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Can definite integral solve the 'aul problem?

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ABSTRACT

There is a similarity in concept between 'aul and the definite integral. 'Aul deals with the issue of the quantity of inheritance which must be divided into smaller quantities in order to be handed over to the heirs, while definite integral relates to the problem of calculating the broad of an area by dividing the area into smaller ones, followed by summing up the broad of each area. Achmad Yani used the definite integral to solve the problem of 'aul. But, in our opinion, the concept he used was wrong and there were still some errors in the construction that he described. This article aims to provide some corrections to such errors and provide some notes that as far as our research, the definite integral has not been able to be used in solving the problem of 'aul yet.

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INTRODUCTION

Islamic inheritance science (*faraidh*) is one of the branches that occupy a very important position in *Fiqh*. The high position of this knowledge can be seen that Allah SWT himself determines the parts for each heir in the Qur'an directly. Even this science was specifically ordered by the Prophet (peace be upon him) to be studied due to his concern that the first science that would be revoked from Muslims was this science (*hadith* from Hakim and Ibn Majah). The application of this science in human life can be seen as an implementation of the purpose of *maqashid syari'ah* in terms of maintaining offspring (*hifdzun nasl*). While ignoring the implementation of this science will

bring a rift in family relations and even cause bloodshed.

One of the problems that exist in the science of Islamic inheritance is the problem of 'aul. This problem occurs when the total share of the heir's rights is more than the total inheritance so if the treasure is distributed according to the portion of the heir's share it will not be enough (Syarifuddin, 2008). In Islamic inheritance, this problem is solved by enlarging the number of denominators in the portion of the heirs in such a way that the sum of the entire number of numerators in the portion of the heir section is equal to the number of denominators.

Research on 'aul is still relatively few compared with the studies on the science of *faraidh*. One of the studies on 'aul was conducted by Fitriyati (2014) who studies the position of *ashabah* on the problem of 'aul in ibn Abbas's view. On the other hand, research around the *radd* issues, as a couple of 'aul, is more done. Some of them are Murlisa (2015) who discusses the compilation of Islamic law (KHI) on the problem of heirs of *radd* recipients, and Juhdi et al. (2017) who researches the relationship between students' knowledge of *radd* concepts and fractional rules in *faraidh* science with their ability to solve the problem of inheritance division.

Yani (2016) offered another way to solve the problem of 'aul by using the concept of indefinite integral. But, in our opinion, the concept he used is wrong and there were still some errors in the construction that he described in the book. This article was written with the aim of providing corrections to the errors contained in the book and providing some notes on the possible use of definite integral in solving the problem of 'aul.

This article is presented in the following order. After the introduction, the next part is the presentation of the basic concepts to be discussed, that is the concept of 'aul and the definite integral. The next part is the main part of this article, which is the correction of the errors contained in the book by Yani (2016), which is followed by several notes on the possible use of definite integral in solving the problem of 'aul. The last part is the conclusion and the open problems offered as further research.

METHOD

The Concept of 'Aul in Islamic Inheritance Law

'Aul, as a word in Arabic has many meanings. Some of its meanings are *dzalim*, rising, increasing, and *fakir*. *Dzalim* as one of the meanings of 'aul is expressed in surah *an-Nisa'*: 3

ذَلِكَ أَذْنَىٰ أَلَّا تَعُولُوا ۝ ٣

"That is closer to not doing evil (persecution)"

The root of the word 'aul is 'ala with the meaning of *iftaqoro* which means poor or *fakir* (Ali & Muhdlor, 1996). 'Aul which has the meaning of ascending can be seen in the sentence '*ala al qadhiyah ila al hakim*, meaning that the matter has been appealed to the judge. While the word 'aul which has the meaning of *al ziyadah* (increased) can be seen in the sentence '*ala al mizan*, meaning that the balance sheet is increasing in weight (Saebani & Djaliel, 2015). Likewise, it can be seen in the sentence '*alal maa'u idza zaada wa irtafa'a 'an haddihi*, meaning that the water rises and increases to higher than its upper limit (Ghomidi, 2011).

In *fiqh*, 'aul means a case in the division of inheritance where the total proportion of *dzawil furud* is more than 1 so that the inheritance they receive becomes less (Syarifuddin, 2008). In line with Sabiq, Syarifuddin (2008) explained the meaning of 'aul is a case of distributing inheritances where the number of *furudh* of the *dzawil furud* is greater than the available heirs which cause the inheritance is not enough to be distributed to all *dzawil furud*. The solution to the 'aul problem is done by enlarging the number of divisors (denominators on fractions representing part of *dzawil furud*) in such a way that the number will be equal to the smallest common multiple of each denominator of *dzawil furud* so that the inheritance becomes sufficient to be shared (Maruzi, 1981).

History records that the case of 'aul has never happened in the time of the prophet Muhammad and the time of the caliphate of Abu Bakr *r.a.* The problem of 'aul began at the time of the caliphate Umar ibn Khattab *r.a.* Ibn Abbas *r.a.* said, "The one who first added the divider (i.e., 'aul) was Umar ibn Khattab. And it is done when the *fardh* that must be given to the heirs increases".

Here is an example of the 'aul problem. A woman (Muslimah) died and left behind the heirs of two sisters and a husband. The woman left an inheritance of Rp 42,000,000. In accordance with the rules in *faraidh*, then the husband's part is $1/2$ and the two sisters' part is $2/3$. So that from Rp 42,000,000, the husband gets $3/6 \times \text{Rp } 42,000,000 = \text{Rp } 21,000,000$ and the two sisters get $4/6 \times \text{Rp } 42,000,000 = \text{Rp } 28,000,000$ with a total inheritance divided is Rp 49,000,000 even though the available inheritance is only Rp 42,000,000. Thus, there is a shortage of inheritance of Rp 7,000,000 to be distributed.

The solution to the problem is done by converting the number 6 (as the denominator of the part of the husband and two sisters) to the number 7 (obtained by summing the numbers 3 and 4). So that a new proportion is obtained for the husband and two sisters as follows: the husband's share = $3/7 \times \text{Rp } 42,000,000 = \text{Rp } 18,000,000$ and the part of two sisters = $4/7 \times \text{Rp } 42,000,000 = \text{Rp } 24,000,000$ with a total amount of Rp 42,000,000. So that the inheritance can be distributed appropriately to all heirs.

The *faraidh* scholars conducted research that the problem of 'aul will only occur in the problem of the heir if one of the heirs has a share of the inheritance with the denominator of the fraction being 6, 12, and 24. The application of the concept of 'aul, in this case is, arranged as follows:

- Denominator 6 is enlarged to 7, 8, 9, or 10.
- Denominator 12 is enlarged to 13, 15, or 17.
- Denominator 24 can only be enlarged to 27.

The Concept Definite Integral

To begin with the definition of definite integral, first the notion of a Riemann sum is introduced (Thomas et al., 2001). This terminology was taken from

the name of German mathematician Bernhard Riemann (Maulidi et al., 2019). Suppose a function f defined on a closed interval $[a, b]$. Then, this interval is subdivided into some subintervals that are not necessarily of equal widths. Next, choose $n-1$ points in interval $[a, b]$ namely x_1, x_2, \dots, x_{n-1} that satisfy

$$a < x_1 < x_2 < \dots < x_{n-1} < b.$$

For easy presentation, denote a by x_0 and b by x_n , so that

$$a = x_0 < x_1 < x_2 < \dots < x_{n-1} < x_n = b.$$

Call the set $P = \{x_0, x_1, \dots, x_{n-1}, x_n\}$ as a partition of $[a, b]$. Another concept of partition can be found in Haryeni et al. (2021). Using this construction, the partition P divides interval $[a, b]$ into n closed subintervals $[x_0, x_1], [x_1, x_2], \dots, [x_{n-1}, x_n]$, where the k^{th} subinterval of P is $[x_{k-1}, x_k]$ for an integer $k \in [1, n]$. Denote the width of k^{th} subinterval by $\Delta x_k = x_k - x_{k-1}$, and if all n subintervals have equal width, then $\Delta x_k = \frac{b-a}{n}$ is obtained. When the width of partition P is not equal then define $\|P\|$ as the largest of all the subinterval widths.

In each subinterval $[x_{k-1}, x_k]$, choose a point x_k^* , where $x_k^* \in [x_{k-1}, x_k]$. Then form a rectangle with height $f(x_k^*)$ and width Δx_k , so the broad of this rectangle is $f(x_k^*) \cdot \Delta x_k$. Next, sum all of the products for all $k \in [1, n]$ to get

$$S_p := \sum_{k=1}^n f(x_k^*) \cdot \Delta x_k.$$

Call the sum S_p as a Riemann sum for f on the interval $[a, b]$.

Next, consider the limit of this Riemann sum when the sub partitions of P

become infinity or the width of Δx_k approaches zero. When the n closed subintervals of P go to infinity ($n \rightarrow \infty$) it results that the width of each Δx_k will approach zero ($\Delta x_k \rightarrow 0$). It leads us to consider the limit of the Riemann sum

$$\lim_{\substack{n \rightarrow \infty \\ \Delta x_k \rightarrow 0}} \sum_{k=1}^n f(x_k^*) \cdot \Delta x_k.$$

If this limit exists then the value of this limit is defined as a definite integral of function f over interval $[a, b]$, and

symbolized by $\int_a^b f(x) dx$. Then, define the definite integral as

$$\int_a^b f(x) dx = \lim_{\substack{n \rightarrow \infty \\ \Delta x_k \rightarrow 0}} \sum_{k=1}^n f(x_k^*) \cdot \Delta x_k.$$

From this construction, it can be interpreted that the concept of definite integral relates to the concept of an area under the graph of a nonnegative function.

RESULTS AND DISCUSSION

Correction in Achmad Yani's Misconception

This section will review some of the misconceptions contained in Yani (2016) entitled "*Faraidh dan Mawaris Bunga Rampai Hukum Islam*", especially in Chapter 15.3. For this reason, in order to get a complete comprehension of Achmad Yani's concept and be able to follow the corrections the author convey here, it is recommended to readers read Chapter 15.3 of the book.

Here are some of the misconceptions contained in Chapter 15.3 of the book.

1. Interpret the concept of derivative as a decrease or inheritance of property from inheritor to heir.

In his book on page 242, the author states:

"Di sini, istilah 'turunan' (derivatif) dari matematika dihubungkan dengan istilah "penurunan" atau "pewarisan" harta dari pewaris (si mayit) kepada ahli waris, sehingga yang dimaksud dengan

dy/dx di sini adalah harta yang diwariskan (diturunkan) kepada semua ahli waris yang nilainya masing-masing belum diketahui."

("Here, the term of 'derivative' in mathematics is associated with the term "inheritance" of property from the inheritor (the corpse) to the heir, so that what is meant here by dy/dx is a property passed down to all heirs whose respective values are not yet known")

As far as our searches for the meaning of derivative concepts go, there are two meanings of derivative, that is in a physical point of view and in a geometric point of view. From a physical point of view, a derivative has a meaning as a change in a quantity expressed by a function to the variables of that function (Apostol, 1991). For example, the distance function that changes against the time variable, the volume function that changes against the time variable, the trader's income function that changes against the variable of the sale of an item, etc. This concept is also known as rate. The changes in the distance function of a moving particle expressed by $f(t)$ with respect to the time variable (t) comes up with the concept of instantaneous velocity. So, in this case, the derivative of the function $f(t)$ with respect to variable t , that is $\frac{d}{dt} f(t) = f'(t)$ gives means the instantaneous velocity of the such particle (Strang, 1991).

Whereas in geometrical views, the derivative means the slope (gradient) of a tangent line (Anton et al., 2021). If there is a curve presented with a function $y = f(x)$, where the variable x is defined in an interval I , and $x_0 \in I$. Then, the line that offends the curve $y = f(x)$ at point $A(x_0, f(x_0))$ is called the tangent line and is noted by the function $g(x)$. Thus, the

derivative of the function $f(x)$ with respect to x , i.e., $\frac{d}{dx}f(x) = f'(x)$ means the slope of the function $g(x)$.

However, the concept of the derivative which is interpreted as the inheritance of property from the inheritor to the heir, the authors do not get in any reference. So, this concept is a new concept as an author's *ijtihad*. If this concept is correct then this *ijtihad* will contribute greatly to the scientific treasures of the meaning of derivative concepts. However, unfortunately, the authors do not explain the meaning of the concept mathematically as is commonly done in references to differential calculus. Furthermore, the meaning of derivative as an inheritance of property from inheritor to heir brings up contradictions in the definition of inherited property functions, as the authors will explain in the next review. In our opinion, the concept is wrong, in the sense that the derivative concept cannot be interpreted as an inheritance of property from the inheritor to the heir.

Presumably, the authors were inspired by the translation of the word derivative which is commonly translated into Indonesian with "turunan". So, the authors interpret the concept of derivative as the inheritance of property from the inheritor to the heir. Whereas although interpreted as "turunan", the concept of the derivative is not related at all to the process of inheritance of something. The object of the derivative is a function. A function that is to be derived does not mean that the function is inherited in its quality or quantity.

2. There is a contradiction in defining the function of inherited treasures.

In his book on page 241, the author states:

"Selanjutnya, untuk dapat dicari penyelesaiannya, maka seharusnya

masalah itu dimodelkan menjadi persamaan berikut ini:")

$$\left(\frac{2}{3}\right)x + \left(\frac{1}{6}\right)x + \left(\frac{1}{6}\right)x + \left(\frac{1}{8}\right)x = 1y \quad (1)$$

Sekarang diasumsikan sebagai berikut:

y = total harta warisan

x = nilai satuan harta yang diwariskan"

("Furthermore, to be able to find a solution, then the problem should be modeled into the following equation:

$$\left(\frac{2}{3}\right)x + \left(\frac{1}{6}\right)x + \left(\frac{1}{6}\right)x + \left(\frac{1}{8}\right)x = 1y \quad (1)$$

Now it is assumed as follows:

y = total inheritance

x = the unit value of inherited treasures")

Next, on the same page, the author also states:

"Secara matematis, hubungan antara variabel x dan y dapat ditulis sebagai berikut:

$$y = f(x). \quad (2)$$

("Mathematically, the relationship between the variables x and y can be written as follows:

$$y = f(x). \quad (2)$$

And on page 242, the author states:

"Sementara itu, harta yang diwariskan (diturunkan) kepada semua ahli waris untuk contoh kasus di atas dapat dituliskan sebagai:

$$f'(x) = \left(\frac{2}{3}\right)x + \left(\frac{1}{6}\right)x + \left(\frac{1}{6}\right)x + \left(\frac{1}{8}\right)x. \quad (3)$$

("Meanwhile, the treasures passed down to all heirs for the above case examples can be written as:

$$f'(x) = \left(\frac{2}{3}\right)x + \left(\frac{1}{6}\right)x + \left(\frac{1}{6}\right)x + \left(\frac{1}{8}\right)x. \quad (3)$$

According to the description submitted by the author, the following facts are obtained. Equation (1) can be written as

$$y = \left(\frac{2}{3}\right)x + \left(\frac{1}{6}\right)x + \left(\frac{1}{6}\right)x + \left(\frac{1}{8}\right)x \quad (4)$$

Next, according to equation (2), then equation (4) can be written as

$$f(x) = \left(\frac{2}{3}\right)x + \left(\frac{1}{6}\right)x + \left(\frac{1}{6}\right)x + \left(\frac{1}{8}\right)x$$

(5)

Then, if the function $f(x)$ is derived with respect to x will be obtained

$$f'(x) = \left(\frac{2}{3}\right) + \left(\frac{1}{6}\right) + \left(\frac{1}{6}\right) + \left(\frac{1}{8}\right) = \frac{27}{24} \quad (6)$$

So, there is a contradiction between equation (3) and equation (6), that is, the function $f'(x)$ in equation (3) is different from the function $f'(x)$ in equation (6).

Similarly, there is a contradiction between equation (3) and equation (5), i.e., the same function has two different names. The existence of contradictions in the description made by the author shows that the concept of derivatives cannot be interpreted as an inheritance of property from the inheritor to the heir.

3. Calculate the total inheritance using the concept of indefinite integral.

In his book on page 241, the author solves the problem of 'aul' by applying the concept of indefinite integral. Whereas if the author consistently defines the variable y as the total of inheritance, then the author should have used a definite integral concept in solving this 'aul' problem. This is because the definite integral concept is related to calculating the area under an integrated function. To calculate the broad of the area, the definite integral uses the Riemann sum approach (Fitzpatrick, 2006), that is, by dividing the calculated area into several parts, then each broad of those parts is calculated and ends by summing the broad of those parts.

In this case, calculating the share of the property to be acquired by the heirs can be analogous to calculating the broad of the parts on a definite integral concept. The difference is that in the concept of the division of the total inherited property the inheritance property has been known and will be sought for the share of the property

to be obtained by the heirs. Whereas in the concept of definite integral what is known is the broad of the parts of the area to be sought for breadth.

Therefore, it is supposed that the author used the concept of definite integral in this 'aul' problem. Although whether the concept of the definite integral can really be applied in solving the 'aul' problem still requires further research.

Can the Definite Integral Solves 'Aul' Problem?

As far as the research that the authors have done, the authors have not been able to construct the use of definite integral to solve the problem of 'aul'. As the author stated in the previous section, there is a similarity of concepts between the problem of the division of inherited property and definite integral, that is, both relate to the problem of a quantity and the parts smaller than that quantity. 'Aul' deals with the issue of the quantity of inherited property which must be divided into smaller quantities in order to be handed over to the heirs, whereas definite integral relates to the problem of calculating the broad of an area by dividing the area into smaller areas, followed by summing up the broad of each smaller area.

If a definite integral construction will be used in solving the problem of the division of inherited property, then the following two equations are made

$$\begin{array}{l} \text{the amount of inherited} \\ \text{property acquired by} \\ \text{all heirs} \end{array} = \begin{array}{l} \text{share of property} \\ \text{for the 1}^{\text{st}} \text{ heir} \end{array} \quad (7)$$

$$\begin{array}{l} + L + \\ \text{share of property} \\ \text{for the } i^{\text{st}} \text{ heir} \end{array} = \begin{array}{l} \text{proportion (faradh)} \\ \text{of } i^{\text{st}} \text{ inheritance} \end{array} \times \begin{array}{l} \text{total of} \\ \text{inherited properties} \end{array} \quad (8)$$

Next, the following variables are defined:

y = the amount of inherited property acquired by all heirs

y_i = share of the property for the i^{th} heir

Δx_i = proportion (*faradh*) of i^{th} inheritance

A = the total of inherited property to be distributed to all heirs

The value of the proportion (*faradh*) of the heirs is between 0 to 1, that is $\Delta x_i \in (0,1]$. Furthermore, based on the variables that have been defined, Equation (7) can be presented with the equation

$$y = y_1 + y_2 + \dots + y_n \quad (9)$$

and Equation (8) can be presented with equation

$$y_i = \Delta x_i \cdot A \text{ or } y_i = A \cdot \Delta x_i. \quad (10)$$

Substitution of Equation (10) to Equation (9) will obtain

$$y = A \cdot \Delta x_1 + A \cdot \Delta x_2 + \dots + A \cdot \Delta x_n \\ = \sum_{i=1}^n A \cdot \Delta x_i \quad (11)$$

For $n \rightarrow \infty$ and $\Delta x_i \rightarrow 0$ then Equation (11) would be a definite integral

$$y = \int_0^1 A \, dx$$

The solution of the definite integral (12) is

$$y = [Ax]_0^1 = A(1) - A(0) = A.$$

The value of 1 in $A(1)$ means the proportion of an heir to the total existing heirs. Proportion 1 means that the heir will inherit the entire existing inheritance. So, $A(1)$ means the total inheritance property distributed to the heir in the proportion of 1, which means that the entire inheritance will be given to him. While the value of 0 in $A(0)$ means that an heir does not have a share (proportion / *faradh*) of the inheritance. So that he does not get a part of the inheritance. This is certainly impossible because all heirs must have a share of the inheritance. It is also in line with $0 \notin (0,1] = \Delta x_i$, that is the value of 0 is not an element of the proportion of heirs (Δx_i).

Until this stage, the authors have managed to construct the definite integral. This construction produces a correct equation, that is

$$y = A$$

which means the number of inherited properties acquired by all heirs = the total of inherited property to be distributed to all heirs. However, the construction cannot be used to solve the problem of the division of inherited property.

CONCLUSIONS AND SUGGESTIONS

There are at least three misconceptions in Achmad Yani's book regarding the use of integral to solve the problem of '*aul*'. That are: 1). Interpreting or interpreting derivative as the inheritance of property from the inheritor to the heir, 2). The existence of contradictions in defining the function of inherited property, and 3). Calculating the total inheritance using the concept of the indefinite integral. The existence of these errors certainly does not mean that the concept of indefinite and finite integral cannot be used to solve the problem of '*aul*' in particular and the problem of the division of inherited property in general. This still needs more research.

But as far as the research that authors have done, the definite integral concept has not been able to be used to solve the problem of '*aul*' yet. So, the issue is an open problem for all readers for further research.

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