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An analysis of genotype inheritance in a trihybrid cross by applying a diagonalization of the matrix method

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ABSTRACT

This research aimed to find out the formulation of inheritance to know the genotype of the n -th generation in trihybrid crosses with controlled parent genotypes and analyze them by applying the diagonalization of a matrix. Matrix diagonalization makes it easier to find out the inheritance genotype of the n -th generation in trihybrid to obtain superior offspring compared with crossing it one by one, which requires a lot of time and cost. Based on the analysis, an equation for the probability of inheritance was obtained for 27 genotypes of the n -th generation, and the resulting offspring in an infinite generation are likely to have the TTKKBB genotype.

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INTRODUCTION

A matrix is a rectangular array of numbers. The numbers in the array are called the entries of the matrix (Anton et al., 2019). The application of several concepts from linear algebra, especially values and eigenvectors, and matrix diagonalization in genetics has been widely discussed (Wijayanti, 1997; Yuliani & Mashuri, 2012). The matrix can be met in a branch of linear algebra that has a very important role in its implementation (Sadhukhan et al., 2022) for the genetic similarity matrix constructed with Jaccard's coefficient using RAPD (Random Amplified Polymorphic DNA).

Diagonalization is the process of forming a diagonal matrix that involves the eigenvalue and eigenvector of the matrix. Searching for eigenvalues and eigenvectors could be a way of using linear algebra that could be used to determine the genotype of offspring for the next generation (Syafwan & Nurwati, 2015). The principles of investigations of genetic code systems from the viewpoint of matrix approaches were published in Petoukhov (2011b, 2011a, 2012). Petoukhov & Petukhova (2016) and Petoukhov (2017) studied the systems of structured alphabets of DNA and RNA in matrix forms of their representations. Paniello (2021)

introduced the concept of in-evolutionary operator ergodicity defined on coalgebras with genetic realization. This concept provides new insights into the correspondence between genetic coalgebra and cubic stochastic matrices, the in-evolution operator emerging as a Markov process defined by the matrix accompanying coalgebra cubic stochastic matrices.

A square matrix $A_{n \times n}$ can be said to be diagonalizable if there is matrix P which has an inverse such that

$$D = P^{-1}AP \quad (1)$$

is a diagonal matrix. Matrix P is called the matrix that diagonalizes matrix A (Anton et al., 2019). The application of diagonalization to the matrix can solve problems in biology, especially genetics, and make it easier to predict genotype inheritance from generation to generation.

Genetics is a branch of biology that studies the inheritance from one generation to the next generation. For example, Gregor Mendel, the father of genetics, studied inheritance by conducting a cross-experiment on peas to determine the genotype distribution of certain traits in a population (Kumari et al., 2018).

In genetics, a cross-term is carried out to combine the traits of two individuals so that new offspring with better quality are produced. One problem in crossing processes is the determination of different properties that require a relatively long processing time if done conventionally (without computer tools). In other words, a computer program package is required to find individuals who are superior to generations via accurate and precise crossing processes.

In the previous research, Kaffah & Romdhini (2015) used the diagonalization of the matrix method to determine the n^{th} individual based on the parent genotype probability, and obtained equation

$$x^n = A^n x^0 \quad (2)$$

so that the genotype probability of an individual in the n th generation can be determined when the limit n goes to infinity. This research also applied diagonalization of the matrix to obtain a solution to the probability equation for the inheritance of normal parental genotypes with all possible trihybrid genotypes.

METHOD

Theoretically, a trihybrid cross will get 2^3 or 8 gametes and 4^3 or 64 cross combinations (Sobir & Syukur, 2015). To solve the probability equation for the inheritance of normal parental genotypes with all possible trihybrid genotypes, this research focus on a diagonalization matrix procedure. The following are the steps required in the procedure.

Consider a square matrix $A_{n \times n}$. The steps in diagonalizing a matrix can be done as follows (Anton et al., 2019):

1. Determine the eigenvalues from the matrix A
2. Define n linearly independent eigenvectors, p_1, p_2, \dots, p_n based on eigenvalue in step 1.
3. Matrix form P where the column vectors are vectors p_1, p_2, \dots, p_n .
4. Find matrix P^{-1} based on the following formula

$$D = P^{-1}AP = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n) \quad (3)$$

with $\lambda_i, i = 1, 2, \dots, n$ is characteristic roots from matrix A .

5. Define the gene symbol in the form of a variable.
6. Conduct a cross-test of three different traits.
7. Identify the form of linear equations based on the probability table, so that obtained a linear equation system in form matrix A .
8. Looking for eigenvalues from matrix A and eigenvectors corresponding

- with the eigenvalues.
- 9. Forming matrix P and determining inverse matrix P .
- 10. Forming a diagonal matrix using the formula in Equation (3).
- 11. Determine the offspring in the n^{th} generation.
- 12. Analyzing results and drawing a conclusion.

For fast computing, all calculations in this research used Wolfram Mathematica programming (see <https://www.wolfram.com/mathematica/online/> for the online version).

RESULTS AND DISCUSSION

Determination of Genotypic Distribution

The genotypic of both parents used in this research is a combination of three traits symbolized by TTKKBB (*homozygote* dominant) and ttkkbb (*heterozygote* recessive) (see Table 1). In Table 1, a pair of three different traits have been marked. The cross produced 64 genotypes, which will appear to consist of 27 different genotypes.

Table 1. A Cross of a Pair of Three Different Traits

	TKB	TKb	TkB	Tkb	tKB	tKb	tkB	tkb
TKB	TTKKBB	TTKKBb	TTKkBB	TTKkBb	TtKKBB	TtKKBb	TtKkBB	TtKkBb
TKb		TTKKbb		TTKkbb		TtKKbb		TtKkbb
TkB			TTkkBB	TTkkBb			TtkkBB	TtkkBb
Tkb				TTkkbb				Ttkkbb
tKB					ttKKbb	ttKKBb	ttKkBB	ttKkBb
tKb						ttKKbb		ttKkbb
tkB							ttkkBB	ttkkBb
tkb								ttkkbb

A cross between parents TTKKBB and TTKKBB has a full genotype probability value of TTKKBB. The probability of genotypes from normal

crosses of TTKKBB with all possible genotypes available is shown in Table 2.

Table 2. Probability of Genotypes from Normal Crosses of TTKKBB with All Possible Genotypes Available

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	a_{n-1}	b_{n-1}	c_{n-1}	d_{n-1}	e_{n-1}	f_{n-1}	g_{n-1}	h_{n-1}	i_{n-1}	j_{n-1}	k_{n-1}	l_{n-1}	m_{n-1}	n_{n-1}
$a_n = \text{TTKKBB}$	1	1/2	1/2	1/4	1/2	1/4	1/4	1/8	0	0	0	0	0	0
$b_n = \text{TTKKBb}$	0	1/2	0	1/4	0	1/4	0	1/8	1	1/2	1/2	1/4	0	0
$c_n = \text{TTKkBB}$	0	0	1/2	1/4	0	0	1/4	1/8	0	0	0	0	1	1/2
$d_n = \text{TTKkBb}$	0	0	0	1/4	0	0	0	1/8	0	1/2	0	1/4	0	1/2
$e_n = \text{TtKKBB}$	0	0	0	0	1/2	1/4	1/4	1/8	0	0	0	0	0	0
$f_n = \text{TtKKBb}$	0	0	0	0	0	1/4	0	1/8	0	0	1/2	1/4	0	0
$g_n = \text{TtKkBB}$	0	0	0	0	0	0	1/4	1/8	0	0	0	0	0	0
$h_n = \text{TtKkBb}$	0	0	0	0	0	0	0	1/8	0	0	0	1/4	0	0

Table 2. Probability of Genotypes from Normal Crosses of TTKKBB with All Possible Genotypes Available (continue)

	15	16	17	18	19	20	21	22	23	24	25	26	27
	o_{n-1}	p_{n-1}	q_{n-1}	r_{n-1}	s_{n-1}	t_{n-1}	u_{n-1}	v_{n-1}	w_{n-1}	x_{n-1}	y_{n-1}	z_{n-1}	α_{n-1}
$a_n = \text{TTKKBB}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$b_n = \text{TTKKBb}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$c_n = \text{TTKkBB}$	1/2	1/4	0	0	0	0	0	0	0	0	0	0	0
$d_n = \text{TTKkBb}$	0	1/4	1	1/2	0	0	0	0	0	0	0	0	0
$e_n = \text{TtKKBB}$	0	0	0	0	1	1/2	1/2	1/4	0	0	0	0	0
$f_n = \text{TtKKBb}$	0	0	0	0	0	1/2	0	1/4	1	1/2	0	0	0
$g_n = \text{TtKkBB}$	1/2	1/4	0	0	0	0	1/2	1/4	0	0	1	1/2	0
$h_n = \text{TtKkBb}$	0	1/4	0	1/2	0	0	0	1/4	0	1/2	0	1/2	1

Identify a Form of Linear Equation

A linear equation is formed based on the probability value obtained from crossing a normal parent with 27 types of genotypes. The linear equation expresses the probability of 27 genotypes in the n^{th} generation. Based on the existing probability value, it is possible to determine each generation's genotype distribution from the previous generation's genotype distribution using the equation. Where the equation state that all the offspring produced are $a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n, j_n, k_n, l_n, m_n, n_n, o_n, p_n, q_n, r_n, s_n, t_n, u_n, v_n, w_n, x_n, y_n, z_n, \alpha_n$ from crosses normal individual TTKKBB with genotype individual TTKKBB, TTKKBb, TTKkBB, TTKkBb, TtKKBB, TtKKBb, TtKkBB, TtKkBb, TTKKbb, TTKkbb, TtKKbb, TtKkbb, TTkkBB, TTkkBb, TtkkBB, TtkkBb, TTkkbb, Ttkkbb, ttKKBB, ttKKBb, ttKkBB, ttKkBb, ttKKbb, ttKkbb, ttkkBB, ttkkBb, and ttkkbb listed in order with $a_{n-1}, b_{n-1}, c_{n-1}, d_{n-1}, e_{n-1}, f_{n-1}, g_{n-1}, h_{n-1}, i_{n-1}, j_{n-1}, k_{n-1}, l_{n-1}, m_{n-1}, n_{n-1}, o_{n-1}, p_{n-1}, q_{n-1}, r_{n-1}, s_{n-1}, t_{n-1}, u_{n-1}, v_{n-1}, w_{n-1}, x_{n-1}, y_{n-1}, z_{n-1}$, and α_{n-1} .

The probability of the initial genotype ($n = 0$) are $a_0, b_0, c_0, d_0, f_0, g_0, h_0, i_0, j_0, k_0, l_0, m_0, n_0, o_0, p_0, q_0, r_0, s_0, t_0, u_0, v_0, w_0, x_0, y_0, z_0$, and α_0 . In addition,

for $n = 1, 2, \dots, N$, the following relationship applies.

$$\begin{aligned}
 & a_n + b_n + c_n + d_n + e_n + f_n + \\
 & g_n + h_n + i_n + j_n + k_n + l_n + \\
 & m_n + n_n + o_n + p_n + q_n + \\
 & r_n + s_n + t_n + u_n + v_n + w_n + \\
 & x_n + y_n + z_n + \alpha_n = 1
 \end{aligned} \tag{4}$$

where

$$\begin{aligned}
 a_n &= a_{n-1} + \frac{1}{2}b_{n-1} + \frac{1}{2}c_{n-1} + \frac{1}{4}d_{n-1} + \\
 & \frac{1}{2}e_{n-1} + \frac{1}{4}f_{n-1} + \frac{1}{4}g_{n-1} + \frac{1}{8}h_{n-1} \\
 b_n &= \frac{1}{2}b_{n-1} + \frac{1}{4}d_{n-1} + \frac{1}{4}f_{n-1} + \frac{1}{4}g_{n-1} + \\
 & \frac{1}{8}h_{n-1} + i_{n-1} + \frac{1}{2}j_{n-1} + \frac{1}{2}k_{n-1} + \\
 & \frac{1}{4}l_{n-1} \\
 c_n &= \frac{1}{2}c_{n-1} + \frac{1}{4}d_{n-1} + \frac{1}{8}h_{n-1} + m_{n-1} + \\
 & \frac{1}{2}n_{n-1} + \frac{1}{2}o_{n-1} + \frac{1}{4}p_{n-1} \\
 d_n &= \frac{1}{4}d_{n-1} + \frac{1}{8}h_{n-1} + \frac{1}{2}j_{n-1} + \frac{1}{2}k_{n-1} + \\
 & \frac{1}{4}l_{n-1} + \frac{1}{2}n_{n-1} + \frac{1}{4}p_{n-1} + q_{n-1} + \\
 & \frac{1}{2}r_{n-1} \\
 e_n &= \frac{1}{2}e_{n-1} + \frac{1}{4}f_{n-1} + \frac{1}{4}g_{n-1} + \frac{1}{8}h_{n-1} + \\
 & s_{n-1} + \frac{1}{2}t_{n-1} + \frac{1}{2}u_{n-1} + \frac{1}{4}v_{n-1} \\
 f_n &= \frac{1}{4}f_{n-1} + \frac{1}{8}h_{n-1} + \frac{1}{4}l_{n-1} + \frac{1}{2}t_{n-1} + \\
 & \frac{1}{4}v_{n-1} + w_{n-1} + \frac{1}{2}x_{n-1} \\
 g_n &= \frac{1}{4}g_{n-1} + \frac{1}{8}h_{n-1} + \frac{1}{2}o_{n-1} + \frac{1}{4}p_{n-1} + \\
 & \frac{1}{2}u_{n-1} + \frac{1}{4}v_{n-1} + y_{n-1} + \frac{1}{2}z_{n-1}
 \end{aligned}$$

$$\begin{aligned}
 h_n &= \frac{1}{8}h_{n-1} + \frac{1}{4}l_{n-1} + \frac{1}{4}p_{n-1} + \frac{1}{2}r_{n-1} + \\
 &\quad \frac{1}{4}v_{n-1} + \frac{1}{2}x_{n-1} + \frac{1}{2}z_{n-1} + \alpha_{n-1} \\
 i_n &= j_n = k_n = l_n = m_n = n_n = o_n = \\
 p_n &= q_n = r_n = s_n = t_n = u_n = v_n = \\
 w_n &= x_n = y_n = z_n = \alpha_n = 0.
 \end{aligned}$$

In Equation (4), the equation with a value of 0 will not have a genotype in this breeding program. Equation (4) could be written in matrix form as:

$$x^n = Ax^{n-1} \forall n \in N \tag{5}$$

In Equation (5), x^n represents the distribution of offspring for the n^{th} generation and can be rewritten as follow.

$$\begin{aligned}
 x^n &= \\
 [a_n & b_n \quad c_n \quad d_n \quad e_n \quad f_n \quad g_n \quad h_n \\
 i_n & j_n \quad k_n \quad l_n \quad m_n \quad n_n \quad o_n \quad p_n \\
 q_n & r_n \quad s_n \quad t_n \quad u_n \quad v_n \quad w_n \quad x_n \\
 & y_n \quad z_n \quad \alpha_n]^T \tag{6}
 \end{aligned}$$

Similar to Equation (6), the form x^{n-1} can be expressed in the following form.

$$\begin{aligned}
 x^{n-1} &= \\
 [a_{n-1} & b_{n-1} \quad c_{n-1} \quad d_{n-1} \quad e_{n-1} \\
 f_{n-1} & g_{n-1} \quad h_{n-1} \quad i_{n-1} \\
 k_{n-1} & l_{n-1} \quad m_{n-1} \quad n_{n-1} \\
 p_{n-1} & q_{n-1} \quad r_{n-1} \quad s_{n-1} \\
 u_{n-1} & v_{n-1} \quad w_{n-1} \quad x_{n-1} \\
 & z_{n-1} \quad \alpha_{n-1}]^T \tag{7}
 \end{aligned}$$

In Equation (5), the genotype distribution of the parents is expressed below:

$$A = [a_{ij}]; i, j = 1, 2, \dots, 27. \tag{8}$$

The results of the elaboration carried out on the genotype distribution of the parents in Equation (8) obtained several possibilities, as follows:

- $a_{ij} = 1$
for $\{i, j\} = \{(1), (1)\}, \{(2), (8)\}, \{(3), (13)\}, \{(4), (17)\},$

- $\{(5), (19)\}, \{(6), (23)\}, \{(7), (25)\}, \{(8), (27)\}$ };
- $a_{ij} = \frac{1}{2}$
for $\{i, j\} = \{(1), (2,3,5)\}, \{(2), (1,10,11)\}, \{(3), (3,14,15)\}, \{(4), (10,14,18)\}, \{(5), (5,20,21)\}, \{(6), (11,20,24)\}, \{(7), (15,21,26)\}, \{(8), (18,24,26)\}$ };
- $a_{ij} = \frac{1}{4}$
for $\{i, j\} = \{(1), (4,6,7)\}, \{(1), (4,6,7)\}, \{(2), (4,6,7,12)\}, \{(3), (4,16)\}, \{(4), (4,12,16)\}, \{(5), (6,7,22)\}, \{(6), (6,12,22)\}, \{(7), (7,16,22)\}, \{(8), (12,16,22)\}$ };
- $a_{ij} = \frac{1}{8}$
for $\{i, j\} = \{(1, \dots, 8), (8)\}$ };
- $a_{ij} = 0$
for the values of i and j other than those stated above.

Eigenvalues and Eigenvectors Matrix A

In this section, the computational results of the eigenvalues and eigenvectors concerning matrix A given in the previous section will be shown.

Consider the genotype distribution of the parents in Equation (8). The eigenvalues of matrix A in Equation (8) are:

$$\{\lambda_i\} = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \tag{9}$$

where $i = 1, 2, \dots, 27$.

Let a matrix P where the columns are eigenvectors corresponding to the eigenvalues in Equation (9) as written in a matrix form below.

$$P = [p_{ij}]; i, j = 1, 2, \dots, 27. \tag{10}$$

The matrix P in Equation (10) is the result of the diagonalization of matrix A from Equation (3). The results of the elaboration carried out on the genotype distribution of the parents in Equation (10) yielded the following possibilities:

- $p_{ij} = 1$
for $\{i, j\} =$
 $\{(1), (1,5,6,7,9,17,23,27)\},$
 $\{(2), (4,8,16,22)\},$
 $\{(3), (3,8,15,26)\},$
 $\{(4), (7,14)\},$
 $\{(5), (2,8,21,25)\},$
 $\{(6), (6,20)\},$
 $\{(7), (5,24)\},$
 $\{(8), (8)\},$
 $\{(9, \dots, 27), (27, \dots, 9)\}.$
 - $p_{ij} = -1$
for $\{i, j\} = \{$
 $\{(1), (2,3,4,8,11,13,19)\},$
 $\{(2), (6,7,10)\}, \{(3), (5,7,12)\},$
 $\{(4), (8)\}, \{(5), (5,6,18)\},$
 $\{(6,7), (8)\}.$
 - $p_{ij} = 2$
for $\{i, j\} = \{ \{(2), (13,19)\},$
 $\{(3), (11,19)\}, \{(4), (10,12)\},$
 $\{(5), (11,13)\}, \{(6), (10,18)\},$
 $\{(7), (12,18)\}.$
 - $p_{ij} = -2$
for $\{i, j\} = \{ \{(2), (9,27)\},$
 $\{(3), (9,23)\}, \{(4), (22,26)\},$
 $\{(5), (9,17)\}, \{(6), (16,25)\},$
 $\{(7), (15,21)\}, \{(8), (14,20,24)\}.$
 - $p_{ij} = 4$
for $\{i, j\} = \{(4,6,7), (9)\}.$
 - $p_{ij} = -4$
for $\{i, j\} = \{ \{(4), (19)\}, \{(6), (13)\},$
 $\{(7), (11)\}, \{(8), (10,12,18)\}.$
 - $p_{ij} = -8$
for $\{i, j\} = \{(8), (9)\}.$
 - $p_{ij} = 0$
for the values of i and j other than those stated above.
- $\{(2), (5,6,7,8,11,12,15,16,18)\},$
 $\{(5), (7,8,12)\},$
 $\{(3), (3,4,7,8,10,12,21,22,24)\},$
 $\{(6), (6,8,16)\},$
 $\{(4), (2,4,6,8,14,16,20,22,26)\},$
 $\{(7), (4,8,22)\}, \{(8), (8)\},$
 $\{(9, \dots, 27), (27, \dots, 9)\}.$
 - $b_{ij} = 2$
for $\{i, j\} = \{(2), (19, \dots, 27)\},$
 $\{(3), (13, \dots, 18, 25, 26, 27)\},$
 $\{(4), (9, \dots, 12, 17, 18, 23, 24, 27)\},$
 $\{(5), (15, 16, 18, 21, 22, 24)\},$
 $\{(6), (11, 12, 18, 20, 22, 26)\},$
 $\{(7), (10, 12, 14, 16, 24, 26)\},$
 $\{(8), (12, 16, 22)\}.$
 - $b_{ij} = 4$
for $\{i, j\} = \{(5), (25, \dots, 27)\},$
 $\{(6), (23, 24, 27)\},$
 $\{(7), (17, 18, 27)\},$
 $\{(8), (12, 16, 22)\}.$
 - $b_{ij} = 8$
for $\{i, j\} = \{(8), (27)\}.$
 - $b_{ij} = 0$
for the values of i and j other than those stated above.

The results obtained concerning the values of a_{ij} , p_{ij} , and b_{ij} for all $\{i, j\}$ can be constructed diagonal matrix D as given in equation (1).

Diagonalization Matrix A

Consider a linear equation that expresses the probability of 27 genotypes in the n^{th} generation in equation (5). Using the results of Equation (10) and Equation (11), Equation (5) can be calculated numerically using the following formula:

$$x^n = A^n x^{(0)} = PD^n P^{-1} x^{(0)}; \quad (12)$$

where $n = 1, 2, \dots, N$.

The factor on the right side of Equation (12) can be expressed in the following form:

$$x^n = PD^n P^{-1} [a_0 \quad b_0 \quad c_0 \quad d_0 \quad e_0 \quad f_0 \quad (13)$$

$$g_0 \quad h_0 \quad i_0 \quad j_0 \quad k_0 \quad l_0$$

Such that matrix P^{-1} is obtained, which is the inverse of matrix P .

$$P^{-1} = [b_{ij}]; \quad i, j = 1, 2, \dots, 27. \quad (11)$$

The results of the elaboration carried out on P^{-1} in Equation (11) yielded the following possibilities:

- $b_{ij} = 1$
for $\{i, j\} = \{(1), (1, \dots, 27)\},$

$$\begin{matrix} m_0 & n_0 & o_0 & p_0 & q_0 & r_0 \\ s_0 & t_0 & u_0 & v_0 & w_0 & x_0 \\ y_0 & z_0 & \alpha_0 \end{matrix}$$

Using the numerical results of Equation (8) and Equation (9), the probability of 27 genotypes in the n^{th} generation in Equation (13) can be calculated in the following way:

$$\begin{aligned} a_n = & a_0 + \left(1 - \left(\frac{1}{2}\right)^n\right) b_0 + \left(1 - \left(\frac{1}{2}\right)^n\right) c_0 + \left(1 - 2\left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n\right) d_0 + \\ & \left(1 - \left(\frac{1}{2}\right)^n\right) e_0 + \left(1 - 2\left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n\right) f_0 + \left(1 - 2\left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n\right) g_0 + \\ & \left(1 - 3\left(\frac{1}{2}\right)^n + 3\left(\frac{1}{4}\right)^n - \left(\frac{1}{8}\right)^n\right) h_0 + \left(1 - \left(\frac{1}{2}\right)^n\right) i_0 + \left(1 + 2\left(\frac{1}{2}\right)^{2n} - \right. \\ & \left. 2\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n\right) j_0 + \left(1 + 2\left(\frac{1}{2}\right)^{2n} - 2\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n\right) k_0 + \left(1 - 2\left(\frac{1}{2}\right)^{3n} + \right. \\ & \left. 4\left(\frac{1}{2}\right)^{2n} - 4\left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n\right) l_0 + \left(1 - 2\left(\frac{1}{2}\right)^n\right) m_0 + \left(1 + 2\left(\frac{1}{2}\right)^{2n} - \right. \\ & \left. 2\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n\right) n_0 + \left(1 + 2\left(\frac{1}{2}\right)^{2n} - 2\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n\right) o_0 + \left(1 - 2\left(\frac{1}{2}\right)^{3n} + \right. \\ & \left. 4\left(\frac{1}{2}\right)^{2n} - 4\left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n\right) p_0 + \left(1 - 4\left(\frac{1}{2}\right)^n - 4\left(\frac{1}{4}\right)^n\right) q_0 + \left(1 - \right. \\ & \left. 4\left(\frac{1}{2}\right)^{3n} + 4\left(\frac{1}{2}\right)^{2n} - 4\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n + 4\left(\frac{1}{4}\right)^n\right) r_0 + \left(1 - 2\left(\frac{1}{2}\right)^n\right) s_0 + \\ & \left(1 + 2\left(\frac{1}{2}\right)^{2n} - 2\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n\right) t_0 + \left(1 + 2\left(\frac{1}{2}\right)^{2n} - 2\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n\right) u_0 + \\ & \left(1 - 2\left(\frac{1}{2}\right)^{3n} + 4\left(\frac{1}{2}\right)^{2n} - 4\left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n\right) v_0 + \left(1 - 4\left(\frac{1}{2}\right)^n - \right. \\ & \left. 4\left(\frac{1}{4}\right)^n\right) w_0 + \left(1 - 4\left(\frac{1}{2}\right)^{3n} + 4\left(\frac{1}{2}\right)^{2n} - 4\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n + \right. \\ & \left. 4\left(\frac{1}{2}\right)^{2n} - 4\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n + \right. \end{aligned}$$

$$\begin{aligned} & \left. 4\left(\frac{1}{4}\right)^n\right) x_0 + \left(1 - 4\left(\frac{1}{2}\right)^n - 4\left(\frac{1}{4}\right)^n\right) y_0 + \left(1 - 4\left(\frac{1}{2}\right)^{3n} + \right. \\ & \left. 4\left(\frac{1}{2}\right)^{2n} - 4\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n + 4\left(\frac{1}{4}\right)^n\right) z_0 + \left(1 - 6\left(\frac{1}{2}\right)^n + 12\left(\frac{1}{4}\right)^n - \right. \\ & \left. 8\left(\frac{1}{8}\right)^n\right) \alpha_0 \end{aligned}$$

$$\begin{aligned} b_n = & \left(\frac{1}{2}\right)^n b_0 + \left(\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right) d_0 + \left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right) f_0 + \left(\left(\frac{1}{2}\right)^n - \right. \\ & \left. 2\left(\frac{1}{2}\right)^{2n} - \left(\frac{1}{8}\right)^n\right) h_0 + \left(2\left(\frac{1}{2}\right)^n\right) i_0 + \left(2\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{2}\right)^{2n}\right) j_0 + \left(2\left(\frac{1}{2}\right)^n - \right. \\ & \left. 2\left(\frac{1}{2}\right)^{2n}\right) k_0 + \left(2\left(\frac{1}{2}\right)^{3n} - 4\left(\frac{1}{2}\right)^{2n} + 2\left(\frac{1}{2}\right)^n\right) l_0 + \left(\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{2}\right)^{2n}\right) n_0 + \\ & \left(2\left(\frac{1}{2}\right)^{3n} - 2\left(\frac{1}{2}\right)^{2n} + \left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right) p_0 + \left(2\left(\frac{1}{2}\right)^n - 4\left(\frac{1}{4}\right)^n\right) q_0 + \\ & \left(4\left(\frac{1}{2}\right)^{3n} - 2\left(\frac{1}{2}\right)^{2n} + 2\left(\frac{1}{2}\right)^n - 4\left(\frac{1}{4}\right)^n\right) r_0 + \left(\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{2}\right)^{2n}\right) t_0 + \\ & \left(2\left(\frac{1}{2}\right)^{3n} - 2\left(\frac{1}{2}\right)^{2n} + \left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right) v_0 + \left(2\left(\frac{1}{2}\right)^n - 4\left(\frac{1}{4}\right)^n\right) w_0 + \\ & \left(4\left(\frac{1}{2}\right)^{3n} - 2\left(\frac{1}{2}\right)^{2n} + 2\left(\frac{1}{2}\right)^n - 4\left(\frac{1}{4}\right)^n\right) x_0 + \left(4\left(\frac{1}{2}\right)^{3n} - 4\left(\frac{1}{2}\right)^{2n} + \right. \\ & \left. \left(\frac{1}{2}\right)^n\right) z_0 + \left(2\left(\frac{1}{2}\right)^n - 8\left(\frac{1}{2}\right)^{2n} + 8\left(\frac{1}{2}\right)^n\right) \alpha_0 \end{aligned}$$

$$\begin{aligned} c_n = & \left(\frac{1}{2}\right)^n c_0 + \left(\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right) d_0 + \left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right) g_0 + \left(\left(\frac{1}{2}\right)^n - \right. \\ & \left. 2\left(\frac{1}{2}\right)^{2n} - \left(\frac{1}{8}\right)^n\right) h_0 + \left(\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{2}\right)^{2n}\right) j_0 + \left(2\left(\frac{1}{2}\right)^{3n} - 2\left(\frac{1}{2}\right)^{2n} + \right. \\ & \left. \left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right) l_0 + \left(2\left(\frac{1}{2}\right)^n\right) m_0 + \left(2\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{2}\right)^{2n}\right) n_0 + \left(\left(\frac{1}{2}\right)^n - \right. \end{aligned}$$

$$\begin{aligned}
& 2 \left(\frac{1}{2}\right)^{2n} o_0 + \left(2 \left(\frac{1}{2}\right)^{3n} - 4 \left(\frac{1}{2}\right)^{2n} + \right. \\
& 2 \left(\frac{1}{2}\right)^n p_0 + \left(2 \left(\frac{1}{2}\right)^n - 4 \left(\frac{1}{4}\right)^n\right) q_0 + \\
& \left. \left(4 \left(\frac{1}{2}\right)^{3n} - 2 \left(\frac{1}{2}\right)^{2n} + 2 \left(\frac{1}{2}\right)^n - \right. \right. \\
& 4 \left(\frac{1}{4}\right)^n r_0 + \left.\left. \left(\left(\frac{1}{2}\right)^n - 2 \left(\frac{1}{2}\right)^{2n}\right) u_0 + \right. \right. \\
& \left. \left. \left(2 \left(\frac{1}{2}\right)^{3n} - 2 \left(\frac{1}{2}\right)^{2n} + \left(\frac{1}{2}\right)^n - \right. \right. \right. \\
& \left. \left. \left(\frac{1}{4}\right)^n v_0 + \left(4 \left(\frac{1}{2}\right)^{3n} - 4 \left(\frac{1}{2}\right)^{2n} + \right. \right. \right. \\
& \left. \left. \left(\frac{1}{2}\right)^n x_0 + \left(2 \left(\frac{1}{2}\right)^n - 4 \left(\frac{1}{4}\right)^n\right) y_0 + \right. \right. \\
& \left. \left. \left(4 \left(\frac{1}{2}\right)^{3n} - 2 \left(\frac{1}{2}\right)^{2n} + 2 \left(\frac{1}{2}\right)^n - \right. \right. \right. \\
& \left. \left. 4 \left(\frac{1}{4}\right)^n z_0 + \left(2 \left(\frac{1}{2}\right)^n - 8 \left(\frac{1}{2}\right)^{2n} + \right. \right. \right. \\
& \left. \left. 8 \left(\frac{1}{2}\right)^n \alpha_0 \right. \right. \\
d_n = & \left(\frac{1}{4}\right)^n d_0 + \left(\left(\frac{1}{4}\right)^n - \left(\frac{1}{8}\right)^n\right) h_0 + \\
& \left(2 \left(\frac{1}{2}\right)^{2n}\right) j_0 + \left(2 \left(\frac{1}{2}\right)^{2n} - \right. \\
& 2 \left(\frac{1}{2}\right)^{3n}\right) l_0 + \left(2 \left(\frac{1}{2}\right)^{2n}\right) n_0 + \\
& \left(2 \left(\frac{1}{2}\right)^{2n} - 2 \left(\frac{1}{2}\right)^{3n}\right) p_0 + \\
& \left(4 \left(\frac{1}{4}\right)^n\right) q_0 + \left(4 \left(\frac{1}{4}\right)^n - \right. \\
& 2 \left(\frac{1}{2}\right)^{3n}\right) r_0 + \left(\left(\frac{1}{4}\right)^n - 2 \left(\frac{1}{2}\right)^{3n}\right) v_0 + \\
& \left(2 \left(\frac{1}{2}\right)^{2n} - 4 \left(\frac{1}{2}\right)^{3n}\right) x_0 + \left(2 \left(\frac{1}{2}\right)^{2n} - \right. \\
& 4 \left(\frac{1}{2}\right)^{3n}\right) z_0 + \left(4 \left(\frac{1}{4}\right)^n - 8 \left(\frac{1}{8}\right)^n\right) \alpha_0 \\
e_n = & \left(\frac{1}{2}\right)^n e_0 + \left(\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right) f_0 + \\
& \left(\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right) g_0 + \left(\left(\frac{1}{2}\right)^n - \right. \\
& 2 \left(\frac{1}{2}\right)^{2n} - \left.\left.\left(\frac{1}{8}\right)^n\right) h_0 + \left(\left(\frac{1}{2}\right)^n - \right. \right. \\
& 2 \left(\frac{1}{2}\right)^{2n}\right) k_0 + \left(2 \left(\frac{1}{2}\right)^{3n} - 2 \left(\frac{1}{2}\right)^{2n} + \right. \\
& \left.\left.\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right) l_0 + \left(\left(\frac{1}{2}\right)^n - \right. \right. \\
& 2 \left(\frac{1}{2}\right)^{2n}\right) o_0 + \left(2 \left(\frac{1}{2}\right)^{3n} - 2 \left(\frac{1}{2}\right)^{2n} + \right. \\
& \left.\left.\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right) p_0 + \left(4 \left(\frac{1}{2}\right)^{3n} - \right. \right. \\
& 4 \left(\frac{1}{2}\right)^{2n} + \left.\left.\left(\frac{1}{2}\right)^n\right) r_0 + \left(2 \left(\frac{1}{2}\right)^n\right) s_0 + \right. \\
& \left. \left(2 \left(\frac{1}{2}\right)^n - 2 \left(\frac{1}{2}\right)^{2n}\right) t_0 + \left(2 \left(\frac{1}{2}\right)^n - \right. \right. \\
& 2 \left(\frac{1}{2}\right)^{2n}\right) u_0 + \left(2 \left(\frac{1}{2}\right)^{3n} - 4 \left(\frac{1}{2}\right)^{2n} - \right. \\
& 2 \left(\frac{1}{2}\right)^n v_0 + \left(2 \left(\frac{1}{2}\right)^n - 4 \left(\frac{1}{4}\right)^n\right) w_0 + \\
& \left(4 \left(\frac{1}{2}\right)^{3n} - 2 \left(\frac{1}{2}\right)^{2n} + 2 \left(\frac{1}{2}\right)^n - \right. \\
& 4 \left(\frac{1}{4}\right)^n x_0 + \left(2 \left(\frac{1}{2}\right)^n - 8 \left(\frac{1}{2}\right)^{2n} + \right. \\
& \left. \left. 8 \left(\frac{1}{2}\right)^n\right) \alpha_0 \right. \\
f_n = & \left(\frac{1}{4}\right)^n f_0 + \left(\left(\frac{1}{4}\right)^n - \left(\frac{1}{8}\right)^n\right) h_0 + \\
& \left(2 \left(\frac{1}{2}\right)^{2n}\right) k_0 + \left(2 \left(\frac{1}{2}\right)^{2n} - \right. \\
& 2 \left(\frac{1}{2}\right)^{3n}\right) l_0 + \left(\left(\frac{1}{4}\right)^n - 2 \left(\frac{1}{2}\right)^{3n}\right) p_0 + \\
& \left(2 \left(\frac{1}{2}\right)^{2n} - 4 \left(\frac{1}{2}\right)^{3n}\right) r_0 + \\
& \left(2 \left(\frac{1}{2}\right)^{2n}\right) t_0 + \left(2 \left(\frac{1}{2}\right)^{2n} - \right. \\
& 2 \left(\frac{1}{2}\right)^{3n}\right) v_0 + \left(4 \left(\frac{1}{4}\right)^n\right) w_0 + \\
& \left(4 \left(\frac{1}{4}\right)^{2n} - 4 \left(\frac{1}{2}\right)^{3n}\right) x_0 + \left(2 \left(\frac{1}{2}\right)^{2n} - \right. \\
& 4 \left(\frac{1}{2}\right)^{3n}\right) z_0 + \left(4 \left(\frac{1}{4}\right)^n - 8 \left(\frac{1}{8}\right)^n\right) \alpha_0 \\
g_n = & \left(\frac{1}{4}\right)^n g_0 + \left(\left(\frac{1}{4}\right)^n - \left(\frac{1}{8}\right)^n\right) h_0 + \\
& \left(\left(\frac{1}{4}\right)^n - 2 \left(\frac{1}{2}\right)^{3n}\right) l_0 + \left(2 \left(\frac{1}{2}\right)^{2n}\right) o_0 + \\
& \left(2 \left(\frac{1}{2}\right)^{2n} - 2 \left(\frac{1}{2}\right)^{3n}\right) p_0 + \left(2 \left(\frac{1}{2}\right)^{2n} - \right. \\
& 4 \left(\frac{1}{2}\right)^{3n}\right) r_0 + \left(2 \left(\frac{1}{2}\right)^{2n}\right) u_0 + \\
& \left(2 \left(\frac{1}{2}\right)^{2n} - 2 \left(\frac{1}{2}\right)^{3n}\right) v_0 + \left(2 \left(\frac{1}{2}\right)^{2n} - \right. \\
& 4 \left(\frac{1}{2}\right)^{3n}\right) x_0 + \left(4 \left(\frac{1}{4}\right)^n\right) y_0 + \\
& \left(4 \left(\frac{1}{4}\right)^n - 4 \left(\frac{1}{2}\right)^{3n}\right) z_0 + \left(4 \left(\frac{1}{4}\right)^n - \right. \\
& \left. \left. 8 \left(\frac{1}{8}\right)^n\right) \alpha_0 \right. \\
h_n = & \left(\frac{1}{8}\right)^n h_0 + \left(2 \left(\frac{1}{2}\right)^{3n}\right) l_0 + \\
& \left(2 \left(\frac{1}{2}\right)^{3n}\right) p_0 + \left(4 \left(\frac{1}{2}\right)^{3n}\right) r_0 + \\
& \left(2 \left(\frac{1}{2}\right)^{3n}\right) v_0 + \left(4 \left(\frac{1}{2}\right)^{3n}\right) x_0 + \\
& \left(4 \left(\frac{1}{2}\right)^{3n}\right) z_0 + \left(8 \left(\frac{1}{8}\right)^n\right) \alpha_0.
\end{aligned}$$

$$i_n = j_n = k_n = l_n = m_n = n_n = o_n =$$

$$p_n = q_n = r_n = s_n = t_n = u_n = v_n =$$

$$w_n = x_n = y_n = z_n = \alpha_n = 0$$

for $n = 1, 2, 3, \dots$

The Result of Genotype Inheritance Analysis

In section 3.4, the probability of 27 genotypes in the n^{th} generation in Equation (13) has been calculated, and x^n is obtained which involves three particular forms of the sequence. In matrix $x^{(n)}$, there are $(\frac{1}{2})^n$, $(\frac{1}{4})^n$, and $(\frac{1}{8})^n$, which tend of approach 0 to n towards infinity, so the concept of limit is used from n to infinity to know the sum of the parts of the genotypes that exist in infinite generations. The limit of Equation (13) is:

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \{ a_0 + (1 - (\frac{1}{2})^n) b_0 + \dots +$$

$$(1 - 6(\frac{1}{2})^n + 12(\frac{1}{4})^n -$$

$$8(\frac{1}{8})^n) \alpha_0 \}$$

$$= a_0 + b_0 + c_0 + d_0 + \dots + y_0 +$$

$$z_0 + \alpha_0$$

$$\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \{ ((\frac{1}{2})^n) b_0 + \dots + (2(\frac{1}{2})^n -$$

$$8(\frac{1}{2})^{2n} + 8(\frac{1}{2})^n) \alpha_0 \} = 0$$

$$\lim_{n \rightarrow \infty} c_n = \lim_{n \rightarrow \infty} \{ ((\frac{1}{2})^n) c_0 + \dots + (2(\frac{1}{2})^n -$$

$$8(\frac{1}{2})^{2n} + 8(\frac{1}{2})^n) \alpha_0 \} = 0$$

$$\lim_{n \rightarrow \infty} d_n = \lim_{n \rightarrow \infty} \{ ((\frac{1}{4})^n) d_0 + ((\frac{1}{4})^n -$$

$$(\frac{1}{8})^n) h_0 + \dots + (4(\frac{1}{4})^n -$$

$$8(\frac{1}{8})^n) \alpha_0 \} = 0$$

$$\lim_{n \rightarrow \infty} e_n = \lim_{n \rightarrow \infty} \{ ((\frac{1}{2})^n) e_0 + \dots + (2(\frac{1}{2})^n -$$

$$8(\frac{1}{2})^{2n} + 8(\frac{1}{2})^n) \alpha_0 \} = 0$$

$$\lim_{n \rightarrow \infty} f_n = \lim_{n \rightarrow \infty} \{ ((\frac{1}{4})^n) f_0 + ((\frac{1}{4})^n -$$

$$(\frac{1}{8})^n) h_0 + \dots + (4(\frac{1}{4})^n -$$

$$8(\frac{1}{8})^n) \alpha_0 \} = 0$$

$$\lim_{n \rightarrow \infty} g_n = \lim_{n \rightarrow \infty} \{ ((\frac{1}{4})^n) g_0 + ((\frac{1}{4})^n -$$

$$(\frac{1}{8})^n) h_0 + \dots + (4(\frac{1}{4})^n -$$

$$8(\frac{1}{8})^n) \alpha_0 \} = 0$$

$$\lim_{n \rightarrow \infty} h_n = \lim_{n \rightarrow \infty} \{ ((\frac{1}{8})^n) h_0 + (2(\frac{1}{2})^{3n}) l_0 +$$

$$\dots + (8(\frac{1}{8})^n) \alpha_0 \} = 0$$

$$\lim_{n \rightarrow \infty} i_n = \lim_{n \rightarrow \infty} j_n = \lim_{n \rightarrow \infty} k_n = \dots =$$

$$\lim_{n \rightarrow \infty} \alpha_n = \lim_{n \rightarrow \infty} 0 = 0$$

Based on the Equation (1) then for

$$a_0 + b_0 + c_0 + \dots + z_0 + \alpha_0 = 1$$

the probability of genotype for infinite offspring is:

$$a_n = a_0 + b_0 + c_0 + \dots + z_0 + \alpha_0 = 1$$

$$b_n = c_n = d_n = e_n = f_n = g_n = h_n$$

$$= i_n = j_n = k_n = l_n = m_n = n_n = o_n$$

$$= p_n = q_n = r_n = s_n = t_n = u_n = v_n$$

$$= w_n = x_n = y_n = z_n = \alpha_n = 0$$

Parents cross BbLL with all possible genotypes of the dihybrid, resulting in the genotype probability for its infinite offspring, that are (Kaffah & Romdhini, 2015):

$$a_n = \frac{1}{4}$$

$$d_n = \frac{1}{2}$$

$$g_n = \frac{1}{4}$$

$$b_n = c_n = e_n = f_n = h_n = 0$$

In this research, all the emerging individuals had a relatively good chance of passing on their seed traits to their infinite offspring. Meanwhile, in this research, the chances of offspring are not fully genotyped TTKKBB, and eight genotypes have a chance of breeding.

CONCLUSIONS AND SUGGESTIONS

Based on the result of this research, it can be concluded that the genotype of the n^{th} generation in trihybrid crosses with controlled parental genotypes could be obtained from the equation obtained by

applying the diagonalization of a matrix. Furthermore, in infinite generation for the n^{th} generation controlled genotype inheritance can be used to calculate the limit n to infinity, and obtained all offspring that will be produced tend to have a genotype TTKKBB.

As for the suggestion for this research, in the genotype inheritance research, using matrix diagonalization could be developed for crosses with more traits.

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