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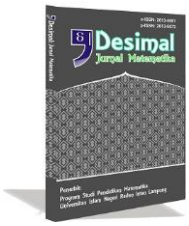

A comparison benefit reserves of an n–year term life insurance between using the vasicek model and cox-ingersoll-ross model

Isti Kamila*, Ani Andriyati, Embay Rohaeti

Universitas Pakuan, Indonesia

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*Correspondence: E-mail: mila.istikamila@gmail.com

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ABSTRACT

The purpose of this study was to determine benefit reserves with the premium sufficiency method that would be applied to types of term insurance with non-constant interest rates by using the Vasicek and Cox-Ingersol-Ross models. The novelty of this study is in determining term insurance benefit reserves by comparing benefit reserves using the Vasicek and Cox-Ingersol-Ross interest rate models so that it can be a decision-maker for insurance companies. The stages of this research activity started by estimating the parameters for the Vasicek and Cox-Ingersoll-Ross (CIR) interest rates. The next step was to estimate the interest rate on Vasicek and CIR. Then, estimate the reserve value of term insurance benefits using the premium sufficiency method with non-constant interest rates. At the final stage, the results of the reserve value of benefits with non-constant interest rates and CIR would be interpreted and compared. The results of the research obtained by the CIR interest rate were always positive, and the difference in interest rates between each time was not too large compared to Vasicek. This was in line with the reserve results of the benefits obtained. The difference in benefit reserves between times using Vasicek was greater than that of the CIR interest rate model.

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INTRODUCTION

Benefit reserve is the amount of money saved by the insurance company in the form of the difference in the cash value of benefits and premiums to be returned to policyholders and other unexpected payments such as claims beyond estimates and termination of premium payments by policyholders (Dewi, Setyahadewi, & Sulistianingsih, 2013; Oktavian, Devianto, & Yanuar, 2014). Effectively managing uncertainty and complexity in the setting of insurance premium reserves poses new challenges for decision-makers and policymakers (Li, Pantelous, & Yang,

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2020). Therefore, determining the right and efficient benefit reserve is an important thing for insurance companies to do in order to carry out their liabilities.

The methods used in determining benefit reserves are prospective and retrospective methods (Iriana & Nasution, 2019). There are several methods that are the development of these two methods, namely the Zillmer method, the Canadian method, the New Jersey method, the Illinois method, the Fackler method, and the Premium method (Mashitah, Satyahadewi, & Mara, 2013; Nurrohmah & Rohaeni, 2022; Rohaeni, 2007; Sari & Darma Ekawati, 2022; Vikrantha, Devianto, & HG, 2019; Widuri & Sari, 2023).

In previous studies, the determination of benefit reserves using the premium sufficiency method has been carried out by Aprijon (2020), who found that benefit reserves increase from the beginning of the year to the 8th year, while after the 8th year, the reserves begin to slow down to reach 0. In another study conducted by Widuri & Sari (2023), using the premium sufficiency and gross premium methods can provide in detail all the costs needed along with the amount of reserves that must be provided by insurance companies. Both studies used constant interest rates, even though interest rates fluctuated close to 2% for more than a century (Negro, Giannone, Giannoni, & Tambalotti, 2019).

In another study, the determination of benefit reserves with the Vasicek interest rate model used two methods, namely the premium sufficiency method and the Zillmer method (Rachman & Pertiwi, 2022). However, this study uses the Vasicek interest rate model, which allows negative interest rate results. This is contrary to the characteristics of interest rates, which are always positive. Therefore, an interest rate model that always produces positive interest rates is needed, namely Cox-Ingersoll-Ross. Therefore, in this study, the problem of determining benefit reserves will be carried out with the premium sufficiency method, which will be applied to the type of term insurance with non-constant interest rates. Non-constant interest rates use the Vasicek and Cox-Ingersol-Ross (CIR) model. The CIR process is the most commonly used model to describe instantaneous dispersion or dynamic interest rates (Thamrongrat & Rujivan, 2022). Applying the CIR to fluctuations in interest may prevent interest rates from going negative (Jhwueng, 2020).

The novelty of this study is in determining term insurance benefit reserves by comparing benefit reserves using the Vasicek and Cox-Ingersol-Ross interest rate models. The benefit reserves generated from both models will be interpreted and compared so that they can be a decision-maker for insurance companies.

METHOD

The data used in this study is secondary data, namely Bank Indonesia's annual interest rate data from 2009 to 2023. The annual interest rate data for Bank Indonesia (BI) from 2009 to 2023 is contained in Table 1.

Table 1. BI Annual Interest Rates

Year	Interest Rates (r)
2009	0.0715
2010	0.065
2011	0.0658
2012	0.0577
2013	0.0648
2014	0.0754
2015	0.0752
2016	0.06
2017	0.0456
2018	0.0506
2019	0.0563
2020	0.0425
2021	0.0352
2022	0.04
2023	0.0581

In addition, this study also used data from the Indonesian Mortality Table (TMI)

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IV of 2019 (Asosiasi Asuransi Jiwa Indonesia (AAJI), 2019), data on two participants of BNI Life term life insurance products in Table 2, and BNI Life loading data, namely the new closing cost of insurance policies of 0.75% of compensation money and premium maintenance costs of 0.15% of compensation money.

Table 2. Data Participants of BNI Life

Name	Gender	Age	Registered	Benefit (Rp)
A	Male	27	2023	26,250,000
B	Female	32	2019	57,750,000

The data processing process is carried out in the following stages:

1. Estimating the parameters of the Vasicek and Cox-Ingersoll Ross interest rate models using the Ordinary Least Square (OLS) method with the following steps (Kamila & Andriyati, 2023).
   a. $U_1 = \sum_{t=1}^{n-1} r_t$ (1)
   b. $U_2 = \sum_{t=1}^{n-1} \frac{1}{r_t}$ (2)
   c. $U_3 = \sum_{t=1}^{n-1} \frac{r_t+1}{r_t}$ (3)
   d. $U_3 = \sum_{t=1}^{n-1} r_t + 1$ (4)
   e. Determining the value of kappa ($\hat{\kappa}$)
   $$\hat{\kappa} = \frac{n^2-2n+1+u_2\,u_3-u_1\,u_3-(n-1)u_4}{(n^2-2n+1-u_3)\Delta t} \quad (5)$$
   f. Determining the value of theta ($\hat{\theta}$)
   $$\hat{\theta} := \frac{(n-1)u_2-u_4u_1}{n^2-2n+1+u_2\,u_3-u_1u_3-(n-1)u_4} \quad (6)$$
   g. Determining the value of sigma ($\hat{\sigma}$)
   $$\hat{\sigma} = \sqrt{\frac{1}{n-2}\sum_{t=1}^{n-1}\left(\frac{r_{t+1}-r_t}{\sqrt{r_t}} - \frac{\theta}{\sqrt{r_t}} + \hat{\kappa}\sqrt{r_t}\right)} \quad (7)$$

Where:
$\hat{k}$: The speed r(t) returns towards $\hat{\theta}$
$\hat{\theta}$: long-term average of interest rates
$\hat{\sigma}$: volatility of interest rates
n: the number of interest rate data
r_t: t-th interest rate

2. Estimating interest rates with the Vasicek interest rate model.

The $\hat{k}$, $\hat{\theta}$, and $\hat{\sigma}$ parameters will then be used to simulate the interest rate values of the Vasicek model. Interest rate estimation uses Monte Carlo to generate standard normal random numbers ε_t. The Vasicek interest rate model is as follows:

$$r_t = r_{t-1} + \hat{k}(\hat{\theta} - r_{t-1})\Delta t + \hat{\sigma}\Delta t\varepsilon_t \quad (8)$$

where,
Δt: time interval
ε_t: normally distributed random variable with $\varepsilon \sim N(0,1)$
r_t: t-th interest rate

3. Estimating interest rates with the Cox-Ingersoll-Ross interest rate model.

The $\hat{k}$, $\hat{\theta}$, and $\hat{\sigma}$ parameters will also be used to simulate the interest rate values of the CIR model. Interest rate estimation uses Monte Carlo to generate standard normal random numbers ε_t. The CIR interest rate model is as follows:

$$r_t = r_{t-1} + \hat{k}(\hat{\theta} - r_{t-1})\Delta t + \hat{\sigma}\sqrt{r_{t-1}}\Delta t\varepsilon_t \quad (9)$$

where,
Δt: time interval
ε_t: normally distributed random variable with $\varepsilon \sim N(0,1)$
r_t: t-th interest rate

4. Compare and analyze the results of interest rate estimates obtained using the Vasicek interest rate model and the Cox-Ingersoll-Ross interest rate model.

5. Determine benefit reserves with the Vasicek interest rate model and the Cox-Ingersoll-Ross interest rate model using the Premium Sufficiency method.
   a. Create a commutation table for each participant from BNI Life term insurance policy owners containing D_x, C_x, M_x, and N_x values determined by the following formula.
   $$D_x = \frac{l_x}{(1+r(x))^x} \quad (10)$$
   $$C_x = \frac{d_x}{(1+r(x+1))^{x+1}} \quad (11)$$

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$$M_x = \sum_{i=x}^{\omega} C_i \qquad (12)$$

$$N_x = \sum_{i=x}^{\omega} D_i \qquad (13)$$

where,
l_x : the number of individuals living to the age of x years based on TMI IV
$r(x)$: x-th interest rate
d_x : the number of individuals who died at the age of x years based on TMI IV
ω : oldest age in TMI IV

b. Determine benefit reserves by using the Premium Sufficiency method.
Next is to determine the benefit reserve using the following commutation formula (Warni, Devianto, & Husna, 2017).
for $t < m$,

$$ {}_t^m V_{x:\bar{n}|}^1 = \frac{M_{x+t} - M_{x+n}}{D_{x+t}} - \left(\frac{M_x - M_{x+n} + \alpha D_x}{N_x - N_{x+m}}\right)\left(\frac{N_{x+t} - N_{x+m}}{D_{x+t}}\right) + \gamma' \left(\frac{N_{x+m} - N_{x+n}}{D_{x+t}} \times \frac{N_{x+t} - N_{x+t}}{N_x - N_{x+m}}\right) \qquad (14)$$

For $t \geq m$,

$$ {}_t^m V_{x:\bar{n}|}^1 = \frac{M_{x+t} - M_{x+n}}{D_{x+t}} - \gamma' \frac{N_{x+t} - N_{x+n}}{D_{x+t}} \qquad (15)$$

where,
m : premium payment period
n : term insurance period
α : the cost of closing the new insurance policy is 0.75% of the compensation money.
γ' : Premium maintenance fee is 0.15% of the compensation money
${}_t^m V_{x:\bar{n}|}^1$: t-th benefit reserve of n-year and term insurance products payment of premiums for m years from insurance policy participants who are aged x years with sum insured of Rp 1.

6. Compare and analyze benefit reserves with the Vasicek interest rate model and the Cox-Ingersoll-Ross interest rate model using the Premium Sufficiency method.

Research flow chart as shown in Figure 1.

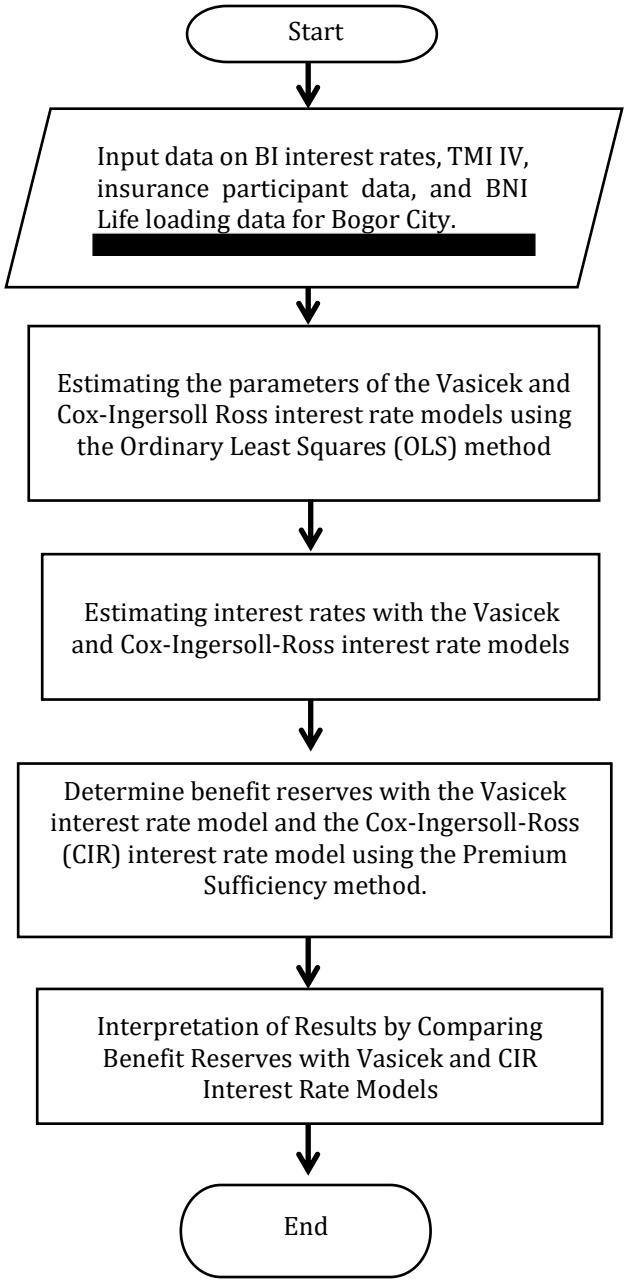

Figure 1. Research Flow Chart

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RESULTS AND DISCUSSION

The term insurance product used in this study is 10-year term insurance with a premium payment period of 5 years. Two participants in BNI Life Insurance counted the benefit reserves.

Results of Estimation of the Vasicek and Cox-Ingersoll-Ross Interest Rate Model Parameters using the Ordinary Least Square (OLS) Method

Using Bank Indonesia's annual interest rate data from 2009 to 2023 and the Ordinary Least Square (OLS) method, the results of estimating the parameters of the Vasicek and Cox-Ingersoll-Ross (CIR) interest rate models are obtained in Table 3.

Table 3. Estimated Value of Interest Rate model parameters

Parameters	Estimated Value
$\hat{k}$	4.2819094
$\hat{\theta}$	0.0548605
$\hat{\sigma}$	0.6416522

The Vasicek interest rate model obtained with the estimated value of these parameters is as follows:

$$r_t = r_{t-1} + 4.2819094(0.0548605 - r_{t-1})\Delta t + 0.6416522\Delta t\varepsilon_t$$

The CIR interest rate model obtained is as follows:

$$r_t = r_{t-1} + 4.2819094(0.0548605 - r_{t-1})\Delta t + 0.6416522\sqrt{r_{t-1}}\Delta t\varepsilon_t$$

Analysis of Interest Rate Estimation Results

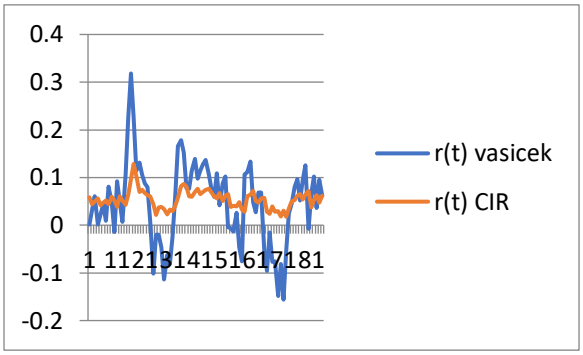

Figure 2. Vasicek and CIR Interest Rate Charts

Figure 2 is a graph of the results of the estimated interest rate using the Vasicek interest rate model and the CIR interest rate model. Based on the graph, the interest rate of the Vasicek model produces negative interest rates that do not match the characteristics of interest rates. In addition, the Vasicek interest rate difference is larger for some point in time compared to the CIR interest rate.

Benefit Reserves Using the Premium Sufficiency Method and the Vasicek and Cir Model

Table 4. Benefit Reserves of Participant A

t	Vasicek (Rp)	CIR (Rp)
0	-196,875.00	-196,875.00
1	1,495,599.16	325,157.13
2	5,411,533.14	1,216,587.75
3	1,175,294.10	2,356,624.10
4	3,899,455.03	2,093,139.38
5	9,363,392.46	3,331,696.31
6	2,949,601.08	3,080,841.37
7	22,756,672.89	1,856,011.28
8	7,784,900.60	1,845,687.39
9	788,190.54	834,582.08
10	0	0

Based on Table 4, benefit reserves using both the Vasicek and Cox-Ingersoll-Ross (CIR) interest rate models are negative at the time participant A enrolls in insurance. Whereas in years 2 to 9, the benefit reserve for both models is positive, while the benefit reserve for both models in year 10 is 0.

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Table 5. Benefit Reserves of Participant B

t	Vasicek (Rp)	CIR (Rp)
0	-433,125.00	-433,125.00
1	1,218,696.89	809,625.02
2	3,802,941.20	2,707,499.95
3	10,911,617.83	7,884,700.90
4	1,865,513.23	9,845,609.21
5	11,136,722.51	7,155,205.50
6	27,664,229.12	7,040,979.01
7	2,886,882.57	7,001,315.51
8	2,614,691.47	3,199,800.90
9	2,020,874.62	1,790,687.04
10	0	0

Based on Table 5, the benefit reserve using both the Vasicek and Cox-Ingersoll-Ross (CIR) interest rate models is negative at the time participant B registers insurance. Whereas in years 2 to 9, the benefit reserve for both models are positive, while the benefit reserve for both models in year 10 is 0. This condition happened because the type of insurance is term insurance, so when the insurance coverage period ends, the insurance company does not pay the sum assured to the insurance participant if the insurance participant is still alive until the insurance coverage period ends (Bowers, Gerber, Hickman, Jones, & Nesbitt, 1997).

CONCLUSIONS AND SUGGESTIONS

Vasicek's interest rate model is as follows:

$$r_t = r_{t-1} + 4.2819094(0.0548605 - r_{t-1})\Delta t + 0.6416522\Delta t\varepsilon_t$$

Whereas the Cox-Ingersoll-Ross (CIR) interest rate model is as follows:

$$r_t = r_{t-1} + 4.2819094(0.0548605 - r_{t-1})\Delta t + 0.6416522\sqrt{r_{t-1}}\Delta t\varepsilon_t$$

The Vasicek model interest rate produces negative interest rates that do not match the characteristics of the interest rate, while the CIR model interest rate produces positive interest rates that match the characteristics of the interest rate. In addition, the Vasicek interest rate difference is larger for some point in time compared to the CIR interest rate.

In addition, each participant's benefit reserve, whether using the Vasicek or CIR interest rate models, is negative at the beginning of the insurance participant's enrollment. Whereas in years 2 to 9, the benefit reserve for both models are positive, while the benefit reserve for both models in year 10 is 0.

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