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# Reversible thinking in solving mathematics problems in terms of cognitive style

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Reversible Thinking.

**Abstract**

**Background:** Reversible thinking, the ability to think bidirectionally, is a crucial component of mathematical problem-solving. Differences in cognitive styles, particularly field-dependent and field-independent characteristics, play a significant role in students' reversible thinking, necessitating a deeper exploration of these relationships.
**Aim:** This study aims to describe students' reversible thinking processes in solving mathematical problems based on their cognitive styles, focusing on field-dependent and field-independent traits.
**Method:** A qualitative descriptive approach was applied to 32 eighth-grade students from a junior high school in Malang City, Indonesia. Data were collected using the Group Embedded Figures Test (GEFT), a reversible thinking test, and semi-structured interviews. Students were categorized into field-dependent and field-independent groups using GEFT before undertaking a reversible thinking test. Semi-structured interviews were conducted to gain deeper insights into their problem-solving approaches.
**Results:** The findings indicate that students with field-independent cognitive styles exhibit better performance in the aspects of negation and reciprocity. They carefully apply problem-solving strategies, consistently reverting to initial values after achieving correct solutions. Conversely, students with field-dependent cognitive styles are more prone to errors, particularly in changing operation signs and applying the concept of reciprocal equivalence.
**Conclusion:** This study highlights significant differences in reversible thinking between students with field-dependent and field-independent cognitive styles. The results suggest the need for tailored teaching methods to enhance reversible thinking based on cognitive styles. Further research is recommended to explore barriers and additional factors influencing reversible thinking.

## INTRODUCTION

As a fundamental part of education, mathematics shapes the way individuals think and approach problems in their daily lives. Mathematics significantly influences various aspects of human life, driving advancements in technology, science, and business (Siregar & Nasution, 2019). Its contribution extends to sharpening logical reasoning and systematic thinking, enabling students to better understand complex concepts. Through mathematics education, students are trained in diverse ways of thinking that prepare them to tackle real-world challenges (Darwanto, 2019). One prominent form of such thinking is reversible thinking, which enhances analytical skills and equips students with the ability to approach problems from different perspectives.

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Mathematics cultivates various types of thinking skills, one of which is reversible thinking, a crucial aspect of problem-solving in mathematical contexts. This cognitive ability allows students to mentally retrace or reverse their thought processes. Inhelder & Piaget (1958) defined reversibility as the capacity to mentally reverse a process to return to its starting point. He identified two main forms: negation and reciprocity. Negation refers to an operation having an inverse that neutralizes the original action, such as division reversing multiplication or exponential functions reversing logarithms. Reciprocity, on the other hand, involves relationships characterized by equivalence or mutual compensation. Krutetskii (1976) further explored reversibility, describing it as a process of shifting from forward to backward thinking. For example, if a student learns a sequence of steps (A, B, C, D, E, F), they master the progression from A to F. By reversing the order—from F back to A—they gain the ability to restructure their reasoning process. Krutetskii's framework highlights that reversibility encompasses reversing operations, recognizing reciprocal relationships, and applying both theorems and their inverses (Muzaini et al., 2021). This cognitive skill underscores the importance of flexible thinking, equipping students to approach mathematical problems with a broader perspective and greater adaptability.

Reversible thinking has been widely examined due to its significance in enhancing students' problem-solving abilities in mathematics. Pebrianti et al. (2023) observed that while many students demonstrated forward-thinking proficiency, some encountered challenges when required to reverse their reasoning to construct answers. This difficulty was linked to a lack of contextual understanding during the initial stages of learning, highlighting the importance of developing more robust conceptual foundations. Similarly, Purwaningrum & Sutiarso (2022) emphasized that strengthening reversible thinking involves nurturing students' mathematical attitudes and reasoning skills through varied instructional approaches, strategies, and models. Moreover, Balingga et al. (2016) identified that students with lower reversible thinking capabilities often struggled with solving unfamiliar problems, leading to confusion and difficulty in adapting to new challenges. These insights underscore the critical role of reversible thinking in equipping students with the flexibility and adaptability required for effective problem-solving.

Reversible thinking offers significant potential in enhancing students' ability to approach and solve mathematical problems effectively. Maf'ulah & Juniati (2020) highlighted that this ability aids in the development of new understandings within a student's cognitive framework, resulting in more meaningful learning experiences. This is particularly valuable when students encounter problems that differ from examples provided by their teachers. Similarly, Sutiarso (2020) emphasized that reversible thinking plays a crucial role in stimulating students' mental knowledge and experiences, ultimately reinforcing their problem-solving confidence. Additionally, Maf'Ulah et al. (2019) noted that reversible thinking enables students to examine problems from both forward and backward perspectives. As a result, students who develop strong reversible thinking skills are better equipped to overcome challenges in mathematics with greater ease and adaptability.

The variation in students' reversible thinking abilities highlights the complexity of this cognitive skill in the context of mathematics learning. Amalia et al. (2024) emphasized that differences in these skills are evident among students. Supporting this, Purwaningrum & Sutiarso (2022) found that among 31 students tested, only 20.96% demonstrated reversible thinking skills, while 79.04% lacked them. Similarly, Sutiarso (2020) revealed that of 40 students tested, 42.5% possessed reversible thinking abilities, whereas 57.5% did not. Amalia (2024) further explained that these differences are primarily influenced by how students comprehend and internalize material during the learning process, which is closely tied to their cognitive styles. Such findings underscore the importance of recognizing individual differences in cognitive processes to enhance learning outcomes.

Cognitive style plays a crucial role in shaping how students approach and solve mathematical problems. Ulya et al. (2014) described cognitive style as a unique way individuals respond to, process, organize, and utilize ideas when facing various situations or phenomena. Ngilawajan (2013) emphasized that each student has distinct potentials, strategies, and thinking styles for addressing mathematical challenges. These differences in cognitive processes are influenced by environmental factors and past educational experiences, which categorize cognitive styles into field dependent and field independent (Siregar & Nasution, 2019). Field independent students tend to adopt a more analytical approach to problem-solving, while field dependent students often struggle with identifying the core elements of a problem (Wulan & Anggraini, 2019). Observations during the preliminary study revealed similar patterns, where some students demonstrated ease in understanding and solving problems, while others faced significant difficulties in interpreting the problem context and executing solutions. Sellah et al. (2017) explained that variations in cognitive style directly influence how students think and complete their tasks. Understanding these differences is crucial for developing teaching strategies that cater to diverse cognitive styles and enhance overall learning outcomes.

Previous research has discussed students' reversible thinking ability at the school level (Balingga et al., 2016; Muzaini et al., 2021; Purwaningrum & Sutiarso, 2022) and college (Sutiarso, 2020). Maf'ulah & Juniati (2020) explored the reversible thinking ability of prospective teacher students through algebraic problem solving tasks, while Ikram et al. (2018) examined students' reversible reasoning in composition function problems. In addition, some studies have also focused on improving reversible thinking skills through learning media/strategies (Bharata et al., 2022). However, this study is different because it seeks to identify how students process the information provided in mathematical problems. This is important, considering that differences in the way students process information can affect their reversible thinking ability (Amalia et al., 2024). The findings from this study are expected to make a significant contribution to mathematics education theory by expanding the understanding of the relationship between information processing and reversible thinking ability. Practically, this research can serve as a basis for designing learning strategies that are more adaptive to the different ways students understand and process lessons so as to improve overall mathematics learning outcomes.

Considering the differences in how students process information and the influence of cognitive styles, further research is essential to understand how reversible thinking interacts with field-dependent and field-independent characteristics. Reversible thinking, as a fundamental mathematical skill, plays a pivotal role in enhancing problem-solving abilities. By addressing these cognitive styles, this study seeks to fill the gap in understanding how students' information processing impacts their reversible thinking. The findings are expected to contribute significantly to mathematics education theory and practice, particularly in designing adaptive instructional strategies that align with students' diverse cognitive profiles.

## METHODS

### *Design*

In this study, researchers used a qualitative approach that aims to describe the phenomena experienced by research subjects, such as perceptions, behavior, actions, and motivation, which are described narratively in natural situations (Waruwu, 2023). The qualitative approach was chosen because it allows researchers to explore in depth the reversible thinking process of students, which is complex and difficult to measure quantitatively. As stated by Rusandi & Rusli (2021), descriptive qualitative research is a research strategy that directs researchers systematically in investigating events, phenomena, and various facts related to the research subject. This research is descriptive with a qualitative approach to provide a detailed description of the reversible thinking ability of students in solving mathematical problems in terms of field dependent and field independent cognitive styles. Descriptive design was chosen because it is relevant to understanding and revealing differences in students' thinking patterns based on their cognitive styles, thus providing a comprehensive picture that supports the development of more effective learning strategies.

### *Participants and Instruments*

This study used GEFT (Group *Embedded Figure Test*) adapted from Witkin's theory to determine the type of cognitive style of students, including field dependent cognitive style and field independent cognitive style. The instrument developed by Witkin has been tested for validity and reliability before, so the GEFT can be used directly without validation (Davis, 2006). Meanwhile, the reversible thinking test used was first validated by an expert in the field of mathematics education. Based on the validation process that has been carried out, the validator states that the reversible thinking test instrument is valid and can be used for research purposes.

The research subjects of this study were eighth-grade students in one of the junior high schools in Malang, Indonesia. The research subjects were based on the results of the Group Embedded Figure Test (GEFT) and the results of solving the reversible thinking test. In the selection of research subjects, the first step is the grouping of cognitive styles with GEFT. This cognitive style test was conducted to categorize students into the type of cognitive style field dependent (FD) and cognitive style field independent (FI). In the next step, students were given a reversible thinking test. Based on the results of these two tests selected 2 students with field dependent cognitive style and 2 students with field independent cognitive style.

### *Data Analysis*

The data collected in this research is qualitative. The analysis was carried out from the beginning to the end of the research. The following are the stages: (1) Data reduction, the data reduced in this study is in the form of GEFT test results and reversible thinking tests, which are used to determine research subjects. At the same time, the interview data is reduced by listening to the recorded interviews conducted, writing a summary of the interview results, and discarding unnecessary data. (2) Data presentation, the reduced data will provide a specific picture of students' reversible thinking in solving mathematics problems. The results of the description of students' reversible thinking are presented in narrative form to faciliatate concluding. (3). Making conclusions, on the results of the presentation of data that has been reduced and presented previously. This stage is done to discover how reversible thinking students in solve mathematical problems regarding field dependent and field independent cognitive styles.

## RESULTS AND DISCUSSION

### *Result*

This section describes the data found in the field. Before being presented, the collected data were selected according to the research needs. The data were selected based on their relevance to the research objectives and their involvement in supporting other findings in this study. Meanwhile, unnecessary data will be excluded and not presented in this study.

Subjek field independent $SFI_1$

Issue 1

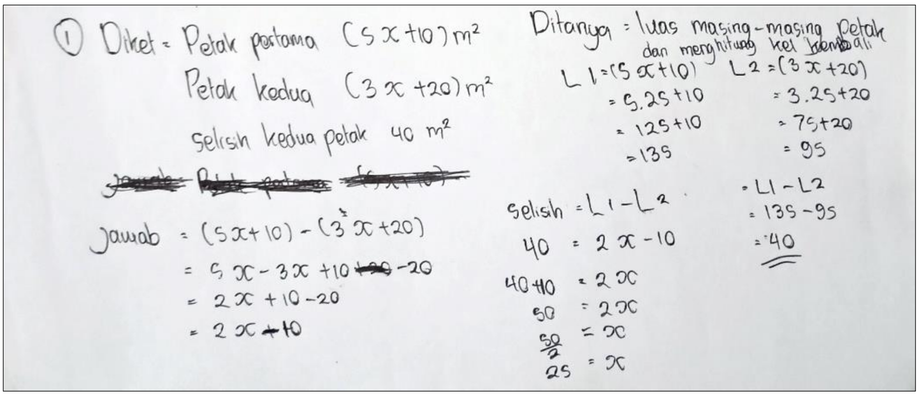

**Figure 1.** $SFI_1$'s first problem

$SFI_1$'s problem-solving activity begins with understanding the problem. In this activity, $SFI_1$ understands important data and information, according to $SFI_1$'s answer in Figure 1. $SFI_1$ wrote that the area of the first plot is $(5x + 10)m^2$, the area of the second plot is $(3x + 20)m^2$, and the difference between the two plots is $40m^2$. Then, the question asks about the area of each plot and the difference between the two plots.

The next step is $SFI_1$ developing a solution plan. This activity is shown by making an equation for the difference between the two plots. $SFI_1$ did the calculation between the two known plots, which $SFI_1$ knew from the problem that the first plot was wider than the second plot, $(5x + 10) - (3x + 20)$. After that, $SFI_1$ made an equation from the results of the previous calculation with the known difference.

Furthemore, $SFI_1$ continued to solve problems that gave rise to reversible thinking indicators. $SFI_1$ brought up the negation indicator when solving the problem, which can be seen when changing the equation $40 = 2x - 10$ to the equation $40 + 10 = 2x$. As is known, the negation aspect appears when students use inversion of related operations; this can be seen when students cancel the subtraction of 10 with $+10$. In addition, there is also the aspect of reciprocity, this can be seen when $SFI_1$ changed the form of the equation $2x = 50$ to $x = \frac{50}{2}$. At the completion, as shown by $SFI_1$, it can be concluded that the reversible thinking indicators that appear are negation and reciprocity.

In the next activity, $SFI_1$ returned to the initial equation by substituting $x = 25$ into each land plot area. Then, $SFI_1$ was verified by recalculating the difference between the two plots of land. This result strengthened $SFI_1$'s belief that the final value he obtained was correct. From this stage, it can be concluded that $SFI_1$ raises an indicator of reversible thinking, namely returning to the initial value after obtaining the result.

Issues 2

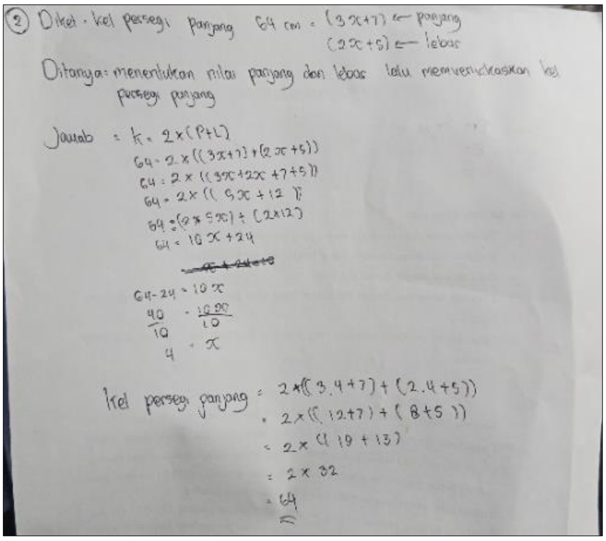


**Figure 2.** $SFI_1$'s second problem answer

$SFI_1$'s problem-solving activity begins with understanding the problem. In this activity, $SFI_1$ could identify the important data in the problem, as shown in $SFI_1$'s answer in Figure 2. $SFI_1$ wrote the perimeter of the rectangle as 64 cm, length (3x+7) $cm$, and width $(2x + 5)$ $cm$. Then, the problem asked for the value of length and width and recalculated the perimeter of the rectangle. $SFI_1$ could understand the purpose of the problem well.

The next step is developing a solution plan. This activity is shown by making the formula for the perimeter of the rectangle. $SFI_1$ used the formula for the perimeter of a rectangle that had been learned previously. After that, he entered important data obtained from the problem, such as length and width. $SFI_1$ can develop a solution plan carefully and accurately.

Furthermore, $SFI_1$ succeeded in bringing up reversible thinking indicators, namely negation and reciprocity. This can be seen when $SFI_1$ changed the form $64 = 10x + 24$ to $64 - 24 = 10x$. As is known, the negation aspect appears when students use inversion of related operations, which, in this case, $SFI_1$ cancels $+24$ with $-24$. In addition, there is also a reciprocity aspect; this can be seen when $SFI_1$ performs an equivalent reciprocal relationship where both segments are equally divided by ten so that the equation is $\frac{40}{10} = \frac{10x}{10}$. If the student is involved a reciprocal relationship, then the student provides the same treatment on both sides. From the stages of solving this problem, $SFI_1$ brought up the reversible thinking indicators of negation and reciprocity.

In the next activity, $SFI_1$ recalculated the perimeter of the rectangle by substituting $x = 4$ in the length and width equations. This result strengthened $SFI_1$'s belief that the result obtained was correct. $SFI_1$ can verify the perimeter of the rectangle again. From this, it can be seen that the reversible thinking aspect emerges as the ability to return to the initial value. This is indicated by the suitability of the answer obtained by $SFI_1$ with the information known from the problem.

Subjek filed independent $SFI_2$

Issue 1

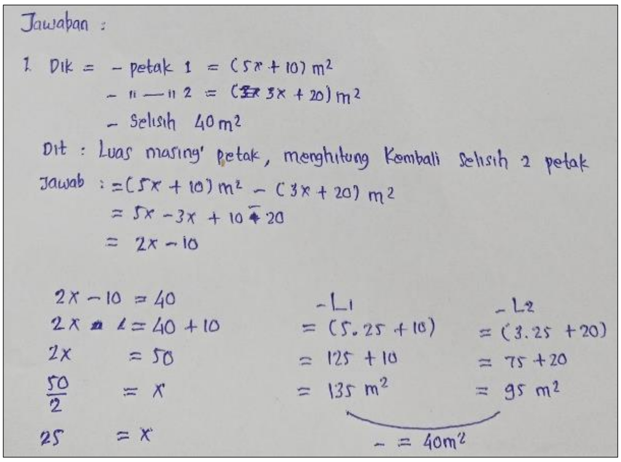

**Figure 3.** $SFI_2$'s first problem answer

$SFI_2$'s problem solving activity begins with understanding the problem. In this activity $SFI_2$ obtained information contained in the problem, according to $SFI_2$'s answer in Figure 3. $SFI_2$ wrote that the area of the first plot is $(5x + 10)m^2$, the area of the second plot is $(3x + 20)m^2$, and the difference between the two plots. $SFI_2$ can understand the purpose of the problem well, so $SFI_2$ managed to write down all the important information known and asked in the problem.

The next step is $SFI_2$ development of a solution plan. This activity is shown by making an equation for the difference between the two plots. $SFI_2$ did the calculation between the two known plots, which $SFI_2$ knew from the problem that the first plot was wider than the second plot. After that, $SFI_2$ made an equation from the results of the previous calculation with the known difference. $SFI_2$ can develop a solution plan well and correctly.

Furthermore, $SFI_2$ solved the problem by displaying indicators of reversible thinking, where $SFI_2$ used aspects of negation and reciprocity in solving the problem. This can be seen when changing the equation $2x - 10 = 40$ to the equation $2x = 40 + 10$. The negation aspect appears when students use inversion of related operations, where $SFI_2$ cancels the substraction of $-10$ with $+10$. In addition, there is also the aspect of reciprocal relationship, which can be seen when $SFI_2$ changed the form $2x = 50$ to $x = \frac{50}{2}$ and obtained the value of $x = 25$. $SFI_2$ can apply the step plan or problem solving strategy well. It can be concluded that the reversible thinking indicators that appear are negation and reciprocity.

In the next activity, $SFI_2$ returned to the initial equation by substituting the value of $x = 25$ into each land plot area. Then, $SFI_2$ was verified by recalculating the difference between the two plots of land. This result strengthened $SFI_2$'s belief that the *results* obtained were correct. $SFI_2$ can verify the value of the difference between the two plots of land again. From this, it can be seen that the reversible thinking aspect emerges as the ability to return to the initial value. This is indicated by the suitability of the answer obtained by $SFI_2$ with what is known from the problem.

Issue 2

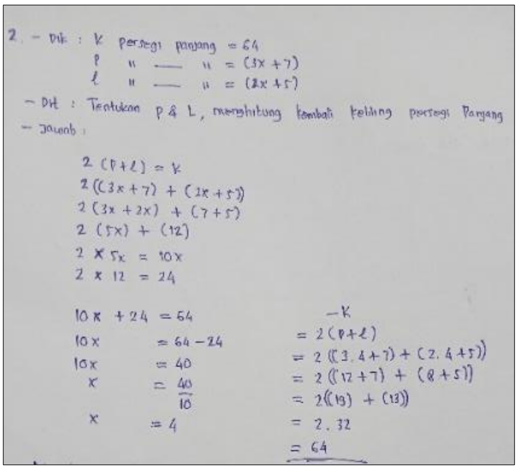

**Figure 4.** $SFI_2$'s second problem answer

$SFI_2$'s problem solving activity begins with understanding the problem. In this activity, $SFI_2$ got important information from the problem, according to $SFI_2$'s answer in Figure 4. $SFI_2$ wrote the perimeter of the rectangle, which is $64$, length $(3x + 7)$, and width $(2x + 5)$. Then, the question asks $SFI_2$ to determine the length and width and recalculate the perimeter of the rectangle. $SFI_2$ understood the purpose of the problem well and managed to explain the information known and asked in the problem.

The next step is $SFI_2$'s development of a solution plan. This activity is shown by making the formula for the perimeter of the rectangle. $SFI_2$ used the formula for the perimeter of a rectangle that had been learnt previously. After that, he entered important information from the problem, such as length and width. $SFI_2$ can develop a solution plan well and correctly.

In solving the problem, $SFI_2$ managed to indentify the indicators of reversible thinking, namely negation and reciprocity. This can be seen when $SFI_2$ changes the form $10x + 24 = 64$ to $10x = 64 - 24$. As is known, the negation aspect appears when students use inversion of related operations, which, in this case, $SFI_2$ cancels $+24$ with $-24$. In addition, there is also a reciprocity aspect, this can be seen when $SFI_2$ performs a reciprocal relationship when changing the form of the equation from $10x = 40$ to $x = \frac{40}{10}$ and obtaining the value $x = 4$. $SFI_2$ can apply the step plan or problem solving strategy well. At the completion stage, $SFI_2$ brought up reversible thinking indicators: negation and reciprocity.

In the next activity, $SFI_2$ recalculated the value of the perimeter of the rectangle by substituting the values $x = 4$ in the length and width equation and immediately recalculated the perimeter of the rectangle. This result strengthened $SFI_2$'s belief that the result obtained was correct. $SFI_2$ can verify the value of the perimeter of the rectangle again. From this, it can be seen that the aspect of reversible thinking emerges as the ability to return to the initial value. This is characterized by the suitability of the answer obtained with the information known from the problem.

Subjek field dependent $SFD_1$
Issue 1

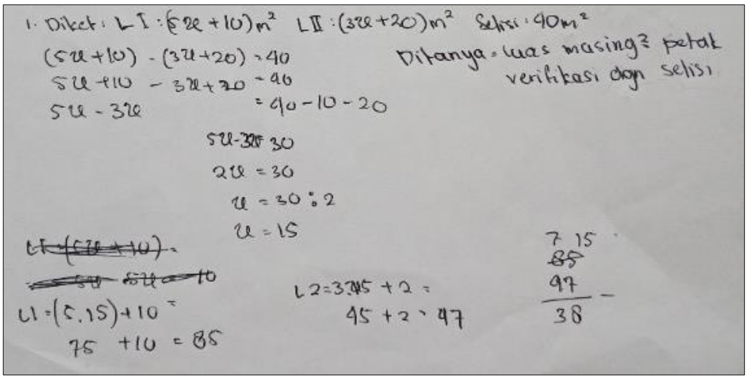

**Figure 5.** $SFD_1$'s first problem answer

$SFD_1$'s problem solving activity begins with understanding the problem. In this activity, $SFD_1$ can get important information contained in the problem, according to $SFD_1$'s answer in Figure 5. $SFD_1$ wrote the area of the first plot as $(5x + 10)m^2$, the area of the second plot as $(3x + 20)m^2$, and the difference between the two plots as $40m^2$. Then, $SFD_1$ wrote down what was asked in the problem, namely the area of each plot and the difference between the two plots. $SFD_1$ understood the meaning of the problem well, and $SFD_1$ explained the important information known and asked in the problem.

The next step is $SFD_1$ developing a solution plan. This activity is shown by making an equation for the difference between the two areas of the plots. $SFD_1$ made the equation for the difference between the two known plots, with the first plot's area minus the second plot's area. $SFD_1$ was able to plan the solution well and correctly.

Furthermore, $SFD_1$ performed algebraic operations by involving reversible thinking indicators. $SFD_1$ used aspects of negation and reciprocity to solve the problem. This can be seen when $SFD_1$ changed the form $5x + 10 - 3x + 20 = 40$ to $5x - 3x = 40 - 10 - 20$. In addition, there is also a reciprocity aspect, this can be seen when $SFD_1$ changes the form $2x = 30$ to $x = \frac{30}{2}$. In the process of solving, $SFD_1$ made mistakes, so this affected the results at the next stage of the work. The mistake $SFD_1$ made occurred when $SFD_1$ was wrong in grouping the terms of the equation. As explained earlier, the reversible thinking indicators, namely negation and reciprocity, can be seen in the working process.

In the next activity, $SFD_1$ returned to the initial equation by substituting each land plot area. Then, $SFD_1$ was verified by recalculating the difference between the two plots of land. $SFD_1$ verified the difference between the value of the two plots. From this, it can be seen that the reversible thinking aspect emerges as the ability to return to the initial value. However, due to the completion process, there was an error in the previous stage, thus making the recalculation of the difference value of the two plots of land wrong. $SFD_1$ also found an error in solving the problem he was working on due to the difference between the final difference value he obtained and the difference known from the problem.

Issue 2

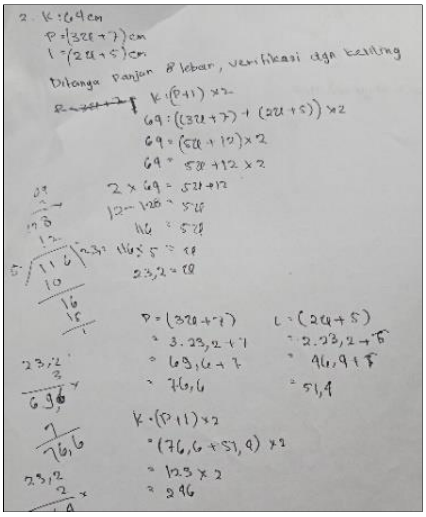

**Figure 6.** $SFD_1$'s second problem answer

$SFD_1$ 's problem solving activity begins with understanding the problem. In this activity, $SFD_1$ got important information about the problem, according to $SFD_1$'s answer in Figure 6. $SFD_1$ wrote the perimeter of the rectangle as $64cm$, length as $(3x + 7)cm$, and width as $(2x + 5)cm$, and the problem asked in the problem is the value of length, width, and re-verification of the perimeter of the rectangle. $SFD_1$ can understand the purpose of the problem well, where $SFD_1$ managed to explain the important information known and asked in the problem.

The next step is $SFD_1$'s development of a solution plan. This activity is shown by making the equation for the perimeter of the rectangle. $SFD_1$ used the formula for the perimeter of a rectangle that he had learned before. After that, he included important information that was known from the problem, such as the length, width, and perimeter values of the rectangle. $SFD_1$ could plan the solution well and correctly.

Furthermore, $SFD_1$ performed algebraic operations involving reversible thinking indicators. However, $SFD_1$ made mistakes in the negation aspect, where $SFD_1$ did not succeed in using the inversion of the related operation, namely canceling the multiplication of 2. Further errors were also seen at the problem solving stage, where $SFD_1$ also failed to use inversion of related operations on $+12$ to $-12$. At the same time, the reciprocity aspect is seen when $SFD_1$ does the equivalent reciprocal relationship, namely when $\frac{116}{5} = x$. At the completion stage, $SFD_1$ only succeeded in using the reciprocity indicator, while for the negation indicator, there were still errors, so further reinforcement was needed in the future negation.

In the next activity, $SFD_1$ recalculated the value of the perimeter of the rectangle *by* substituting $x = 23{,}2$ in the length and width equations, and $SFD_1$ immediately recalculated the perimeter of the rectangle. $SFD_1$ can verify the value of the perimeter of the rectangle again. From this, it can be seen that the aspect of reversible thinking emerges as the ability to return to the initial value. However, an error in the previous work made $SFD_1$ less confident about the results of his work.

Subjek field dependent $SFD_2$

Issue 1

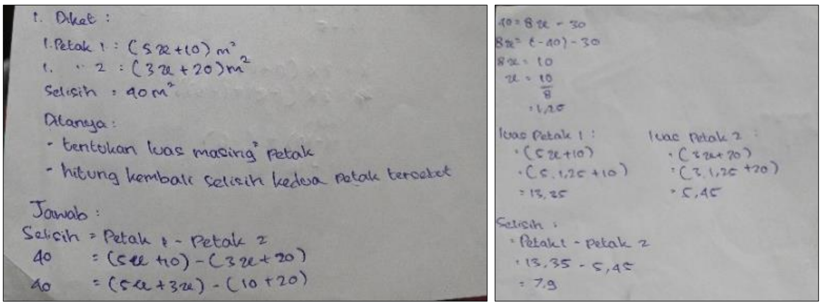


**Figure 7.** $SFD_2$'s first problem answer

$SFD_2$'s problem solving activity begins with understanding the problem. In this activity, *$SFD_2$* obtained important information about the problem, according to $SFD_2$'s answer in Figure 7. $SFD_2$ wrote the area of the first plot is $(5x + 10)m^2$, the area of the second plot is $(3x + 20)m^2$, and the difference between the two plots is $40m^2$. Then, write down what is asked in the problem: determine the area of each plot and recalculate the difference between the two plots. $SFD_2$ understood the meaning of the problem well, where $SFD_2$ explained what information was known and asked in the problem.

The next step is $SFD_2$ developing a solution plan This activity is shown by making an equation for the difference of the two plots. $SFD_2$ made the equation for the difference of the two known plots, with the first plot's area minus the second plot's area. $SFD_2$ can develop a solution plan.

Furthermore, $SFD_2$ performs algebraic operations by involving reversible thinking indicators, but there are errors in the appearent aspect of reciprocity. This error occurred when $SFD_2$ changed the equation from $40 = 8x - 30$ to $8x = (-40) - 30$. In the next stage, $SFD_2$ succeeded in using an aspect reciprocity, namely when $SFD_2$ changed the equation $8x = 10$ to $x = \frac{10}{8}$. At the completion stage, $SFD_2$ made a mistake, which affected the next stage of work.

In the next activity, $SFD_2$ returned to the initial equation by substituting $x = 1{,}25$ into each land plot area. Then, $SFD_2$ was verified by recalculating the difference between the two plots of land. $SFD_2$ can verify the value of the difference *between* the two plots of land. From this, it can be seen that the reversible thinking aspect emerges as the ability to return to the initial value. However, due to the completion process, there was an error in the previous stage, making the recalculation of the difference value of the two plots of land wrong.

Issue 2

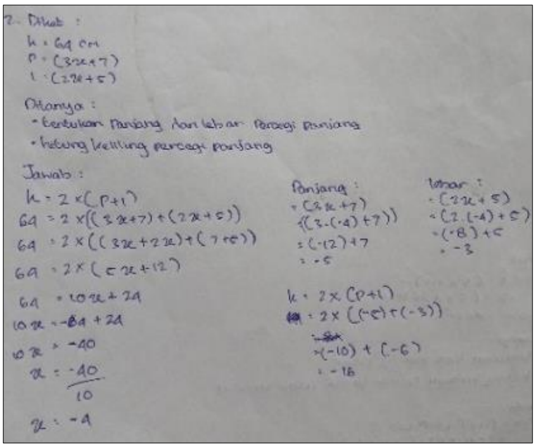

**Figure 8.** $SFD_2$'s second problem answer

$SFD_2$'s problem solving activity begins with understanding the problem. In this activity, *$SFD_2$* got important information on the problem, according to $SFD_2$'s answer in Figure 8. $SFD_2$ wrote the perimeter of the rectangle as $64cm$, length as $(3x + 7)cm$, and width as $(2x + 5)cm$. Then, what is asked in the problem is to determine the length and width of the rectangle and recalculate the perimeter of the rectangle. $SFD_2$ can understand the meaning of the problem well, whereas $SFD_2$ explains what information is known and asked about the problem.

The next step is $SFD_2$ developing a solution plan. This activity is shown by making the equation for the perimeter of the rectangle. $SFD_2$ used the formula for the perimeter of the rectangle obtained previously. After that, he included important information that was known about the problem. Furthermore, $SFD_2$ performed algebraic operations by involving reversible thinking indicators, but there was an error in using equivalent reciprocal relationships, where when $SFD_2$ changed the equation from $64 = 10x + 24$ to $10x = -64 + 24$. At a later stage, $SFD_2$ used equivalent reciprocal relationships, namely when $SFD_2$ changed the equation $10x = -40$ to $x = -\frac{40}{10}$. $SFD_2$ was confused about moving $10x$ segments. It can be concluded that at the problem solving stage $SFD_2$ only uses the reciprocity indicator, but there are still errors.

In the next activity, $SFD_2$ recalculated the value of the perimeter of the rectangle by substituting $x = -4$ in the length and width equation, and $SFD_2$ immediately recalculated the perimeter of the rectangle. $SFD_2$ can verify the value of the perimeter of the rectangle again. From this, it can be seen that the aspect of reversible thinking emerges as the ability to return to the initial value. However, an error in the previous work made $SFD_2$ less confident about the results of his work.

### *Discussion*

Field independent subjects solve problems by involving aspects of reversible thinking. The aspects of reversible thinking that appear in the problem solving process of field independent subjects are negation and reciprocity. The negation aspect appears when students use inversion of related operations, while the reciprocity aspect appears when the subject performs compensation or equal reciprocity by giving the same treatment on both sides (Maf'ulah & Juniati, 2020).

In the first problem, the negation aspect appears when the field independent subject cancels the subtraction of $-10$ with $+10$. This can be seen from the answer of the field independent subject, namely when the field independent subject changes the form of the equation from $40 = 2x - 10$ to $40 + 10 = 2x$. While the reciprocity aspect appears when the field independent subject divides the two segments by 2. We can see this when the field independent subject changes the form of the equation from $50 = 2x$ to $x = \frac{50}{2}$ so that the final value of $x = 25$ is obtained. Based on this, it can be said that $SFI_1$ and $SFI_2$ fulfill the indicators of reversible thinking, namely negation and reciprocity. This is in line with research (Maf'ulah et al., 2023), which states that students with good mathematical ability development will solve problems correctly. In line with that, his research (Hasan, 2020) revealed that students with field independent cognitive styles have a meticulous nature in explaining the problem and can organize the information obtained so that in the process of solving the problem, they will get the right end.

The next aspect of reversible thinking appears to be a return to the initial value after obtaining the results. This can be seen from the field independent subject's answer when recalculating the difference between the two plots. The field independent subject first calculates each plot area; after finding the area of each plot, the field independent subject immediately calculates the difference. In the second problem, the activity of rechecking the answer results was carried out by recalculating the formula for the perimeter of a rectangle. This can be seen from the answer of the field independent subject, who recalculates the perimeter of the rectangle by changing the value that has been obtained. Field independent subjects can believe in the truth of the results they have obtained due to the suitability of the final value obtained with the information known from the problem. Therefore, it can be concluded that the field independent subject fulfills all indicators of reversible thinking and solves the problem correctly. This is in line with research Sutiarso (2020), which reveals that reversible thinking is very influential for someone in solving mathematical problems. In line with that, in her research stated that in solving mathematical problems, students who have the ability to think reversibly will find it easier to deal with it (Purwaningrum & Sutiarso, 2022).

Field dependent subjects perform problem solving according to the plan that has been formulated at the previous stage. However, $SFD_1$ and $SFD_2$ failed to solve the problem according to the plan, where the field dependent subject could not perform the solution steps properly. Students often make mistakes in solving problems due to a lack of care or thoroughness when dealing with problems (Ramadhani & Roesdiana, 2023). Errors made by field dependent subjects occur when changing the sign of operations and when bringing up aspects of reversible thinking.

In the first problem, $SFD_1$ made a mistake in changing the sign of the operation, where when multiplying the operation $(-1)$ with $(-20)$, $SFD_1$ still wrote the result $(+2)$. While in the reversible thinking aspect, $SFD_1$ managed to bring up the negation aspect and the reciprocity. The negation aspect occurs when $SFD_1$ uses inversion of related operations, where $SFD_1$ cancels the addition operation $(+10)$ and $(+20)$ with $(-10)$ and $(-20)$. Likewise, for the reciprocity aspect, this can be seen when changing the form of the equation from $2x = 30$ to $x = \frac{30}{2}$. On the other hand, $SFD_2$ made a mistake when changing the form of the equation $40 = 8x - 30$ to $8x = (-40) - 30$. Here, $SFD_2$ is also wrong in the equivalent reciprocal relationship but is correct when using the negation aspect, which cancels the operation $(+40)$ with $(-40)$.

In the second problem, $SFD_1$ made mistakes in bringing up aspects of reversible thinking. The first mistake made by $SFD_1$ was when canceling the multiplication operation of 2, where $SFD_1$ was wrong in changing the form of the equation from $64 = (5x + 12) \times 2$ to $2 \times 64 = 5x + 12$. The second mistake made by $SFD_1$ was when canceling the operation of $+12$, where $SFD_1$ mistakenly changed the form of the equation from $128 = 5x + 12$ to $12 - 128 = 5x$. However, at a later stage, $SFD_1$ managed to come up with an equivalent reciprocal relationship, namely dividing both segments by 5. This can be seen from SFD1's answer, which changes the form of the equation from $116 = 5x$ to $\frac{116}{5} = x$. Similarly, the mistake made by $SFD_2$ failed to bring up the reversible aspect of thinking. The error occurred when SFD2 was wrong in the equivalent reciprocal relationship but was correct when using the negation aspect, which canceled the operation $(+64)$ with $(-64)$. The mistakes made by $SFD_2$ in the second problem are the same as the mistakes he made in the first problem. From these errors, we can see that the field dependent subject is less careful when performing mathematical operations, so the final value obtained is incorrect. This is in line with his research (Suraji et al., 2018), which states that there are students who understand the problem but are less careful when performing calculations. This is also in line with research (Alifah & Aripin, 2018), which states that students with field dependent cognitive styles tend not to show a coherent line of thinking, make inappropriate steps, and skip some important steps. As a result, the final value obtained is not based on strong arguments, so it can be said that they failed in solving mathematical problems.

In the final stage, the field dependent subject checked the answers that had been obtained. The aspect of reversible thinking that appears at this stage is to return to the initial value after obtaining the results. However, at this stage, the field dependent subject realized that there was an error in his work, wherein, in the first problem, there was a difference in the final value obtained with the information in the problem. $SFD_1$ obtained the final difference result of 38, and $SFD_2$ obtained the final difference result of 7.9, while the difference known from the problem is 40. Likewise, with the second problem, the field dependent subject realized the mistakes made, because there was a difference in the circumference value obtained with the circumference known from the problem. SFD1 obtained the final perimeter result of 246, and $SFD_2$ obtained the final perimeter result $(-16)$, while the perimeter value known from the problem is 64. From this, it can be concluded that the field dependent subject has not mastered the ability of reversible thinking well enough to solve mathematical problems. This is in line with research Maf'ulah et al. (2017), which states that some students still fail to bring up reversible thinking skills in solving mathematical problems. In line with that, in research (Nurlatifah & Hakim, 2024), stated that students who do not have good abilities in reversible thinking will have difficulty solving problems correctly.

The theoretical difference between field independent and field dependent cognitive styles significantly affects their reversible thinking ability. Field independent students' analytical and problem-solving abilities enable them to utilize reversible thinking effectively, ensuring accurate solutions and logical consistency. In contrast, field dependent students' reliance on external guidance and difficulty in separating relevant details often lead to errors in mathematical operations and incomplete problem solving. While field independent students verified their results systematically, field dependent students found that they made mistakes in the solution process due to the difference between the final result and the initial known value.

The results of this study are in line with research findings by Pebrianti et al. (2023) and Amalia et al. (2024), who stated that students with well-developed reversible thinking ability can solve problems more effectively, while those without this ability face significant challenges. By understanding this difference, educators can design various approaches, strategies or learning models targeted at developing reversible thinking ability, such as visual-media-based learning, which will potentially improve their mathematics problem-solving performance.

### *Limitation and Suggestion for Further Research*

This study has several limitations that may affect the results. This study only focuses on reversible thinking and cognitive style, with subjects limited to certain groups. This may affect the diversity of the data and limit the generalizability of the results. In addition, data collection using the GEFT and reversible thinking test also has the potential for bias. For example, reversible thinking tests can be influenced by the way students understand the questions or external factors such as test anxiety, potentially reducing the accuracy of the results. To overcome these limitations, it is recommended that future studies expand the scope of subjects, including students from different levels of education and socio-economic backgrounds, so that the results of the study are more varied and can be better generalised. In addition, future research could also involve more varied instruments to reduce potential bias, such as a combination of performance-based tests with in-depth interviews. The identification of other factors, such as barriers to reversible thinking, logical-mathematical intelligence, maths anxiety or personality type, could also enrich the understanding of the factors that influence reversible thinking in solving mathematics problems.

## CONCLUSIONS

There are significant differences in reversible thinking between students with field independent and field dependent cognitive styles. Students with field independent cognitive styles successfully used aspects of negation and reciprocal relationships in problem solving. In contrast, students with field dependent cognitive styles had more difficulty in using aspects of reversible thinking, especially in using inversion of related operations. In the final stage of completion, both students with field independent and field dependent cognitive styles attempted to return to the initial value after obtaining the result. Students with field independent cognitive styles do it in a structured and systematic of the answer results with the information in the problem is obtained. While students with field dependent cognitive styles get the difference in the final answer with the information in the problem, they are unsure of the answers they get. From this, it shows that the ability to think reversibly students with cognitive style field independent have a better tendency in solving math problems than students with cognitive style field dependent.

## AUTHOR CONTRIBUTIONS STATEMENT

All authors contributed to this research. HRF followed the guidance of each supervisor, such as compiling research instruments, conducting data collection, reporting routine activities, and compiling the final report. EH coordinates and is responsible for the research process, as well as guiding and directing research both technically and substantively. TDC assists the lead researcher, checks research instruments, and guides students in data collection activities, data analysis processes, reporting research results, and publication processes.

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