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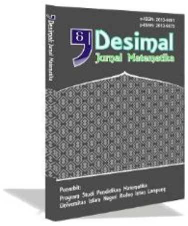

Hydra effects predator-prey bazykin's model with stage-structure and intraspecific for predator

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ABSTRACT

Bazykin's predator-prey population model is considered to represent the exchange stability condition of population growth. The existence of the hydra effect and, at the same time, analyzing its influence on population growth. The condition of the model divides the species into a stage structure, namely, prey, immature predators, and mature predators. The population growth of the three species has its own characteristics. This research revealed that the Holling type II and intraspecific predatory function responses together induce the Hydra effect. In the model formed, there are 12 equilibrium points, with details for every seven points of negative imaginary equilibrium and five points of non-negative equilibrium. The findings of research studies center on five points of non-negative equilibrium. All real roots that interpret the species population's growth conditions are taken and tested for long-term stability. The test results show one point of equilibrium that meets the Routh-Hurwitz criteria and their characteristic equations. In numerical simulations, the maximum sustained yield is in the local asymptotic stable state. The growth of prey trajectories increased significantly, although at the beginning of the interaction there was a slowdown in population growth. Meanwhile, the population of immature predators and mature predators was not significantly different. Both populations grow steadily toward the point of population stability. It turns out that the two populations grow inversely, the faster the rate of predation by predators, the faster the growth rate of the prey population.

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INTRODUCTION

Naturally, every living being will lead to the fate of his death. The logical approach to knowledge of death provides a scientific description of the causes or factors of the event. Species deaths due to predation interactions, natural disasters, harvest, eradication programs and disease are some examples of causes of death in living ecosystems (Pratama et al., 2022). The increasing death rate will cause the population density to decrease. This is a simple ecological principle that often occurs in wild species. In some types of species, there are those that show the opposite phenomenon (Lu et al., 2022). The continuous fluctuating growth of a homogeneous species population is demonstrated by discrete mathematical modeling (Martcheva, 2015). The increase in the average population size of a species due to a higher mortality growth rate is known as the hydra effect.

In the case of the hydra effect, the species grows twice every time one dies. The philosophy of this principle was first used in ancient Greek mythology, namely the Hydra of Lerna (Sieber & Hilker, 2012). The mythological beast, when decapitated, grows back into two heads. The concept of the hydra effect has practical implications for chemical pest control (Cortez & Abrams, 2016). Callosobruchus maculatus, for example, is a pest that is parasitized by Anisopteromalus calandrae. The hydra effect occurred in pest species in the harvested metapopulation and was compared to the unharvested metapopulation (Liz & Sovrano, 2022). The hydra effect condition occurs when the average density of species increases as the species' mortality rate increases (Cortez & Yamamichi, 2019). Meanwhile, the effect of population density corresponding to the hydra effect has been explored by many new studies (Iskin da S. Costa & Dos Anjos, 2018). In fact, ecological knowledge has not yet found a strong supporting factor why the hydra effect is needed in a natural ecosystem.

The discussion of the effect of the hydra on the dynamic predator-prey stage structure model of population dynamics has been carried out. The hydra effect has also been studied in a stable state model of a mature and immature predator-prey population dynamics model for prey in a stable state (Anjos et al., 2020). When the number of predatory species that prey on immature species increases, the overall biomass of immature and mature species also increases. The placement of hydra effect in the population dynamics model is mostly placed specifically on predator species, prey species, or sub-species of both (Bajeux & Ghosh, 2020).

The results of other studies reveal that the population model experiencing the hydra effect occurs when the mature species population is exploited. Meanwhile, the non-stage-structure predator-prey model (Weide et al., 2019) depicts the effect of hydra on predators in population density. Equilibrium stability limits the unstable balance of the equilibrium composition of the population model. The condition of the mathematical model has also been carried out using the hydra effect on an unstructured population model. There is a stable equilibrium condition, and the predator population's sustainability is guaranteed in the long term (Garain & Mandal, 2021). The hydra effect has also been tested on different population dynamics models, such as a simple predator-prey system, a prey-two-predator system, and a simple three-species food chain model. The hydra effect under more complex conditions, namely the dual hydra effect and multistability, also demonstrates the long-term stability of the predator-prey population.

Species that live in the tropics tend to have very high exploitation behavior. Excessive exploitation behavior of predators also creates a hydra effect which is interdependent between predator-prey species (Iskin da S. Costa & Dos Anjos, 2018). For example, the exploitation of aquaculture predatory fish in intensive pond types has a tendency to destroy predator populations (Pal et al., 2019). The hydra effect is also introduced in the eco-evolution system, which shows that the rate of extinction of predatory species becomes greater with this evolution (Esteves & Caxias, 2021; Lee et al., 2020). The phenomenon of the hydra effect over a continuous period of time in the predator-prey population dynamics model also shows a persistent condition. Analytically, many developed models prove that the model is stable over a continuous period of time.

The mathematical form of the predator-prey population dynamics model formulation is based on assumptions that have been developed from theoretical studies. The involvement of predatory species is given an intra-specific nature. This is considered because predator populations have a competitive nature from childhood. The nature of predators from childhood to maturity tends to be the same, only getting more agile when the predator has become more mature. In the second, a predator-prey dynamics model will be given in accordance with the research assumptions. A dimensional analysis is also given to the model that is formed so that it is realistic in accordance with the real conditions in the formed ecosystem. In part three, equilibrium analysis will be shown. Meanwhile, in part four, the model will also be analyzed using trajectories. At this stage, a numerical simulation is also provided to see simple conditioned events.

METHOD

Literature studies are the basis for this type of research modeling the actions of armed criminal groups to be carried out. The research stages are arranged based on the following research steps; assumption testing, unit analysis, model formulation, stability analysis, eigenvalues, and numerical simulation.

The effect of hydra on the differential equation model has been widely studied in the development of science in recent years. The formulated modeling adopts a lot of relevant research (Adhikary et al., 2021). Some of the theoretical studies have been widely expressed, as theories that support the developed model. The hydra effect that will be shown in the model is only in stable conditions. In unstable conditions, the hydra effect is ignored, because it considers ecological factors, namely population sustainability. Population growth followed a logistic growth model, which was in agreement with Lotka-Volterra predator-prey model. Meanwhile, the predator and prey functional response adopts the predator Holling Type II function. The population of predatory species is divided into immature and mature, each of which has intra-specific characteristics. The modified model is known as the predator-prey Bazykin's model, known as:

$$\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{k}\right)-\frac{\beta xz}{h+x},\\ \frac{dy}{dt} &= a\frac{\beta xz}{h+x}-my^2-dy-\delta_1 y,\\ \frac{dz}{dt} &= dy-my^2-\delta_2 z,\end{aligned} \tag{1}$$

where, $\frac{dx}{dt}\geq 0$, $\frac{dy}{dt}\geq 0$ and $\frac{dz}{dt}\geq 0$, where the variable x,y and z represents the population of prey, immature predator, and mature predator at time t. Parameters r and k are the intrinsic growth rate and carrying capacity of the prey population, respectively. Parameter β represents the interaction rate of prey and mature predator, while parameter a is the rate of change of prey biomass to immature predator biomass. Parameter h as half-saturation in response function Holling type II. Parameter d is the coefficient of the rate of transfer of immature biomass to mature predators. The natural mortality rates for immature and mature predatory species are δ_1 and δ_2. The strength of intraspecific competition among predators is represented by parameter m. The interaction conditions are assumed to be the same for both immature and mature predator biomass. Traits like this are really important to consider because predators tend to have the same fighting power from birth to maturity. Model (1) also considers the dimensional analysis of the model, shown in Table 1.

Tabel 1. Unit of Parameters in Model

No.	Parameters	Unit
1.	r	$[T]^{-1}$
2.	k	$[T]^{-1}$
3.	β	$[N]$
4.	a	$[T]^{-1}\,[N]^{-1}$
5.	δ_1	$[T]^{-1}$
6.	δ_2	$[T]^{-1}$
7.	m	$[T]^{-1}\,[N]$
8.	h	-

RESULTS AND DISCUSSION

1. Equilibrium

Model (1) is operated with a system of differential equations in order to obtain a solution that can represent the system in the model. Each equilibrium value that appears will represent the variables arranged in the model. In Model (1), there are 12 equilibrium points, with details of each of the seven points of negative-imaginary equilibrium and the five points of non-negative equilibrium. The focus of this research discussion is on five points of non-negative equilibrium. These 5 equilibrium points become the basis for the next test, which is to ensure that the equilibrium point meets stability in the long term. A stability analysis was carried out considering the sustainability of all populations of the constituent species in the model. Model (1) has a trivial solution, extinction equilibrium, and existence equilibrium. The form of the trivial solution in Model (1) is $E_0 = (0,0,0)$, while the extinction equilibrium solution is $E_1 = (k,0,0)$. The corresponding differential equation at equilibrium values is the septic equation, as follows,

$$x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0 \qquad (2)$$

From this septic equation, three other equilibrium points are obtained, including $E_2 = (x_2, y_2, z_2)$, $E_3 = (x_3, y_3, z_3)$, and $E_4 = (x_4, y_4, z_4)$. This equilibrium is part of the root values of the septic equation. Basically, septic equations allow 7, 5, 3, and 1 real roots to be calculated based on multiples. The number of complex non-real roots in a septic equation is the sum of the total roots minus the real roots of the equation. In simple terms, septic equations are always similar to any other odd-power polynomial, quintic, or cubic. Equation (2) in the end only shows three real roots, each of which will be tested for stability through the Jacobian matrix. The Jacobian matrix corresponding to Equation (2) is as follows:

$$J(E_i) = \begin{bmatrix} j_{11} & 0 & j_{13} \\ j_{21} & j_{22} & j_{23} \\ 0 & j_{32} & j_{33} \end{bmatrix}, \qquad (3)$$

where,

$$j_{11} = r\left(1 - \frac{x}{k}\right) - \frac{rx}{k} - \frac{\beta z}{h+x} + \frac{\beta xz}{(h+x)^2},$$

$$j_{13} = -\frac{\beta x}{h+x}, \qquad j_{21} = \frac{a\beta z}{h+x} - \frac{a\beta xz}{(h+x)^2},$$

$$j_{22} = -2my - b - d, \qquad j_{23} = \frac{a\beta z}{h+x}, \qquad j_{32} = d,$$

$$j_{33} = -2mz - n.$$

The 3 points found in the equilibrium analysis are substituted as x, y, and z, which become the solution of Model (1). After each variable value is substituted, a characteristic equation will be shown that is associated with the eigenvalues.

$$f(\lambda) = \lambda^3 + N_1\lambda^2 + N_2\lambda + N_3 \qquad (4)$$

The eigenvalues that emerge from the characteristic equation are part of the fulfillment of the criteria for Routh-Hurwitz stability as local asymptotic stable equilibrium values. The 3-point equilibrium test will be continued in the numerical simulation section as the main discussion of the research.

2. Numerical Simulation

Simulation Model (1) is given to provide a mathematical description of the numerical model form. The parameter values taken have been considered ecologically similar to the actual condition of the ecosystem. Taking parameter values and analyzing unit Model (1) is highly dependent, therefore some of the parameter values are taken from previous relevant research of Adhikary et al. (2021); Lu et al. (2022); and Pratama et al. (2022). The parameters taken are based on the rational assumptions of the model (1) and are also supported by parameters from previous studies. The parameters of Model (1) are $r = 0.5$, $k = 100$, $\beta = 0.95$, $b = 0.025$, $d = 0.3$, $\delta_1 = 0.4$, $\delta_2 = 0.2$, $m = 0.045$, $n = 0.0003$, and $h = 5$. Equilibrium analysis, the focus is on non-negative equilibrium values, in order to obtain 5 equilibrium $E_0 = (0{,}0{,}0)$, $E_1 = (100{,}0{,}0)$, $E_2 = (0.879, 0.905, 2.454)$, $E_3 = (13.974, 7.093, 6.873)$, and $E_4 = (77.798, 8.994, 7.740)$.

The numerical simulation equilibrium values E_4 have shown a good equilibrium point for population growth to be maintained. Meanwhile, E_0 and E_1 are not analyzed further because they contradict the sustainability of life for all species carrying Model (1). The first test is on the equilibrium value E_2 of each variable value, which are $x_2 = 0.8794$, $y_2 = 0.906$, and $z_2 = 2.454$. Each will be substituted in the Jacobian matrix so that the matrix becomes,

$$J(E_2) = \begin{bmatrix} 0.056 & 0 & -0.142 \\ 0.32 & -0.407 & 0.135 \\ 0 & 0.3 & -0.221 \end{bmatrix}, \quad (5)$$

the associated characteristic equation is,

$$\lambda^3 + 0.057\lambda^2 + 0.014\lambda + 0.011 = 0, \quad (6)$$

the eigenvalues of equation (6) are,

$\lambda_1 = 0.004 + 0.137\,\mathrm{I}$, $\lambda_2 = 0.004 - 0.137\,\mathrm{I}$, $\lambda_3 = -0.579$.

it is clear from these conditions that the conditions for the eigenvalues ($\lambda_1 < 0$, $\lambda_2 < 0$, and $\lambda_3 < 0$) are not met. Equilibrium E_2 does not qualify as an asymptotic stable equilibrium. The second test, on the equilibrium values E_3 of the components of the variable values, are $x_3 = 13.974$, $y_3 = 7.092$, and $z_3 = 6.873$. Each will be substituted in the Jacobian matrix so that the matrix becomes,

$$J(E_3) = \begin{bmatrix} 0.198 & 0 & -0.699 \\ 0.086 & -0.963 & 0.665 \\ 0 & 0.3 & -0.619 \end{bmatrix}, \quad (7)$$

the associated characteristic equation is,

$$\lambda^3 + 1.385\lambda^2 + 0.084\lambda - 0.06 = 0 \quad (8)$$

So, the eigenvalues of equation (8) are,

$\lambda_1 = 0.172$, $\lambda_2 = -0.274$, $\lambda_3 = -1.282$.

It is clear from these conditions that the conditions for the eigenvalues ($\lambda_1 < 0$, $\lambda_2 < 0$, and $\lambda_3 < 0$) are not met. Equilibrium E_3 does not qualify as an asymptotic stable equilibrium.

The third test, on the equilibrium values E_4 of the components of the variable values are $x_4 = 77.798$, $y_4 = 8.994$, and $z_4 = 7.740$. Each will be substituted in the Jacobian matrix so that the matrix becomes,

$$J(E_4) = \begin{bmatrix} -0.2278 & 0 & -0.893 \\ 0.005 & -1.135 & 0.848 \\ 0 & 0.3 & -0.697 \end{bmatrix}. \quad (9)$$

the associated characteristic equation is,

$$\lambda^3 + 2.059\lambda^2 + 0.953\lambda + 0.123 = 0 \quad (10)$$

So, the eigenvalues of equation (10) are,

$\lambda_1 = -0.236$, $\lambda_2 = -0.356$, $\lambda_3 = -1.467$.

From these conditions, it is clear that the conditions for eigenvalues meet $\lambda_1 < 0$, $\lambda_2 < 0$, and $\lambda_3 < 0$. Equilibrium E_4 fulfills an asymptotic stable equilibrium.

In Model (1), the intervention with hydra produces one asymptotic stable point of equilibrium, namely equilibrium E_4. Population growth in all species will be stable in the long term. In prey species, the growth reaches equilibrium point $x_4 = 77.798$, in immature predator species $y_4 = 8.994$, and in mature predator species $z_4 = 7.740$. While maintaining the growth of the three populations, of course, in accordance with the basic ecological concept, preventing the extinction of species is necessary to maintain the balance of the ecosystem.

A trajectories analysis is also provided to see growth on the corresponding curve. The initial values for the surrounding E_4 are $x_4 = 10$, $y_4 = 3.7$ and $z_4 = 2.7$. The t growth time of all species is used to calculate all initial values. The population growth trajectories can be seen in Figures 1 to 3.

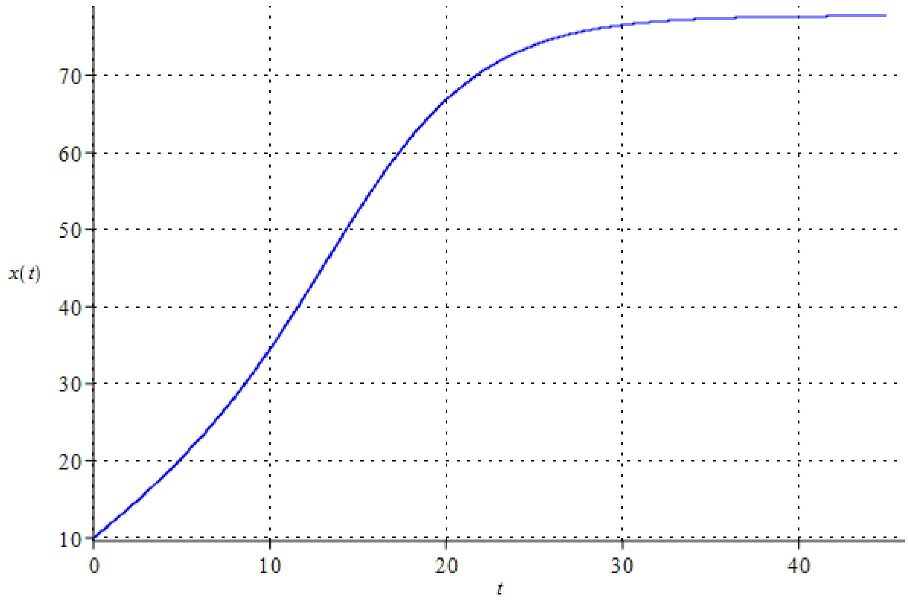

Figure 1. Trajectories Population Prey

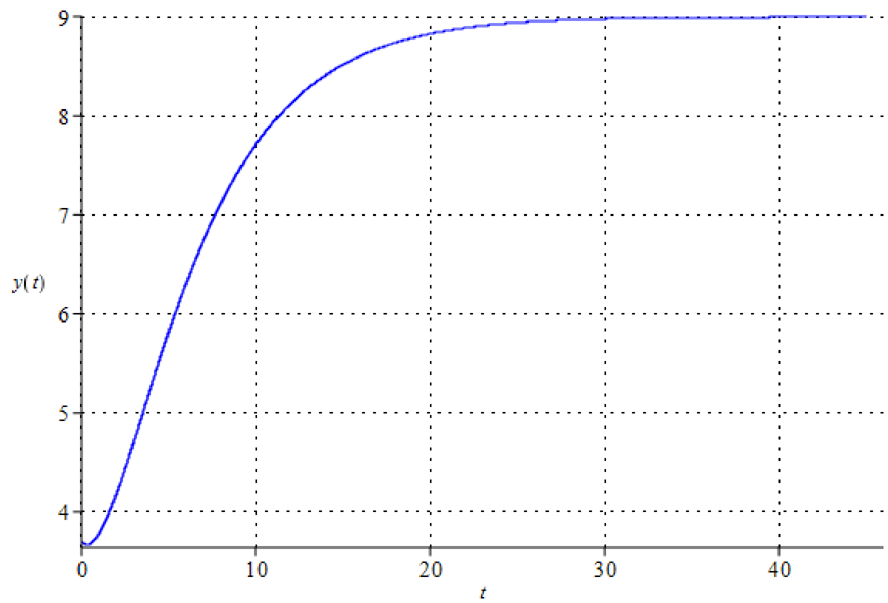

Figure 2. Trajectories Population Immature Predator

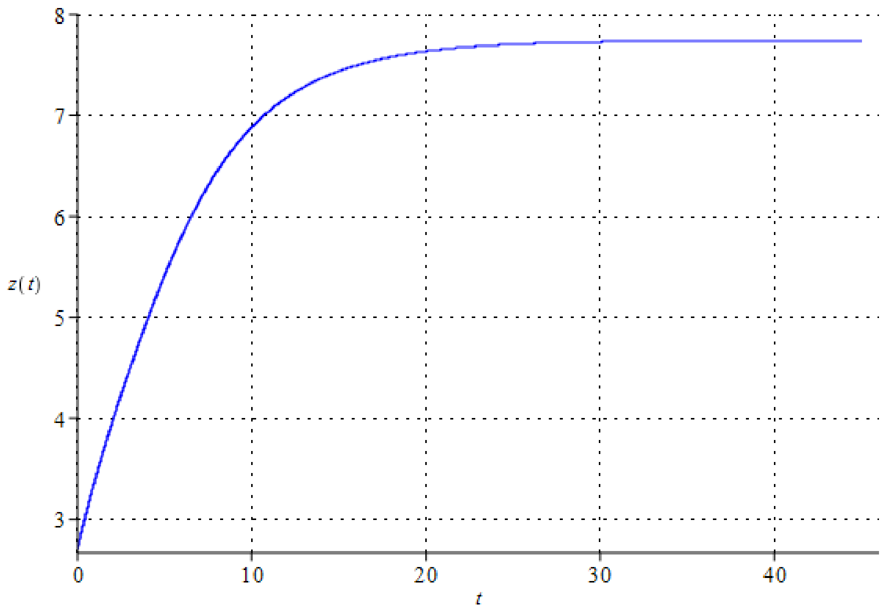

Figure 3. Trajectories Population Mature Predator

Figure 1, Figure 2, and Figure 3 show steady growth over a long period of time. The characteristics of prey population growth have a slow rate at the beginning of the interaction. Growth in the prey population is still considered significant. The slope of prey population growth at the beginning can be influenced by the interaction of attacks carried out by mature predators. Meanwhile, the population growth of immature predators generally experienced growth, but at the beginning of the interaction, there was a decline before the population grew significantly (Garain & Mandal, 2021). In mature populations, predators continue to grow significantly, because the initial assumption is that predators have a very aggressive nature in their attacks. Overall population growth is increasing, but what is interesting is that the growth rate of predators is low when compared to prey growth (Adhikary et al., 2021). According to the theoretical study, attacks by predators on prey species will increasingly make prey growth accelerate. This case is very much in agreement with the hydra effect, which is very evident in the trajectories analysis.

CONCLUSIONS AND SUGGESTIONS

Mathematical modeling with the hydra effect forms the main framework for the mathematical model of predator-prey populations. The growth model is based on realistic assumptions to be developed in the world of ecosystems. The model is formed with the composition of prey species, immature predators, and mature predators. All growth and predation species have different characteristics. The characteristics of predators that are active in predation and the division of stage structure, namely immature predators and mature predators. The model formed was analyzed by including unit variables. Unit variables are analyzed by forming realistic assumptions with the value parameters. Model (1) generates an overall equilibrium point made up of real and imaginary points. In this research, the focus of stability testing is on non-negative equilibrium. The stability criteria use Routh-Hurwitz to determine the sustainability of the population in the ecosystem. The real roots that appear in the model are five equilibrium points. The test is carried out at these 5 equilibrium points, and there is only one equilibrium point that satisfies the stability of the population. The point of equilibrium that satisfies stability is $E_4 = (77.798, 8.994, 7.740)$. The eigenvalues that arise from the Routh-Hurwitz characteristic equation satisfy the principles $\lambda_1 < 0$, $\lambda_2 < 0$, and $\lambda_3 < 0$. The trajectories analysis also shows the sustainability of each species. All species are not extinct and will continue to live in a balanced ecosystem.

In the development of further research, it is very dependent on the species, which is the formulation of the mathematical model. Species that can be used as medicine or as food can be given variable exploitation or harvesting. Further research Model (1) can be given exploitation variables and can be analyzed for optimum profit. Harvesting intervention development will result in a more realistic model scheme for applied mathematics.

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