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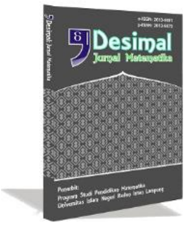

The model of goods delivery using multi depot vehicle routing problem at PT X

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ABSTRACT

Vehicle Routing Problem (VRP) is a problem in shipping that focuses on distributing goods from a depot to customers. There are several developments from VRP, one of which is the Multi Depot Vehicle Routing Problem (MDVRP). The MDVRP model has the same goal as the VRP, which is to minimize travel costs. The difference between VRP and MDVRP depends on the depot used. In VRP, only one depot is used. Whereas in MDVRP, there is more than one depot used. This research discussed the delivery of goods to two depots. The aim of this research is to form a model for shipping goods using two depots, determine the total travel cost, and determine the optimal route to delivery of the goods. The data used in this research is secondary data. The result of this research is that the model for the MDVRP aims to minimize the total travel cost by using two depots and serving 10 customer locations. The total cost of the trip is IDR 390,000, with a total distance traveled as far as 300 km, and the optimal routes for delivering goods involve each depot making two trips. The first depot covers distances of 57 km and 48 km, and the second depot covers distances of 92 km and 103 km.

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INTRODUCTION

Operations research is a branch of mathematics. Operations research is a science that applies analytical and mathematical methods that are useful for finding optimal values in decision-making (Silalahi, Sulistyono, & Bukhari, 2022). The focus of operations research is to use mathematical models and optimization techniques in formulating and solving problems that occur in various fields such as manufacturing, logistics, transportation, scheduling, and etc. (Agatz, Bouman, & Schmidt, 2018; Ba, Prins, & Prodhon, 2016; Ghilas, Demir, & Van Woensel, 2016; Goh, Lim, & Meng, 2007; Grazia Speranza, 2018; Mahardika & Marcos, 2017).

Linear programming is a technique in operations research that is used to solve optimization problems (Nurmayanti &

Sudrajat, 2021). Linear programming is very useful in various fields such as operations management, economics, computer science, and social sciences (Febrianti & Harahap, 2021; Li & Lam, 2002; Purnomo, Werdiastu, Raissa, Widodo, & Wijayaningrum, 2019; Rumahorbo & Mansyur, 2017). The goal of linear programming is to find the best solution with various constraints. Linear programming developed very rapidly after being discovered by George Dantzig in 1947 (Luenberger & Ye, 2021). In today's digital era, linear programming is also the basis for optimization in technology. Linear programming algorithms are used in a variety of applications, such as network routing, task scheduling, and the management of computing resources. Several methods have been developed to solve linear programming problems. One method that can be used is the VRP.

The VRP is a problem in delivery that focuses on the distributing of goods from a depot to a number of customers (Arvianto, Setiawan, & Saptadi, 2014). The delivery starts with a vehicle that delivers goods from the depot to all customers and returns to the depot after completing the delivery. The main goal of VRP is to minimize travel costs that are used during the delivery of goods (Alfiansah & Abdulrahim, 2023). There are several developments from VRP, one of which is the MDVRP (Making, Silalahi, & Bukhari, 2018).

The model of MDVRP has the same goal as the VRP, which is to minimize travel costs. In MDVRP, there is more than one depot that is used to distribute goods. The applications of MDVRP have very a wide scope, especially in the logistics industry. For example, if a company is planning the delivery of goods with some depots, it can use MDVRP to plan effective delivery routes and optimize vehicles at each depot.

In the research of Sulistyono (2022), an analysis was done about the route of delivery of goods using VRP. The mathematical model was useful in determining the optimal route to the problem. Based on these results, we obtained four delivery times. Ferdiansyah et al. (2021) also analyze travel routes by minimizing travel costs by using VRP.

The research of Sulistyono and Ferdiansyah is a model of the delivery of goods using one depot. However, this model cannot be used if there are more than one depot. So VRP cannot be used for these problems, and the method must be developed. This research discussed the delivery of goods to two depots. The aim of this research is to form a model for shipping goods using two depots, determine the total travel cost, and determine the optimal route to delivery of the goods.

METHOD

The first step taken in this research is to identify the problem. The next step is to do a literature study related to MDVRP. Then look for the right data based on the problem. Then form a model based on the problem. Based on the formed model, we find the optimal route that vehicles must travel at each depot using Lingo 11. The next step is to calculate the total distance traveled by all depots. Then determine the costs incurred by the company based on the total distance traveled by the company. The completion steps are described in Figure 1.

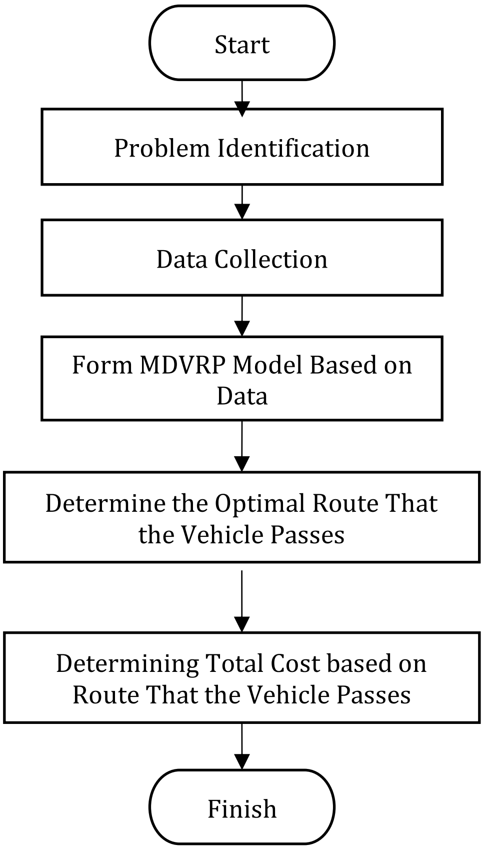

Figure 1. Flowchart

The data used in this research is secondary data derived from Sulistyono's research (Sulistyono, 2022). In Sulistyono's research, the data used was shipping data from TV. The vehicle used is a CDE Jumbo truck with a capacity of 271 units at once. Furthermore, we present the distance between the depots and 10 destination locations. Based on that data, we add one depot and assume the distance between the second depot and 10 other destination locations. The following data shows the distance data between the two depots with 10 destination locations (in km).

DC 1 and DC 2 are the depot, and then SSK, SE, EC, SEJ, TUCR, ERC, TEH, GP, ECT, and TPE are the customers. The travel costs incurred by the company are IDR 1300/Km. The following data shows the total demand of each customer.

Table 1. Distance between Depot and Customers

No.	To	1	2	3	4	5	6
	From	DC 1	DC 2	SSK	SE	EC	SEJ
1	DC 1	0	13	23	17	28	19
2	DC 2	13	0	43	38	20	17
3	SSK	23	43	0	9	38	9
4	SE	17	38	9	0	33	10
5	EC	28	20	38	33	0	26
6	SEJ	19	17	9	10	26	0
7	TUCR	32	25	5	12	30	11
8	ERC	22	38	16	15	24	7
9	TEH	56	24	51	45	38	38
10	GP	48	11	17	30	27	24
11	ECT	27	17	16	18	19	12
12	TPE	53	48	33	44	53	41

Table 2. Distance between Depot and Customers

No.	To	7	8	9	10	11	12
	From	TUCR	ERC	TEH	GP	ECT	TPE
1	DC 1	32	22	56	48	27	53
2	DC 2	25	38	24	11	17	48
3	SSK	5	16	51	17	16	33
4	SE	12	15	45	30	18	44
5	EC	30	24	38	27	19	53
6	SEJ	11	7	38	24	12	41
7	TUCR	0	27	45	17	16	29
8	ERC	27	0	36	21	9	47
9	TEH	45	36	0	23	33	67
10	GP	17	21	23	0	11	33
11	ECT	16	9	33	11	0	38
12	TPE	29	47	67	33	38	0

Tabel 3. The Number of Demand Goods

No.	Source	Total
1	SSK	55
2	SE	90
3	EC	87
4	SEJ	120
5	TUCR	80
6	ECR	100
7	TEH	96
8	GP	78
9	ECT	105
10	TPE	135

RESULTS AND DISCUSSION

Mathematical Models

In this research, the MDVRP model uses two depots as delivery locations. These two depots deliver goods to 10 customer locations. Each depot has a vehicle with a one-time transport capacity of 271 units. It is assumed that delivery can be done up to four times in one day. Delivery at each depot is divided into four delivery times.

The model formed aims to minimize travel costs. So that the objective function formed is as follows:

$$\min Z = \sum_{i=1}^{12}\sum_{j=1}^{12}\sum_{k=1}^{8} c_{ij}x_{ij}^{k}$$

Furthermore, from the MDVRP, a number of constraints are formed, which are then modeled into a mathematical equation as follows:

1. Delivery of goods can be done at a predetermined delivery time, and the vehicle starts its journey from each depot.

$$\sum_{j=3}^{12} x_{ij}^{k} = s^{k}, \qquad i = 1,2, \qquad k = 1,2,\dots,8$$

2. Vehicles at depot 1 cannot be used at depot 2. Likewise, vehicles at depot 2 cannot be used by depot 1.

$$\sum_{j=3}^{12} x_{1j}^{k} = 0, \qquad k = 5,6,7,8$$

$$\sum_{j=3}^{12} x_{2j}^{k} = 0, \qquad k = 1,2,3,4$$

3. During the delivery of goods, vehicles at depot 1 cannot return to depot 2. Furthermore, vehicles at depot 2 cannot return to depot 1.

$$\sum_{j=3}^{12} x_{j1}^{k} = 0, \qquad k = 5,6,7,8$$

$$\sum_{j=3}^{12} x_{j2}^{k} = 0, \qquad k = 1,2,3,4$$

4. There are no trips that may be made by vehicles from the depot to the depot

$$x_{ii}^{k} = 0, \qquad i = 1,2, \qquad k = 1,2,\dots,8$$

5. Vehicles that have left the depot must return to the depot.

$$\sum_{j=3}^{12} x_{ji}^{k} = s^{k}, \qquad i = 1,2,$$

$$k = 1,2,\dots,8$$

6. Service to customers is done exactly once

$$\sum_{j=1}^{12}\sum_{k=1}^{8} x_{ijk} = 1, \qquad i = 3,4,\dots,12$$

$$\sum_{i=1}^{12}\sum_{k=1}^{8} x_{ijk} = 1, \qquad j = 3,4,\dots,12.$$

7. The number of customer requests does not exceed the capacity of the vehicle at a certain delivery time

$$\sum_{j=3}^{12} q_j \sum_{i=1}^{12} x_{ij}^{k} \le Q^{k}, \qquad k = 1,2\dots,8.$$

8. Vehicle travel routes may not run from a customer to the same customer

$$x_{jj}^{k} = 0, \qquad j = 3,4,\dots,10,$$

$$k = 1,2,\dots,8.$$

9. Vehicles that have visited a customer must leave the customer if the customer has been served

$$\sum_{\substack{i=1\\ i\neq h}}^{12} x_{ih}^{k} = \sum_{\substack{j=1\\ j\neq h}}^{12} x_{hj}^{k}, \qquad h = 3,4,\dots,12,$$

$$k = 1,2,\dots,8.$$

10. Check the subtour; if there is a subtour, then subtour elimination is carried out.

$$m_i^{k} - m_j^{k} + Px_{ij}^{k} \le P - 1$$

11. Customers are always visited by vehicles

$$x_{ij}^{k} \le s^{k}.$$

Information

- c_{ij} are the travel costs incurred by the company from *i* to *j*
- x_{ij}^k is a decision variable with a value of 0 or 1
- x_{ij}^k is a trip at the *k-th* time made by the vehicle from *i* to *j*
- s^k is the *k-th* delivery time
- q_j is the *j-th* customer request
- Q^k is the capacity of the *k-th* vehicle in one transport

Travel route

Based on the model that has been formulated, problem solving is carried out with the help of the program so that the following solutions are obtained.

a. Depot Vehicle Travel Route 1

At Depot 1, the vehicle travels twice. The first trip starts from depot 1 to SE customers by delivering 90 units of goods. Then continue the journey to TUCR by delivering 80 units of goods. Then the vehicle travels to SSK, delivering 55 units of goods. Because the remaining quantity of goods cannot meet the next customer demand, the vehicle must return to depot 1 to restock the goods. The total goods delivered on the first trip were 225 units, with 3 customers served.

The second trip was then carried out from depot 1 to the ERC customer by handing over 100 units of goods. Then continue the journey to SEJ by delivering 120 units of goods. The total goods delivered on the second trip were 220 units, with 2 customers served.

b. Depot Vehicle Travel Route 2

At Depot 2, the vehicle travels twice. The first trip starts from depot 2 to GP customers by delivering 78 units of goods. Then continue the journey to TEH by delivering 96 units of goods. Furthermore, the vehicle traveled to EC, delivering 87 units of goods. Because the remaining quantity of goods cannot meet the next customer demand, the vehicle must return to Depot 2 to restock the goods. Total goods delivered on the first trip were 261 units, with 3 customers served. The second trip was then carried out from Depot 2 to the TPE customer by handing over 135 units of goods. Then continue the journey to ECT by delivering 105 units of goods. The total goods delivered on the second trip were 240 units, with 2 customers served.

Vehicle travel routes at each depot are presented in Table 4.

Table 4. The Routes of Vehicle at Each Depot

No.	Initial Depot	Travel Route	Number of Demand
1.	Depot 1	DC1-SE-TUCR-SSK-DC1	225
2.	Depot 1	DC1-ERC-SEJ-DC1	220
3.	Depot 2	DC2-GP-TEH-EC-DC2	261
4.	Depot 2	DC2-TPE-ECT-DC2	240

The total distance covered by the vehicles at depot 1 is 105 kilometers; furthermore, the total distance covered by the vehicles at depot 2 is 195 kilometers. Therefore, the total distance traveled to serve all customers is 300 kilometers, with a corresponding travel cost of IDR 390,000. The following table presents the total distance and total cost for each travel route.

Table 5. The Routes of Vehicle at Each Depot

No.	Travel Route	Total Journey	Total Cost (IDR)
1	DC1-SE-TUCR-SSK-DC1	57	74.100
2	DC1-ERC-SEJ-DC1	48	62.400
3	DC2-GP-TEH-EC-DC2	92	199.600
4	DC2-TPE-ECT-DC2	103	133.900
	Total	**300**	**390.000**

CONCLUSIONS AND SUGGESTIONS

The model for the MDVRP aims to minimize the total travel cost by using two depots and serving 10 customer locations. There are 11 constraints in this problem, which are then formulated into mathematical equations. Subsequently, the problem is solved to obtain the total travel cost. The total travel cost incurred is IDR. 390,000, with a total distance traveled of 300 km. The optimal routes for delivering goods involve each depot making two trips. The first depot travels through the routes DC1-SE-TUCR-SSK-DC1 and DC1-ERC-SEJ-DC1, covering distances of 57 km and 48 km, respectively. Meanwhile, the second depot travels through the routes DC2-GP-TEH-EC-DC2 and DC2-TPE-ECT-DC2, covering distances of 92 km and 103 km, respectively.

This research discusses goods delivery, but the delivery time to each customer is not explained. This research would be better if it discussed the delivery time for each customer. So that the goods delivery case is closer to real-world problems.

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