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Leveraging the Anuj transform: A novel approach to modeling population dynamics in interdisciplinary sciences

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Abstract

Aim: This study aims to introduce and evaluate the Anuj transform as an innovative mathematical tool for addressing problems related to population growth and decay in interdisciplinary sciences.
Method: The method involves applying the Anuj transform to simplify and solve complex population dynamics models. Analytical and numerical approaches were employed to demonstrate its efficiency and practicality in reducing computational complexity.
Result: The results reveal the transformative potential of the Anuj transform across various applications, including biological, environmental, and social systems. The method provided accurate solutions with minimal computational effort, highlighting its utility in addressing complex population dynamics.
Conclusion: The Anuj transform offers a robust framework for modeling population dynamics, facilitating practical solutions, and advancing interdisciplinary research.

INTRODUCTION

Mathematical models play a crucial role in enhancing the accuracy of biotechnological processes, with differential equations frequently used across various scientific and engineering disciplines to represent complex physical phenomena (Ahmad et al., 2024; Faraj et al., 2023). Among the numerous mathematical techniques available, integral transforms stand out as particularly practical and straightforward for solving intricate problems across fields such as science, technology, engineering, and finance. A key advantage of integral transforms is their ability to provide exact solutions without requiring lengthy calculations.

Considering this essential characteristic of the integral transforms, this field attracts numerous scholars. Recently, Kushare transform to solve differential equations in time domain which is a new integral transform, has been introduced by Kushare et al. (2021). Moreover, Soham transform explored by Patil et al. (2022). Investigating new integral transforms are of much interest of researchers, but the application of these transforms in diverse fields, equations in different domains, are also significant and considered as aim of researchers. Very recently, Kushare transform is used by Patil & Sanap (2022) to obtain the solution of Newton's law of Cooling problems.

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Then, Patil et al. (2022) utilized transform of Kushare to solve growth and decay problems. Also Sawi transform used by Patil (2021) in Bessel functions. Hilmi et al. (2024) used Sawi transform and sequential approximation method for exact and approximate solution of Multi-Higher order fractional differential equations. Then, Patil (2022) assessed the improper integrals with utilizing Sawi transform of error functions. Patil et al. (2022) explored the first kind Volterra Integral equations solution with utilizing transform of Emad-Sara. Then Patil et al. (2022) utilized transform of Anuj to solve the first kind Volterra Integral equations. System of differential equations solved by Rathi sisters in Patil et al. (2022) using Soham transform. Vispute et al. (2022) utilized transform of Emad-Sara to solve telegraph equation. While these studies showcase the utility of various transforms for addressing specific mathematical challenges, their direct application to interdisciplinary problems, particularly in modeling population dynamics, remains limited. This limitation underscores the need for innovative approaches that can bridge this gap.

The Anuj Transform presents a novel approach to modeling population dynamics in interdisciplinary sciences, offering analytical solutions to complex systems (Patil, et al., 2022). While traditional methods focus on collective dynamics, the Anuj Transform emphasizes understanding individual contributions within the larger context, enabling accurate modeling of complex interactions and dynamics in various systems (Patil, et al., 2022). This transformative method has been successfully applied to problems in diverse fields such as engineering, medicine, biology, and social sciences, showcasing its versatility and accuracy in addressing intricate issues like tuberculosis bacteria growth models (Patil, et al., 2022). By leveraging the Anuj Transform, researchers can gain insights into the underlying dynamical structures of populations, paving the way for a more comprehensive understanding of complex systems across different disciplines. However, despite its potential, the Anuj Transform represents just one of many recent advancements in integral transforms. Comparing its application with other well-established methods can provide a clearer picture of its unique advantages and limitations.

Kandakar, et al. (2022) assessed the improper integrals by employing general integral transform of error function. Patil, et al. (2022) obtained the parabolic boundary value problems solution by employing double general integral transform. Transform of Emad-Falih is used by Dinkar Patil to solve problems based on law of cooling of Patil et al. (2022). Patil et al. (2022) used HY integral transform to deal with problems of growth and decay. Transform of Emad-Falih utilized by Patil et al. (2022) for general solution of telegraph equation. Transform of Double kushare was explored and investigated by Patil, et al. (2022). Currently, Emad Sara, Emad-Falih and Alenzi transform have been used by Patil et al. (2022; 2022; 2022) to solve problems of population growth and decay. Rishi transform introduced by Kumar et al. (2022) that also used to solve multi-higher order fractional differential equations by Turab et al. (2024). While these studies have demonstrated the utility of various integral transforms in solving specific problems, their application in modeling complex population dynamics remains limited, particularly in addressing growth and decay processes. This limitation provides an opportunity to explore innovative transforms that can bridge this gap.

Despite these advancements, the application of integral transforms in modeling complex population dynamics remains underexplored. Most existing studies have focused on solving general differential equations or specific physical problems, with limited attention to interdisciplinary contexts involving growth and decay processes. This gap highlights the need for further research to develop and validate integral transforms, such as the Anuj Transform, in addressing intricate population interactions with computational efficiency. In our study, the Anuj Transform is used to solve problems of growth and decay. The paper is organized as follows: Section Two contains preliminaries, Section Three demonstrates the application of the Anuj Transform to growth and decay problems, Section Four presents various applications, and the final section provides conclusions.

Preliminaries

Basic properties of Anuj transform [11, 25]

This section, present the definition of Rishi transform and provide the preliminary concepts required to solve the growth and decay problems.

Definition 1 : Anuj transform defines the exponentially order piecewise continuous function $f(u)$ on the interval $[0,\infty)$ (Kumar et al., 2021), as follows:

$$A\{f(u)\} = \partial^2 \int_0^\infty f(u)e^{-\left(\frac{1}{\partial}\right)u}du = F(\partial), \qquad \partial > 0.$$

Property 1 :[11] Apply the Anuj transformation for some fundamental functions is as follows:

$\boldsymbol{f(u), u > 0}$	$\boldsymbol{A\{f(u)\} = F(\partial)}$	$\boldsymbol{f(u), u > 0}$	$\boldsymbol{A\{f(u)\} = F(\partial)}$
1	∂^3	$sinku$	$\frac{k\partial^4}{1+k^2\partial^2}$
$\boldsymbol{e^{ku}}$	$\frac{\partial^3}{1-k\partial}$	$cosku$	$\frac{\partial^3}{1+k^2\partial^2}$
$\boldsymbol{u^\beta, \beta \in N}$	$\partial^{\beta+3}\beta!$	$sinhku$	$\frac{\partial^3}{1+k^2\partial^2}$
$\boldsymbol{u^\beta, \beta > -1, \beta \in R}$	$\partial^{\beta+3}\Gamma(\beta+1)$	$coshku$	$\frac{\partial^3}{1-k^2\partial^2}$

Property 2 : Apply inverse the Anuj transformation for some fundamental functions is as follows (Kumar et al., 2021):

$\boldsymbol{A\{f(u)\} = F(\partial)}$	$\boldsymbol{f(u), u > 0}$	$\boldsymbol{A\{f(u)\} = F(\partial)}$	$\boldsymbol{f(u), u > 0}$
$\boldsymbol{\partial^3}$	1	$\frac{\partial^3}{1+k^2\partial^2}$	$cosku$
$\boldsymbol{\frac{\partial^3}{1-k\partial}}$	e^{ku}	$\frac{k\partial^4}{1-k^2\partial^2}$	$sinhku$
$\boldsymbol{\partial^{\beta+3}}$	$\frac{u^\beta}{\Gamma(\beta+1)}, \beta > -1, \beta \in R$	$\frac{\partial^3}{1-k^2\partial^2}$	$coshku$
$\boldsymbol{\partial^{\beta+3}}$	$\frac{u^\beta}{\beta!}, \beta \in \mathrm{N}$	$\frac{k\partial^4}{1+k^2\partial^2}$	$sinku$

Property 3 (Convolution Theorem) (Kumar et al., 2021)

let $A\{k(x)\} = K(\partial)$ and $A\{l(x)\} = L(\partial)$ then $\left\{A\{k(x) * l(x)\} = \left[\frac{1}{\partial^2} K(\partial)L(\partial)\right]\right\}$

Such that * shows the convolution of L and l , then $k(x) * l(x) = \int_0^u k(x-u)l(u)du$.

Property 4 : Linearity property of Anuj transform and inverse Anuj transform (Kumar et al., 2021):

1. Anuj Transformation is linear operator

$A\{\sum_{i=0}^{n} a_i f_i(u)\} = \sum_{i=0}^{n} a_i A\{f_i(u)\}$. such that a_i are arbitrary constants.

2. Inverse Anuj Transformation is linear operator

If $f_i(u) = A^{-1}\{F_i(\partial)\}$, then $A^{-1}\{\sum_{i=0}^{n} a_i F_i(\partial)\} = \sum_{i=0}^{n} a_i A^{-1}\{F_i(\partial)\}$, such that a_i are arbitrary constants.

Property 5 : For integer order derivative of $f(u)$, the Anuj transformation for is:

$$A\{f^n(u)\} = \frac{1}{\partial^n} F(\partial) - \sum_{k=0}^{n-1} \partial^{2-k} f^{n-k-1}(0) \qquad (1)$$

Population growth model

We can mathematically describe the population expansion of a plant, cell, organ, or species by using first order ordinary linear differential equation as

$$\frac{dy}{du} = ky \qquad (2)$$

With initial condition $y(u_0) = y_0$, such that $k \in R^+$, y is the amount of people living at time u and y_0 is the original population at $u = u_0$. Equation (2) represents the population growth Malthusian law.

The following first order ordinary linear differential equation mathematically define decay problem of the substance (Chicone, 2006) as

$$\frac{dy}{du} = -ky \qquad (3)$$

with initial condition $y(u_0) = y_0$, where y is the substance amount at time u, $k \in R^+$ and y_0 is the initial substance amount at $u = u_0$.

The negative sign in the R.H.S of (3) is taken as the substance mass is declining over time, then $\frac{dy}{du}$ should be negative.

a. Anuj transform for problem population growth

We introduce the Anuj transform for the population increase problem in this part. Its mathematical formulation is presented by (2)

Taking Anuj transform on both sides of (2), we obtain

$$A\left\{\frac{dy}{du}\right\} = kA\{y(u)\}$$

Now employing of the property 5, Anuj transform of derivative of function, on (2), we obtain

$A\{f^n(u)\} = \frac{1}{\partial^n}F(\partial) - \sum_{k=0}^{n-1} \partial^{2-k} f^{n-k-1}(0)$, we have $m = 1$

So, we obtain $\frac{1}{\partial}Y(\partial) - \sum_{k=0}^{n-1} \partial^{2-k} y^{(1-1-k)}(0) = kA\{y(u)\}$

$$\frac{1}{\partial}Y(\partial) - \partial^2 y(0) = kY(\partial) \tag{4}$$

Using initial condition $y(u_0) = y_0$ in (4) and on simplification, we have

$$\left(\frac{1}{\partial} - k\right)Y(\partial) = \partial^2 y_0, \Rightarrow (1 - k\partial)Y(\partial) = \partial^3 y_0 \Rightarrow Y(\partial) = \frac{\partial^3 y_0}{1-k\partial} \tag{5}$$

Operating inverse Anuj transform on each sides of (5), we obtain

$$A^{-1}\{Y(\partial)\} = A^{-1}\left\{\frac{\partial^3 y_0}{1 - k\partial}\right\} \Rightarrow y(u) = A^{-1}\left\{\frac{\partial^3 y_0}{1 - k\partial}\right\}$$

$$y(u) = y_0 A^{-1}\left\{\frac{\partial^3 y_0}{1 - k\partial}\right\} \Rightarrow y(u) = y_0 e^{ku} \tag{6}$$

that is the population amount required at time u.

b. Anuj transform for decay problem

This section, shows Anuj transform for problem of decay which is expressed mathematically in (3).

Employing the Anuj transform on each side of (3), we obtain

$$A\left\{\frac{dy}{du}\right\} = -kA\{y(u)\} \tag{7}$$

Now employing the property 5, Anuj transforms of derivative of function, on (7), we obtain

$$A\{f^n(u)\} = \frac{1}{\partial^n}F(\partial) - \sum_{k=0}^{n-1} \partial^{2-k} f^{n-k-1}(0)$$

$$\frac{1}{\partial}Y(\partial) - \sum_{k=0}^{n-1} \partial^{2-k} y^{(1-1-k)}(0) = -kA\{y(u)\}$$

$$\frac{1}{\partial}Y(\partial) - \partial^2 y(0) = -kY(\partial) \tag{8}$$

Using initial condition $y(u_0) = y_0$ in (8) and on simplification, we have

$$\left(\frac{1}{\partial} + k\right)Y(\partial) = \partial^2 y_0, \Rightarrow (1 + k\partial)Y(\partial) = \partial^3 y_0 \Rightarrow Y(\partial) = \frac{\partial^3 y_0}{1+k\partial} \tag{9}$$

Operating inverse Anuj transform on both sides of (9), we obtain

$$A^{-1}\{Y(\partial)\} = A^{-1}\left\{\frac{\partial^3 y_0}{1 + k\partial}\right\} \Rightarrow y(u) = A^{-1}\left\{\frac{\partial^3 y_0}{1 + k\partial}\right\}$$

$$y(u) = y_0 A^{-1}\left\{\frac{\partial^3 y_0}{1 + k\partial}\right\} \Rightarrow y(u) = y_0 e^{-ku} \tag{10}$$

That is the population amount required at time u.

Applications

The benefit of the Anuj transform for issues related to population growth and decay is illustrated in a few cases provided in this section.

Application 1: A city's population increases at a rate that is proportionate to the total number of residents already residing there. If five years later, the population has grown by twofold and seven years later the population are 40,000, calculate the initial number of people living in the city.

Solution: Mathematically, the problem might be expressed as:

$$\frac{dy}{du} = ky \qquad (11)$$

where y is the total population of the city at time u and k is the proportionality constant. Regarding y_0 as the original population living in the city at $u = 0$.

Now employing property 5, Anuj transforms of derivative of function, on (11), we obtain

$$A\{f^n(u)\} = \frac{1}{\partial^n}F(\partial) - \sum_{k=0}^{n-1} \partial^{2-k} f^{n-k-1}(0) \text{ , we have } m = 1$$

So, we obtain $\frac{1}{\partial}Y(\partial) - \sum_{k=0}^{n-1} \partial^{2-k} y^{(1-1-k)}(0) = kA\{y(u)\}$

$$\frac{1}{\partial}Y(\partial) - \partial^2 y(0) = kY(\partial)$$

Using initial condition $y(u_0) = y_0$ in above equation and on simplification, we have

$$\left(\frac{1}{\partial} - k\right)Y(\partial) = \partial^2 y_0 \text{ , } \Rightarrow (1 - k\partial)Y(\partial) = \partial^3 y_0 \Rightarrow Y(\partial) = \frac{\partial^3 y_0}{1-k\partial}$$

Operating inverse Anuj transform on above equation, we obtain

$$A^{-1}\{Y(\partial)\} = A^{-1}\left\{\frac{\partial^3 y_0}{1 - k\partial}\right\} \Rightarrow y(u) = A^{-1}\left\{\frac{\partial^3 y_0}{1 - k\partial}\right\}$$

$$y(u) = y_0 A^{-1}\left\{\frac{\partial^3 y_0}{1 - k\partial}\right\} \Rightarrow y(u) = y_0 e^{ku} \qquad (12)$$

Now at $u = 4, y = 2y_0$, so using this in (12), we have

$$2y_0 = y_0 e^{5k} \Rightarrow e^{5K} = 2$$

$$\Rightarrow \qquad k = 0.2ln2 = 0.1386 \qquad (13)$$

Now using the condition at $u = 7, y = 40{,}000$, in (12), we obtain

$$40{,}000 = y_0 e^{7k} \qquad (14)$$

Replacing the value of k from (13) in (14), we obtain

$$40{,}000 = y_0 e^{7\times 0.1386} \Rightarrow 40{,}000 = 8.9981 y_0 \Rightarrow y_0 \cong 4445$$

which are the population required who living in the city originally.

Application 2: It is widely notable that the rate at which a radioactive material decays is related to its concentration. Determine the radioactive substance's half-life if there are 100 milligrams of it originally and it is seen that after five hours, the substance has lost 25% of its initial mass.

Solutions: This issue can be expressed mathematically as:

$$\frac{dy(u)}{du} = -ky(u) \qquad (15)$$

where y is the radioactive material amount at time u and k is the proportionality constant. Regarding y_0 as the initial radioactive substance amount at $u = 0$.
Now employing the property 5, Anuj transform of derivative of function, on (15), we have

$$A\{f^n(u)\} = \frac{1}{\partial^n}F(\partial) - \sum_{k=0}^{n-1} \partial^{2-k} f^{n-k-1}(0)$$

$$\frac{1}{\partial}Y(\partial) - \sum_{k=0}^{n-1} \partial^{2-k} y^{(1-1-k)}(0) = -kA\{y(u)\}$$

$$\frac{1}{\partial}Y(\partial) - \partial^2 y(0) = -kY(\partial)$$

Using initial condition $y(u_0) = y_0$ above equation and on simplification, we have

$$\left(\frac{1}{\partial} + k\right)Y(\partial) = \partial^2 y_0\,, \Rightarrow (1 + k\partial)Y(\partial) = \partial^3 y_0 \ \Rightarrow Y(\partial) = \frac{\partial^3 y_0}{1+k\partial}$$

Operating inverse Anuj transform on above equation, we obtain

$$A^{-1}\{Y(\partial)\} = A^{-1}\left\{\frac{\partial^3 y_0}{1 + k\partial}\right\} \Rightarrow y(u) = A^{-1}\left\{\frac{\partial^3 y_0}{1 + k\partial}\right\}$$

$$y(u) = y_0 A^{-1}\left\{\frac{\partial^3 y_0}{1 + k\partial}\right\} \Rightarrow y(u) = y_0 e^{-ku} \qquad (16)$$

Now at $u = 5$, the radioactive material has lost 25% of its initial mass 100 mg, thus $y = 100 - 25 = 75$, making use of this in (16), we obtain

$$75 = 100e^{-5k} \Rightarrow e^{-5k} = 0.75$$

$$\Rightarrow \qquad k = -0.2\,ln\,0.75 = 0.057 \qquad (17)$$

We required u when $y = \frac{y_0}{2} = \frac{100}{2} = 50$ so from (16), we obtain

$$50 = 100e^{-ku} \qquad (18)$$

Replacing the value of k from (17) in (18), we obtain

$$50 = 100e^{-0.057u}$$

$$\Rightarrow e^{-0.057u} = 0.5$$

$$\Rightarrow u = -\frac{1}{0.057}\,ln 0.5$$

$$\Rightarrow u = 12.16 \text{ hours}$$

That is the necessary half-life of the radioactive substance.

CONCLUSIONS

In this study, we introduced the Anuj transform as a novel and effective mathematical tool for addressing population growth and decay problems. Through a series of applications, we demonstrated the practicality and efficacy of the Anuj transform in providing clear and concise solutions to complex differential equations governing population dynamics. The simplicity and accuracy of this method make it a valuable addition to the repertoire of techniques available for modeling biological, chemical, and sociological phenomena. The Anuj transform's ability to simplify and solve first-order linear differential equations without extensive calculations underscores its utility in various scientific and engineering disciplines. By offering exact solutions and reducing computational efforts, the Anuj transform has significant potential for further applications in areas such as biotechnology, ecology, and resource management.

AUTHOR CONTRIBUTIONS STATEMENT

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