. The subjects of this research were three students from the Mathematics Education Study Program who were one subject pre-service and two subjects in-service mathematics teachers at a university in Malang. The sampling technique uses *nonprobability sampling, namely a purposive* technique, because each individual does not have the same opportunity but is selected based on specific considerations. Research participants are students who have completed discrete mathematics courses. The data collection technique is a reasoning test question.

This research flow was carried out in two stages. The first stage is observation during lectures, carried out offline for three credits on graph material, especially graph coloring. The lecture process starts with an explanation of graph coloring, practice questions, and examples of the application of graph coloring in everyday life. At this stage, researchers recorded the flow of the learning process, documented learning, collected teaching materials, and collected student lecture notes. The second stage is an assessment carried out in the form of individual tests for students. Students are asked to take an individual test in the form of one test question related to proving the four-color theorem on planar graphs. This assessment form is carried out because it assumes students already understand graph coloring and its application. Students are asked to write answers describing the entire series of proof processes. The answer sheets that have been collected will go through the data analysis stage.

Information:
Please answer the complete question based on the definition of graph coloring and the four-color theorem of planar graphs. Complete the work with the proof process and completion steps.

Question:
Make a color of the planar graph shown with this algorithm, calculate the vertex chromatic number, and prove the four-color theorem of planar graphs!

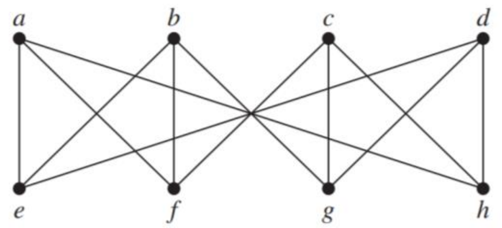
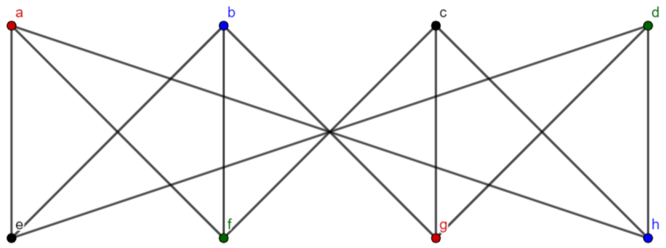


Table 1. Deductive Reasoning Indicators

No.	Deductive Reasoning Indicators	Answer Description
1	Carry out the calculation process based on specific formulas and rules	It is known that the definition of graph coloring includes: Definition 1: Simple graph coloring assigns a color to each vertex so that no two neighboring vertices have the same color. Definition 2: The chromatic number in a graph is the minimum number of colors needed to color a graph. The chromatic number of a graph G is denoted by $\chi(G)$. Then, the four-color theorem is related to graph coloring. Four-color theorem: The chromatic number in a planar graph is not more than 4.
2	Compile direct or indirect evidence	The proof is prepared based on the given planar graph. The maximum chromatic number in a planar graph G is four colors because vertices a, e, f, and h must be different colors because they are neighbors according to the definition of graph coloring. To see that, graph G will be colored with four colors. Node A is red. Node e is black. Node f is green. The h node is blue. In such a way, node b cannot be black and green because the nodes are neighbors. So, node b is colored blue because it is not adjacent to node h. Then, node g cannot be black, green, or blue because the nodes are neighbors. So, node g is red because it is not adjacent to node a. Then, node c cannot be green, red, or blue because the nodes are neighbors. So, node c is colored black because it is not adjacent to node e. Then, node d cannot be black, red, or blue because the nodes are neighbors. So, node d is colored green because it is not adjacent to node f. 
3	Drawing logical conclusions	It can be proven that the planar graph G uses no more than four colors, namely red, blue, green, and black. The chromatic number graph G is$\chi(G) = 4$.

Data analysis based on deductive reasoning indicators will follow the test results and observations. The test results are described based on evidentiary findings and student difficulties. The data validity checking technique uses triangulation techniques. The data analysis technique consists of three stages: 1) reducing data, sorting and organizing information; 2) presenting data and attaching the collected data; 3) drawing conclusions and verifying based on the study results.

RESULTS AND DISCUSSION

First stage: Learning the process of graph coloring theorem

Before the learning process, the lecturer provided material and video explanations related to the topic of discussion that would be studied. The learning process in discrete mathematics courses for graph coloring topics consists of several stages. The first stage is when the lecturer explains the definition of graph coloring. The definitions of graph coloring are as follows:

Definition 1. Simple graph coloring assigns a color to each vertex so that no two neighboring vertices have the same color.

Definition 2. The chromatic number in a graph is the minimum number of colors needed to color a graph. The chromatic number of a graph G is denoted by $\chi(G)$.

After the lecturer explained the definition of graph coloring, he continued with practice questions related to types of graph coloring. There are three types of graph coloring: vertex coloring, edge coloring, and plane coloring. All types of graph coloring apply the same theory, according to the definition. While working on practice questions, the lecturer is open and allows students to see if there is something they need help understanding. The lecturer guides the process of working on practice questions so that students can understand the concept of graph coloring. While explaining and practicing questions, several students took notes on the lecture process.

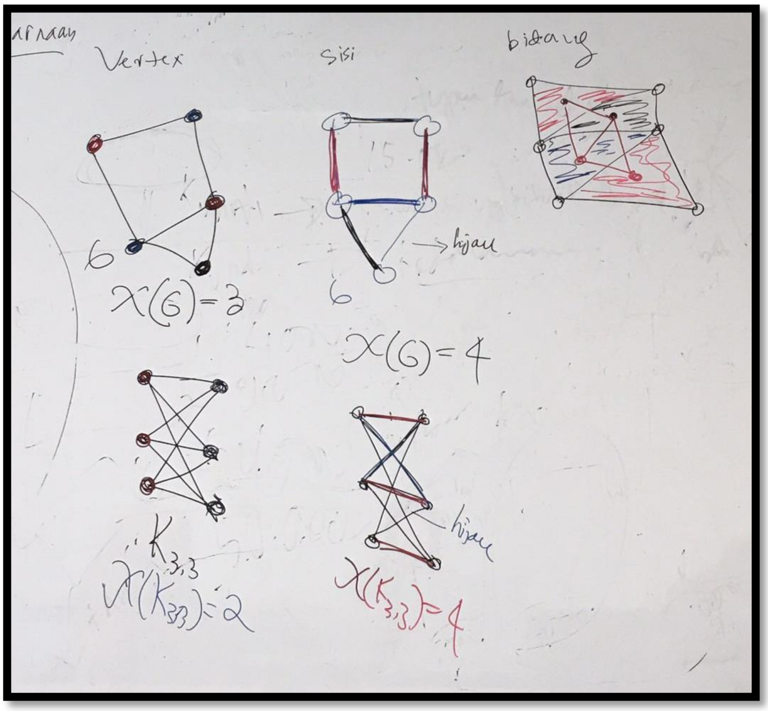


Figure 1. Graph Coloring Theorem Practice Questions

After working on the practice questions, the lecturer explained the application of graph coloring theory in everyday life. The graph-coloring application prepares a schedule and coloring map areas and is related to the primary colors used in human life. Then, the lecture process continues with independent study based on the module teaching materials, PowerPoint, and learning videos provided previously. One of the theorems related to graph coloring is the four-color theorem.

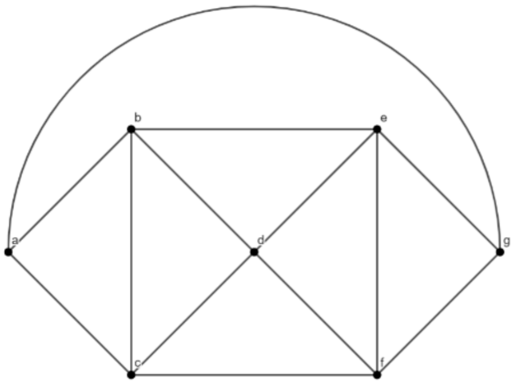
Four-color Theorem. The chromatic number in a planar graph is not more than 4 (Jofie et al., 2021).

The module explains that the four-color theorem began to appear in the 1850s. The proof of the four-color theorem is done by proving that the four-color theorem is wrong using around 2000 different types of planar graphs. Then, it can be proven that there is nothing wrong with all these types. So, it can be proven that the four-color theorem is true.

Proving the four-color theorem took more than 1000 hours by relying on computers.

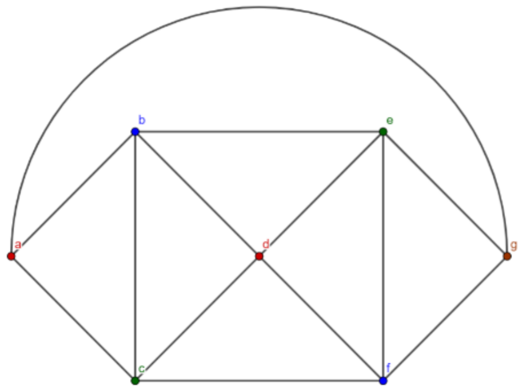
It should be noted that the four-color theorem only applies to planar graphs. So, to determine the chromatic number and prove the four-color theorem, you need to know first that the graph being tested is planar. Examples of practice questions on the Four-color Theorem will be written below.

Prove the four-color theorem on graph G below:



Proving process:
Based on the definition of a planar graph, it can be proven that graph G is a planar graph. It is said to be a planar graph because there are no intersecting edges.
The maximum chromatic number in graph G is four because vertices a, b, c, and g must be different colors because they are neighbors according to the definition of graph coloring. To see that, graph G will be colored with four colors.
Node A is red.
Node b is blue.
Node c is green.
Node g is brown.
In such a way, node d cannot be blue and green because the nodes are neighbors. So, node d is red because it is not adjacent to node a.
Then, node e cannot be blue, red, or brown because the nodes are neighbors. So, node e is colored green.
Then, node f cannot be green, red, or brown because the nodes are neighbors. So, node f is colored blue.

It can be proven that the planar graph G uses no more than four colors, namely red, blue, green, and brown. The chromatic number of graph G is $\chi(G) = 4$.



In the discrete mathematics lecture process, lecturers emphasize the proof aspect based on definitions, theorems, or mathematical statements. So, learning discrete mathematics, especially graph theory, requires aspects of deductive Reasoning. Judging from the deductive aspect of Reasoning, students need to connect previous factual statements to a conclusion.

Second stage: Analysis of test results

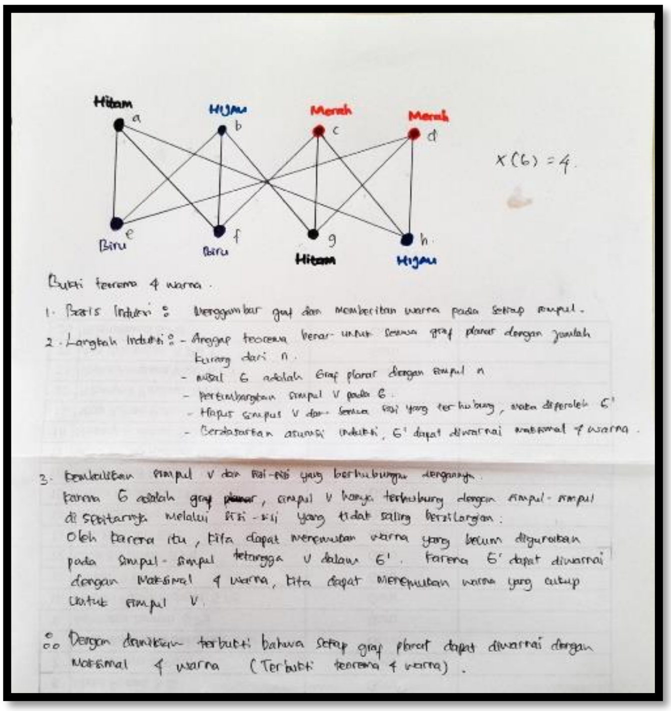


Figure 1. First Subject Test Results

Judging from the results of the analysis of the description test of the first research subject, it is known that the first subject successfully fulfilled the deductive reasoning indicators correctly. The first subject understands the proof process that meets the first indicator: a calculation process based on specific formulas and rules. Furthermore, the indicators for preparing direct or indirect evidence have also been explained coherently. In the end, it is completed by drawing appropriate logical conclusions based on the evidence that has been collected.

Based on the results of the description, it is known that the first subject can understand, apply, and analyze graph coloring theory. The proof process carried out provides a coherent explanation. It starts by drawing a planar graph. Then, it continues with the proof process, which provides the correct conclusion. So, the first subject has reached the stage of high-level thinking.

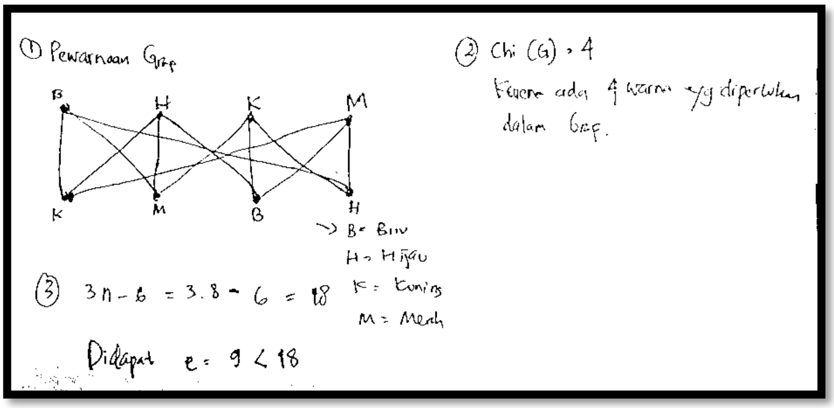


Figure 3. Second Subject Test Results

Judging from the results of the analysis of the description test of the second research subject, it is known that the second subject succeeded in fulfilling the deductive reasoning indicators correctly. The second subject understands the first indicator by calculating it based on specific formulas and rules. The second subject also succeeded in compiling evidence. At the end of the research, the subjects drew the correct conclusions.

The proof process found a different strategy in the second subject. The second subject completes the proof using the Euler inequality formula. Euler's inequality is used to prove the planarity of a graph. This flexibility is unexpected because the subject proves the planarity of graphs to strengthen the graph coloring theorem.

Euler inequality proof of planar graph: $e \leq 3n - 6$

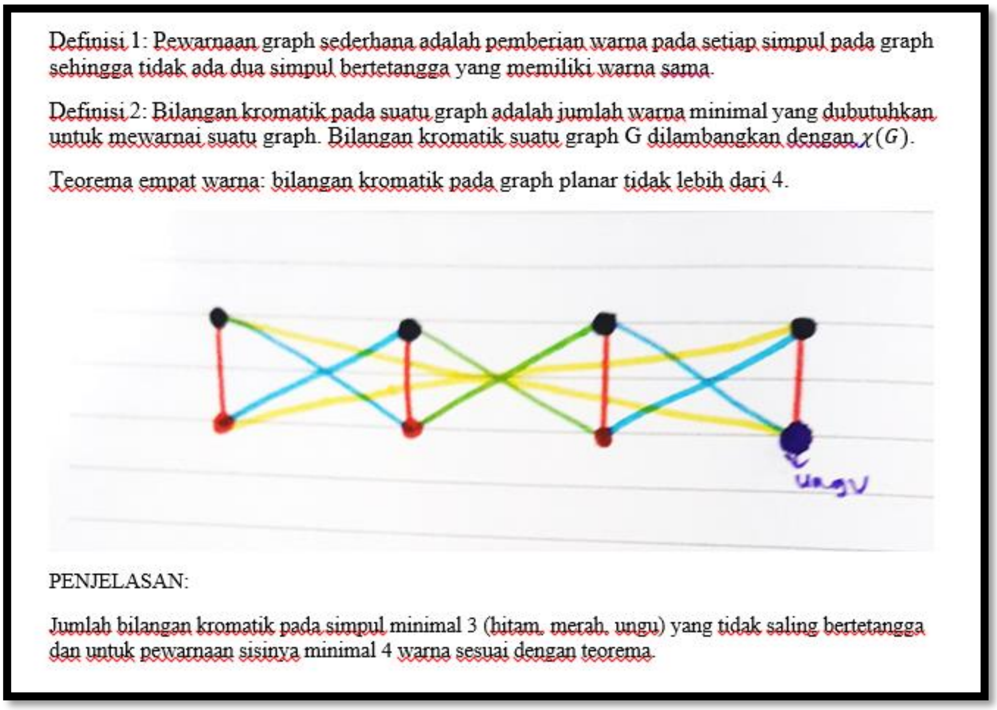


Figure 4. Third Subject Test Results

Judging from the analysis results of the third research subject's description test, it is known that the third subject could only fulfill one of the three indicators of deductive Reasoning correctly. The third subject understands the first indicator, namely, carrying out a calculation process based on definitions and related theorems. However, it could be more precise in proving and drawing conclusions. The third subject carries out the proof process on the vertices and edges. The chromatic number obtained from vertex coloring is three colors, and the side coloring is four. This is unsuitable because the four-color theorem must prove that the chromatic number in a planar graph is, at most, four colors.

Based on the results of the written description test analysis related to the Four-color Theorem, it is presented in Table 2.

Table 2. Results of Deductive Reasoning Indicator Analysis

Deductive Reasoning Indicators	Subject 1	Subject 2	Subject 3
Carry out the calculation process based on specific formulas and rules	√	√	√
Compile direct or indirect evidence	√	√	
Draw logical conclusions	√	√	

Table 2 shows that of the three students who took the test, two met all the deductive reasoning indicators, and one could only meet one. The first subject successfully carried out the proof process through deductive Reasoning. The second subject also successfully carried out the proof process through deductive Reasoning. Meanwhile, the third subject only succeeded in the first indicator, namely, carrying out the calculation process based on specific rules, but preparing the proof needed to be more precise.

The first and third subjects are in-service mathematics teachers. The second subject is a pre-service mathematics teacher. As seen from the results, the first and second subjects fulfilled all the indicators of deductive reasoning. There is no difference between teachers who have taught or have not taught mathematics. However, the crucial difference is that the third subject is an in-service mathematics teacher, but it has yet to be able to meet the deductive reasoning indicators. These results indicate that the learning process in discrete mathematics courses, using a deductive reasoning process, is generally successful in impacting students in mathematics education study programs.

To maintain teacher quality, pre-service and in-service teachers must be equipped with the main competencies, including pedagogical, professional, social, and personality competencies (Gultom & Mampouw, 2019). Deductive reasoning is one of the mathematical abilities most students need today (Aprisal & Arifin, 2020). According to NCTM (2000), the ability to reason is the goal of mathematics learning. Reasoning cannot be separated from mathematics because Reasoning is needed to understand mathematics. Through Reasoning, academic abilities can be improved. This is supported by research results that explain that the mathematics learning process, namely graph theory, impacts students' deductive reasoning. Learning graph theory facilitates students' ability to reason. So that the goal of mathematics learning, namely the development of reasoning abilities, can be achieved.

Reasoning aims to draw conclusions based on evidence that has been found (NCTM, 2000). Students' conclusions are formed from knowledge and the application of previously applicable rules. Specifically, Reasoning is a research process to obtain an appropriate conclusion. Reasoning is used to solve problems because it requires logical thinking and clear thinking to get results (Ardiansyah, Wahyuningrum, & Rumanta, 2022).

Efficiency and correctness in solving problems are influenced by Reasoning. This is done through the verification process carried out by the three research subjects. The deeper the reasoning process is carried out, the closer you are to the truth in solving the problem. The deductive reasoning process carried out by research subjects leads them to the correct conclusions. According to (Fadillah, 2019), mathematics is a reasonable and logical study, it requires understanding, proving, and evaluating. Reasoning is the foundation for acquiring knowledge and plays a vital role in solving problems (Octaviyunas & Ekayanti, 2019).

CONCLUSIONS AND SUGGESTIONS

It was concluded that the process of learning graph theory in discrete courses was through explanations related to the definition of graph coloring, examples of the application of theorems, practice questions through guided proof with lecturers, and the application of graph coloring theory in everyday life. Overall, the graph theory learning process emphasizes deductive reasoning abilities. This deductive Reasoning is needed when proving theorems and applying them to practice problems. Furthermore, it was concluded from the analysis of the four colors that they generally succeeded in impacting students in the mathematics education study program.

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