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The potrrait of prospective mathematics teacher in critical thinking through problems with contradictory information: A view from prior knowledge

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Abstract

Background: The development of critical thinking skills in prospective mathematics teachers is essential for their future effectiveness in the classroom. Understanding how these individuals process and resolve problems that contain contradictory information provides insight into their critical thinking abilities. Previous research has highlighted the significant role of prior knowledge in problem-solving and critical thinking.
Aim: This study aims to explore the critical thinking processes of prospective mathematics teachers when faced with problems that contain contradictory information. Specifically, it seeks to determine the influence of prior knowledge on their ability to navigate and resolve these complex problems.
Methods: The study employed a sequential explanatory design. Initially, quantitative data from prerequisite skill and critical thinking tests (specifically, problems with contradictory information) were collected from 68 participants. Simple regression analysis informed the selection of six participants (two each with high, medium, and low prerequisite abilities) for the subsequent qualitative phase. In-depth interviews and problem-solving tasks were conducted, prompting participants to articulate their thought processes. Data analysis focuses on identifying patterns and themes in their use of prior knowledge and critical thinking strategies.
Results: The findings reveal that prior knowledge plays a pivotal role in how prospective mathematics teachers approach and resolve problems with contradictory information. Those with a strong foundation in mathematical concepts and problem-solving strategies are better equipped to identify inconsistencies and develop logical solutions. Conversely, participants with limited prior knowledge struggle to reconcile conflicting information and often resort to less effective problem-solving methods.
Conclusion: This study underscores the importance of prior knowledge in the development of critical thinking skills among prospective mathematics teachers. Educator preparation programs should emphasize the cultivation of a robust knowledge base and provide opportunities for students to engage in complex problem-solving tasks. By doing so, future teachers will be better prepared to navigate the challenges of the classroom and foster critical thinking in their own students.

INTRODUCTION

The Pandemic has brought about significant transformations in the education system, particularly impacting Mathematics Education students at both public and private universities academic levels (Anugraheni, 2019; Bellaera et al., 2021; Perkins & Murphy, 2006). Through critical thinking, students establish conceptual connections with reflective decision-making,

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issue understanding, high-level thinking, logical reasoning, decision-making, problem-solving, and scientific methods (Giancarlo & Facione, 2001; Kim et al., 2013). For instance, when students are presented with information whose truth is not yet certain, they require the ability to gather relevant concepts associated with that information. Subsequently, they need to reflect on the actions they should take, whether to start identifying the proof of its truth or merely believe it. When engaging in proof, it is essential to ascertain whether they thoroughly understand the proving tools and the logical reasoning they will employ. This process continues until they are capable of solving the problem.

Problem solving and critical thinking abilities are highly essential in the 21st-century market. This signifies that simply mastering knowledge and information alone is insufficient. This competition underscores the need for a generation that is collaborative, creative, innovative, communicative, and analytically critical in their thinking. To prepare them to effectively address the complexity of problems, both in the workplace and in personal life (Clarisa et al., 2021; Peter, 2012; ŽivkoviĿ, 2016).

The process of honing critical thinking skills is undoubtedly supported by the vital role of teachers as the vanguard of educational progress (Alsaleh, 2020; Lorencová et al., 2019; Sahika, 2018). It is significant whether individuals hold the belief that their fundamental traits are determined by nature (referred to as an entity theory or fixed mindset) or whether they believe that their traits can be cultivated (referred to as an incremental theory or growth mindset) (Dweck, 2012; Dweck & Yeager, 2019; Farrington et al., 2012). However, altering the fixed mindset and behavior of teachers to become critical thinkers is not a simple matter, as they are already adults and resistant to change (Hargreaves, 2005; Holloway & Gouthro, 2011; Ketelhut et al., 2020; Wright, 2021). Therefore, preparing future teachers who are capable of critical thinking is more strategic than the arduous journey of training existing teachers (As'ari et al., 2017). Additionally, the education of prospective teachers, among others, plays a crucial role in developing students' critical thinking abilities (Rochmad et al., 2018; Zayyadi & Subaidi, 2018). If prospective teachers do not master this skill well, it will be difficult for them to fulfil their role in advancing critical thinking in their students.

Critical thinking is essential in learning mathematics because mathematics is not just about memorizing formulas and procedures but also about understanding concepts, applying logical principles, and solving problems systematically and creatively (Celik & Ozdemir, 2020; Wulan & Ilmiyah, 2022). With critical thinking, students can develop the ability to analyze situations, evaluate information, and make appropriate decisions within the context of mathematics (Erdoğan, 2020; Jablonka, 2020; Lin et al., 2021; Romero Ariza et al., 2021). This not only helps them master mathematics but also develops thinking skills relevant to various aspects of life.

Critical thinking skills can be honed through stimuli in the form of complex and non-routine problems that must be integrated into the learning process (Ismail & Bempah, 2018). Non-routine problems serving as learning stimuli consist of problems-to-find and problems-to-prove (Polya, 2014). Non-routine problems, more commonly studied, include problem-to-find (Ismail & Bempah, 2018; Mujib et al., 2021; Rochmad et al., 2018; Widodo et al., 2019; Zayyadi & Subaidi, 2018). Subsequently, problems categorized by cognitive levels in secondary schools have been examined and analyzed based on the theory of high-level thinking in general and problem-item development studies (Angriani et al., 2018; Arifin & Retnawati,

2017; Kurniasi & Arsisari, 2020; Masitoh & Aedi, 2020). When compared, problems-to-prove have yet to be explored.

One type of problem to prove is known as a truth-seeking problem. The concept of truth-seeking problems has been developed as a predictor of critical thinking (Kurniati et al., 2019). Truth-seeking behavior emerges when an individual consistently strives to discover the truth in the information provided and seeks evidence to arrive at a precise solution. Truth-seeking problems can manifest as problems with contradictory information nuances, known as Problems with Contradictory Information (PWCI) (Kurniati et al., 2019). PWCI presents information content that contradicts each other (Ardiansyah et al., 2022; Hariati et al., 2022). A sceptical individual will engage in verification efforts before trusting in the information presented within a problem (Primiero et al., 2017). Therefore, presenting problems in the form of PWCI can enhance an individual's sensitivity to selecting information and accompany the development of critical thinking skills.

An essential asset for someone to successfully solve a problem is an awareness of relevant prior knowledge (Razak, 2017). Inadequate or fragmented prior knowledge becomes a significant challenge to consider. If there is a mismatch between the required prior knowledge and an individual's actual knowledge base, the learning process can be hindered immediately (Hailikari et al., 2008). Contact with an individual's prior knowledge can easily become engaged during the learning process (Schwartz et al., 2007) and when faced with problems. Prior knowledge, often referred to as prerequisites, has been studied in secondary school students in relation to their motivation and learning outcomes in mathematics, as well as other abilities such as logical reasoning and critical thinking (Lestari, 2017; Pamungkas et al., 2017; Razak, 2017).

Research related to PWCI has been extensively conducted among mathematics-based school students (Amalia, 2020; Aminudin & Maharani, 2021; Ardiansyah et al., 2022; Hariati et al., 2022; Mutmainah et al., 2021; Rohmah et al., 2022). Furthermore, most mathematics teachers tend to solve problems directly without recognizing the contradictions in the questions provided (Hasanah et al., 2022). Moreover, research in higher education suggested that 73% of students needed help to identify contradictory information in mathematical induction problems (Wulan & Ilmiyah, 2022). Whereas in previous studies, the focus was on exploring students' critical thinking processes when solving PWCI or recognizing existing contradictions, this study aims to discuss its connection with prerequisite knowledge further. Here, prerequisite knowledge serves as the initial basis for someone's thinking before they attempt to solve a problem. We are following up on the research by Wulan & Ilmiyah (2022), and the PWCI being utilized is a mathematical induction problem. In addition, these problems frequently arise in mathematical proof topics, such as number theory, algebra, and analysis. Mathematical induction problems presented in the form of PWCI are rarely studied.

Therefore, this study aims to investigate whether there is a significant influence of prior knowledge in mathematical induction on the critical thinking abilities of students. The next objective is to comprehensively describe the characteristics of critical thinking among prospective mathematics teachers when solving PWCI at each level of prior knowledge abilities. The portrait of the relationship between prior abilities and critical mathematical thinking abilities at the higher education level becomes an essential matter for study, primarily in the context of preparing programs that support students' critical thinking abilities.

METHODS

This research employs a mixed-method design, which includes both quantitative and qualitative approaches. Specifically, it utilizes a sequential explanatory design, where data is collected over a specified period in two consecutive phases (Ivankova et al., 2006). In the first phase, quantitative data is gathered and analyzed, consisting of students' prerequisite skill test results and critical thinking ability test results in PWCI. Following this analysis, the study proceeds to the second phase, which involves collecting qualitative data through interviews and relating it to the findings from the initial phase.

The population of this research comprises all the students of the batch 2020 of the Mathematics Education Study Program at IAIN Kediri, who are currently enrolled in the Real Analysis course and have studied mathematical induction. A saturation sampling technique is employed, resulting in a sample size of 68 students. Subsequently, research subjects are selected purposively, with six students chosen from each category, representing high, medium, and low levels of prerequisite abilities. Subject selection is based on the characteristics of critical thinking test responses that align with the first phase of the study.

The data was collected through testing and interviews. The instruments include critical thinking test questions in PWCI, as illustrated in Figure 1, prior knowledge test questions related to the Principle of Mathematical Induction, as shown in Figure 2, and a semi-structured interview guide presented in Table 1. The instruments were developed through peer discussions among mathematics education faculty members and underwent content validity testing by mathematics education experts. Inter-rater agreement among validators was assessed using the Gregory Model.

Unno has been assigned a college assignment in the Real Analysis course. He was asked to create a sequence of numbers that is always negative. After experimenting with various patterns, he eventually obtained the following sequence calculation.

$$1! - 3 = -2$$
$$2! - 3^2 = -7$$
$$3! - 3^3 = -21$$
$$4! - 3^4 = -57$$

Please help Unno investigate whether the pattern can be used to fulfil the assignment. If it can, explain your reasoning. Suppose it cannot, use the Principle of Mathematical Induction to strengthen your answer.

Figure 1. Critical Thinking Ability Test: Mathematical Induction in PWCI

1. It is given a statement:
$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$
Prove that the statement above is true for every natural number n.
2. Consider the following statements.
$P(1)$: $1^3 + 2^3 + 3^3$ is divisible by 9
$P(2)$: $2^3 + 3^3 + 4^3$ is divisible by 9
$P(3)$: $3^3 + 4^3 + 5^3$ is divisible by 9
Prove your answer. Is the statement $P(n)$ formed according to the pattern above true for every natural number n? Prove your answer.
3. Prove that $2n - 3 \leq 2^{n-2}$ for all ≥ 5, $n \in \mathbb{N}$.

Figure 2. Prior Knowledge Ability Test: Mathematical Induction

Table 1. Interview Guidelines Based on Critical Thinking Indicators

Stage	Description	Indicator	Interview Questions
Clarification	Stating, clarifying, describing (not explaining), or defining the problem under discussion.	• Presenting a debated issue • Analyzing, negotiating, or discussing the meaning of the issue • Identifying one or more assumptions behind the statements being discussed • Identifying relationships between statements or assumptions • Defining terms relevant to the problem	• What information do you know from the problem? Explain. • What do you know regarding what is being asked or requested in the problem? Explain. • What mathematical concepts/ideas do you think are relevant to solve the problem? Explain.
Assessment	Evaluating various debated aspects, making decisions in a situation, proposing evidence for arguments, or their connection to other problems.	• Providing/requesting reasons that the presented evidence is valid and relevant. • Establishing criteria used to assess a condition. • Making a judgment of true or false regarding a criterion, situation, or topic. • Presenting evidence for the choice of assessment criteria.	• What are the conditions that must be met to solve the problem? Explain. • Why are these conditions necessary to solve the problem? Explain. • What needs to be proven from the conditions you've stated? Explain. • Why is it important to conclude at each stage of your proof?
Inference	Demonstrating the relationship between ideas; drawing appropriate conclusions through deduction or induction, generalizing, explaining (not describing), and making hypotheses.	• Making appropriate deductions. • Drawing appropriate conclusions. • Arriving at a final conclusion. • Making generalizations. • Summarizing the relationships between ideas.	• What initial conclusions do you draw from the question? • Are there any specific patterns/generalizations you make when solving the problem? If so, explain. • What conclusions do you arrive at at each stage of your proof?
Strategy	Suggesting, discussing, or evaluating possible actions.	• Taking an action. • Describing possible actions. • Evaluating potential actions. • Predicting the outcomes of proposed actions.	• Explain the steps you took to solve the problem. • Why did you choose these steps? • What results did you expect from these steps? • Did you consider other approaches to solving it? If so, explain why.

(Perkins & Murphy, 2006)

The quantitative data analysis technique is performed through simple regression analysis. Assumption tests are carried out, including tests for normality and linearity. The quantitative data analysis technique used in this research is simple regression analysis using SPSS 24.0. Qualitative data analysis involves (1) data reduction, (2) data presentation, and (3) drawing conclusions or verification (Miles & Huberman, 1994). The research method flow is as Figure 3.

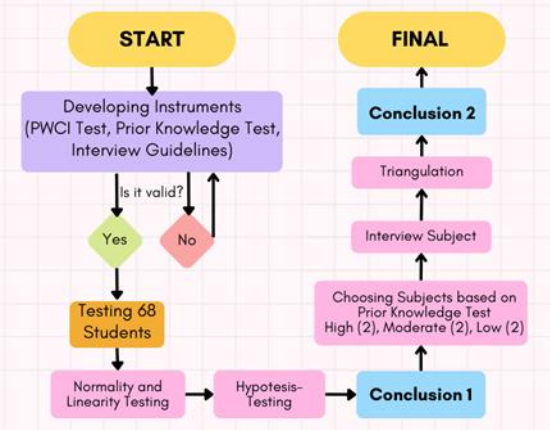

Figure 3. Research Method Flow

RESULTS AND DISCUSSION

The results of quantitative data can be presented in two parts, i.e. the description of student prior knowledge skills data and the description of student critical thinking abilities data when solving mathematical induction in PWCI. The next is the results of hypothesis testing. Furthermore, the results of the qualitative data are elaborated according to the level of the student's prior knowledge.

Descriptive data of students' prior knowledge skills and critical thinking abilities.

The data on students' prior knowledge skills were obtained from the results of students' tests. The data description was performed using SPSS 24.0. The results of the descriptive data of the prerequisite skill test are shown in Table 2.

Table 2. Descriptive Data of Students' Prior Knowledge Skills

	N	Minimum	Maximum	Mean	Std. Deviation
Prior Knowledge Skills (PNS)	68	10.00	100.00	66.6667	20.04142

Table 2 shows that the results of the prior knowledge skill test for 68 students have a minimum value of 10.00, a maximum value of 100.00, an average value of 66.67, and a standard deviation (SD) of 20.04. Furthermore, based on the test results, student prerequisite skills were categorised for the 68 samples in Table 3.

Table 3. Categorization of Prior Knowledge Skill Groups

Score		Category	Number of Students
Interval Notation	**Conversion Interval**		
PNS $\geq \bar{x} + SD$	PNS $\geq$ 86.7081	High	11
$\bar{x} - DS \leq$ KPM $< \bar{x} + SD$	62,6253 $\leq$ PNS $<$ 86.7081	Moderate	45
KAM $< \bar{x} - SD$	PNS $<$ 62.6253	Low	12

The data on students' critical thinking abilities were obtained from the results of the students' critical thinking skill tests when solving mathematical induction of PWCI. The data description was carried out with the assistance of SPSS 24.0. The test results for students'

critical thinking abilities are presented in Table 4. Table 4 shows that the test results for students' critical thinking abilities among 68 students have a minimum value of 0.00, a maximum value of 100.00, an average value of 62.62, and a standard deviation (SD) of 20.49. This indicates a wide range of scores in students' critical thinking and significant variation in critical thinking abilities among students.

Table 4. Descriptive Data of Students' Critical Thinking Abilities

	N	Minimum	Maximum	Mean	Std. Deviation
Critical Thinking Abilities	68	0.00	100.00	62.6225	20.48807

Results of the assumption test and research hypothesis testing

Quantitative data obtained from the research, derived from the prior knowledge skill test administration for students and the critical thinking ability test, were analyzed using Simple Regression Analysis, with tests for normality and linearity.

The normality test in the regression model is used to determine whether the residual values are normally distributed. In this case, the normality of the residuals produced by the regression model is being tested, not the individual independent and dependent variables. The Kolmogorov-Smirnov test, as adapted by Anwar (2009), is employed: if the probability > the significance level ($\alpha = 0.05$), then H_0 is accepted, and if the probability ≤ the significance level ($\alpha = 0.05$), then $H_0$is rejected. The results of the normality test of the residual values with the assistance of SPSS 24.0 are shown in Table 5.

Based on Table 5, the interpretation of the normality test results indicates that $(0{,}200) > 0{,}05$, meaning that H_0 is accepted, confirming that the residual values follow a normal distribution. Furthermore, Figure 4, using the probability-plot technique, shows that the plotted points consistently follow the diagonal line, suggesting that the residual values are normally distributed.

Table 5. Results of the Residual Value Normality Test (Source: SPSS 24.0 Output)

One-Sample Kolmogorov-Smirnov Test		
		Unstandardized Residual
N		68
Normal Parameters[a,b]	Mean	.0000000
	Std. Deviation	18.15491310]
Most Extreme Differences	Absolute	.094
	Positive	.061
	Negative	-.094
Test Statistic		.094
Asymp. Sig. (2-tailed)		.200[c,d]

a. Test distribution is Normal.
b. Calculated from data.
c. Lilliefors Significance Correction.
d. This is a lower bound of the true significance.

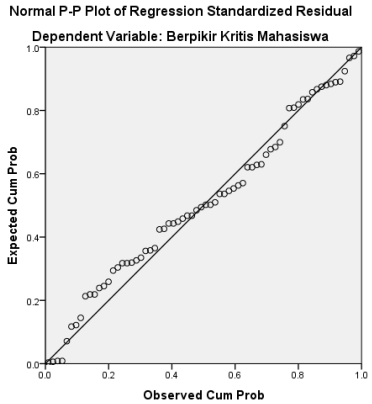

Figure 4. Results of the Normal P-P Plot of Residual Values (Source: SPSS 24.0 Output)

To determine whether a linear model is applicable, we assess whether the $F_{table} < F_{score}$ at a 5% significance level and evaluate the significance value for linearity, which should be less than 0.05. The results of the linearity test, conducted with the assistance of SPSS 24.0, are depicted in Table 6. From Table 6, it is evident that the F_{table} for $F(0{,}05; 1; 31) = 4.16 <$ 27.409, which is less than 27.409, and sign.(0.00) < 0.05. This implies a linear relationship between the students' prerequisite skill variable and the students' critical thinking skill variable when solving mathematical induction in PWCI.

Table 6. Results of the Linearity Test (Source: SPSS 24.0 Output)

			Sum of Squares	df	Mean Square	F	Sig.
Critical Thinking Ability * Prior Knowledge Skills Students	Between Groups	(Combined)	21291.803	36	591.439	2.684	.003
		Linearity	6040.720	1	6040.720	27.409	.000
		Deviation from Linearity	15251.082	35	435.745	1.977	.029
	Within Groups		6832.176	31	220.393		
	Total		28123.979	67			

Based on the results of the simple linear regression analysis assisted by SPSS 24.00, as shown in Table 7, we obtained values from the unstandardized coefficient – B column, which is $a = 31.037$ and $b = 0.474$. The Regression Model derived is $Y = 31.037 + 0.474X$, where Y represents the dependent variable of students' critical thinking abilities, and X represents the independent variable of students' prior knowledge skills. In other words, for each unit increase in prerequisite skill scores, there is an increase of 0.474 in critical thinking skill scores from the baseline of 31.037.

Table 7. Results of Simple Regression Analysis and t-Test (Source: SPSS 24.00 Output)

		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95,0% Confidence Interval for B	
Model		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	31.037	7.758		4.001	.000	15.549	46.526
	Prior Knowledge Skills Students	.474	.112	.463	4.249	.000	.251	.696

a. Dependent Variable: Critical Thinking Ability

Next, a t-test can be conducted to test the hypothesis. Based on Table 7, it is found that Sign. (0.000) < 0.05 (α). Furthermore, by comparing the result $t_{score} = 4{,}249$ with $t_{table} = 1{,}9966$ on $df = 66$ and $\alpha = 0{,}05$. It is obtained that $t_{table} \leq t_{score}$. It is concluded that H_0 is

rejected, and it is concluded that students' prerequisite skills have a significant influence on their critical thinking abilities when solving mathematical induction in PWCI. This is supported by the result of the F-test in Table 8, which shows that Sign.(0,000)$< 0{,}05$ (α). The result of $F_{score} = 18{,}054$ is compared to $F_{table} = 3{,}986$ on $df_1 = 1$, $df_2 = 66$, and $\alpha = 0{,}05$, then $F_{score} > F_{tabel}$, meaning H_0 is rejected. Therefore, students' prior knowledge skills significantly influence their critical thinking abilities when solving mathematical induction in PWCI.

Table 8. Results of F-test (Source: SPSS 24.00 Output)

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	6040.720	1	6040.720	18.054	.000[b]
	Residual	22083.258	66	334.595		
	Total	28123.979	67			

a. Dependent Variable: Critical Thinking Ability. b. Predictors: (Constant), Prior Knowledge Skills

In Table 9, a correlation coefficient (R) of 0,463 is obtained. This indicates that the strength of the relationship between the prior knowledge skills of students and their critical thinking abilities falls into the moderate category. The coefficient of determination (R^2) reveals the extent of the contribution of students' prior knowledge skills to their critical thinking abilities. In Table 9, $R^2 = 0{,}215$. Based on this result, it can be interpreted that 21.5% of the variation in critical thinking abilities can be attributed to prerequisite skills, while the remaining 79.5% is influenced by other variables. On the other hand, due to its low magnitude, it suggests that the formed regression line is not a very good fit.

Table 9. Results of Model Summary (Source: SPSS 24.00 Output)

					Change Statistics				
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	R Square Change	F Change	df1	df2	Sig. F Change
1	.463[a]	.215	.203	18.29193	.215	18.054	1	66	.000

a. Predictors: (Constant), Prior Knowledge Skills

Qualitative Data Findings

In this section, the results of critical thinking ability tests and interviews are presented and coded according to critical thinking indicators, which include clarification, evaluation, inference, and strategy, as defined by Perkins & Murphy (2006). A total of six students were chosen as interview subjects, comprising two students from each of the high, medium, and low prior knowledge skill categories. In addition, consideration was given to selecting subjects with good communication and interpersonal skills. Data from the six subjects can be seen in Table 10.

Critical thinking of prospective mathematics teachers on PWCI with high prior knowledge

S1 and S2 demonstrate varying responses, typically due to differences in their high prior knowledge skills. S1 shows an ability to identify contradictory information, while S2 tends to overlook it. This is because S2's response mainly revolves around the information they already possess compared to exploring the pattern or other strategies. These distinctions are reflected in the conclusions drawn by S1 and S2, as illustrated in Figure 5. Figure 6 provides a side-by-side evaluation of their critical thinking abilities.

Table 10. Interview Subject Data

No	Subject	Score Prior Knowledge Test	Score Critical Thinking Test	Subject Category
1.	S1	98.33	100	High
2.	S2	93.33	66.67	High
3.	S3	80	75	Moderate
4.	S4	85	91.67	Moderate
5.	S5	45	58.33	Low
6.	S6	40	66.67	Low

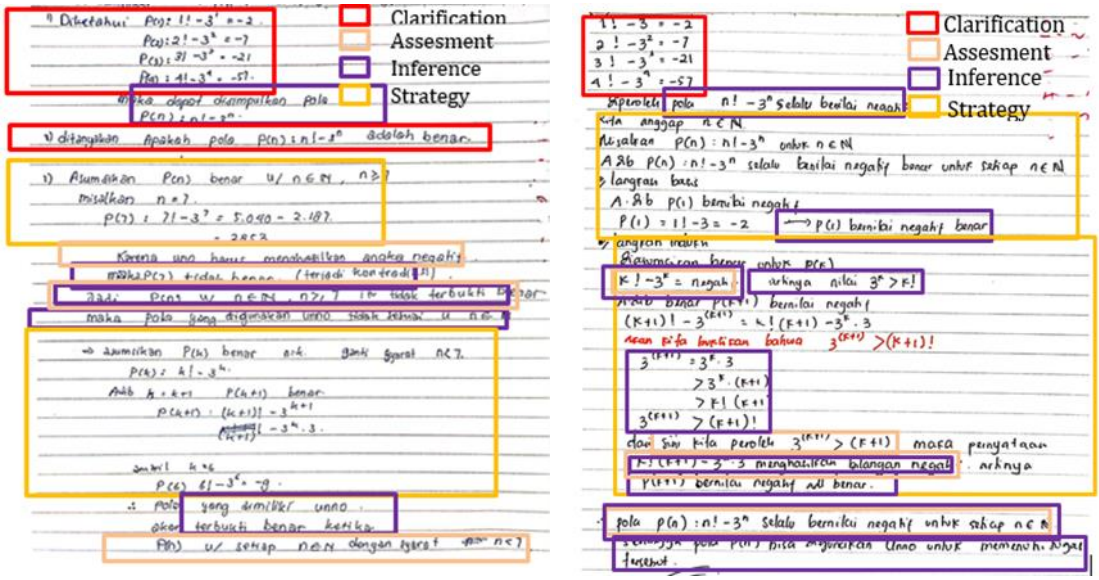

Figure 5. The Responses of Critical Thinking Tests for S1 and S2 Respectively

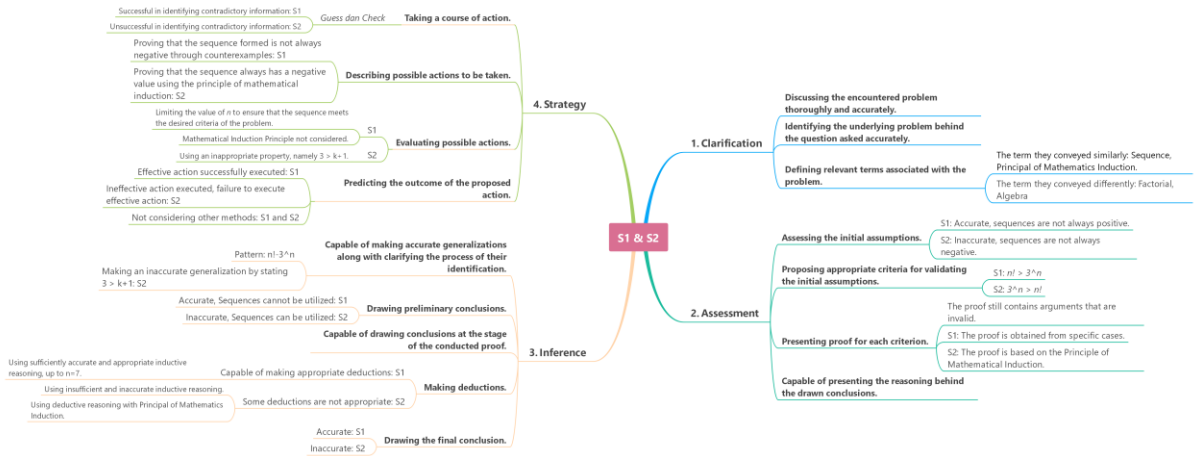

Figure 6. Critical thinking of students on PWCI with high prior knowledge

In the *Clarification* phase, based on the responses of S1, it can be deduced that they possess the capacity to articulate the problem they are confronted with by drawing upon their existing knowledge and the specific queries raised. Conversely, S2's response mainly revolves around the information they already possess. Subsequent confirmation during interviews revealed that both individuals could provide comprehensive explanations. They were presented with a sequence consisting of four terms and tasked with determining whether this sequence

consistently yielded negative results. This insight was acquired by analyzing how S1 and S2 approached the problem, indicating their shared ability to discern the fundamental issues embedded within the presented questions. Additionally, S1 and S2 furnished definitions of pertinent mathematical concepts relevant to the problem, albeit with variations in the concepts they conveyed.

In the *Assessment* phase, both S1 and S2 exhibited their capacity to provide rationales supporting their reached conclusions. The interviews revealed their initial different hypotheses. S1 argued that considering the problem with contradictory information, Unno's pattern does not invariably produce negative values. Conversely, S2 upheld the opposing stance, asserting that the sequence consistently leads to negative values. S1's supposition was grounded in the idea that $n!$ (n factorial) yields factors that are invariably greater than 3^n (3 power n), with these factors exclusively being 3. This inference led S1 to speculate that the generated pattern is not consistently negative. In contrast, S2, based on the provided information, drew an inductive inference that the sequence is consistently negative. Starting from their initial hypotheses, both S1 and S2 formulated criteria to validate their respective suppositions. They subsequently provided evidence for each of these criteria, even though some of the supporting arguments were not entirely sound. Furthermore, S1 and S2 each furnished justifications for their respective conclusions.

During the *Strategy* phase, it became evident from the responses of both S1 and S2 that they could correctly generalise the generated sequence pattern, which is $n! - 3^n$. However, S2 faced challenges in making appropriate deductions during the induction proof steps. This difficulty was compounded by the fact that S2 drew conclusions through inductive reasoning from a limited number of cases, preventing the formulation of a correct final conclusion. Notably, in the context of the mathematical induction stage, some of the conclusions reached were accurate, but there were instances of incorrect generalizations, such as the case $3 > k + 1$, which resulted in a contradiction and an inaccurate conclusion. In contrast, S1's approach to reaching conclusions began with inductive reasoning grounded in a more comprehensive set of cases. S1 also employed deductive reasoning to deduce the relationship between $n!$ and 3^n based on an established definition, leading to the belief that $n! > 3^n$. It should be noted, however, that a formal proof using mathematical induction was not executed by S1. Additionally, S1 did not explore alternative methods for finding the solution. The fundamental difference between S1 and S2 is that S1 recognizes the contradictions that occur, while S2 does not. This influences their journey in critical thinking.

Critical thinking of prospective mathematics teachers on PWCI with moderate prior knowledge

S3 and S4 possess moderate levels of prior knowledge. In general, both of them provide different answers. S3 cannot recognize the contradictory information provided, while S4 can identify it. This is indicated by the conclusions given by S3 and S4 in the subject's answers as shown in Figure 7. A comparison of the critical thinking skills of S3 and S4 is illustrated in Figure 8.

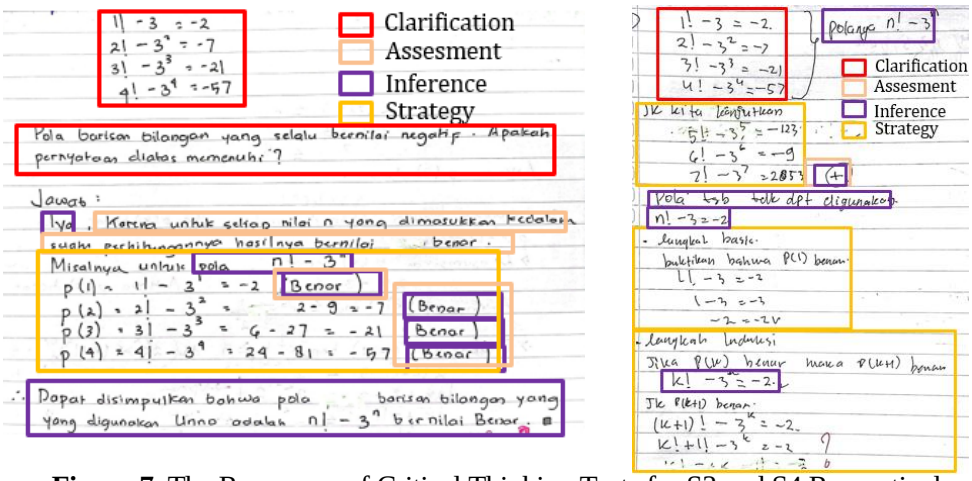

Figure 7. The Responses of Critical Thinking Tests for S3 and S4 Respectively

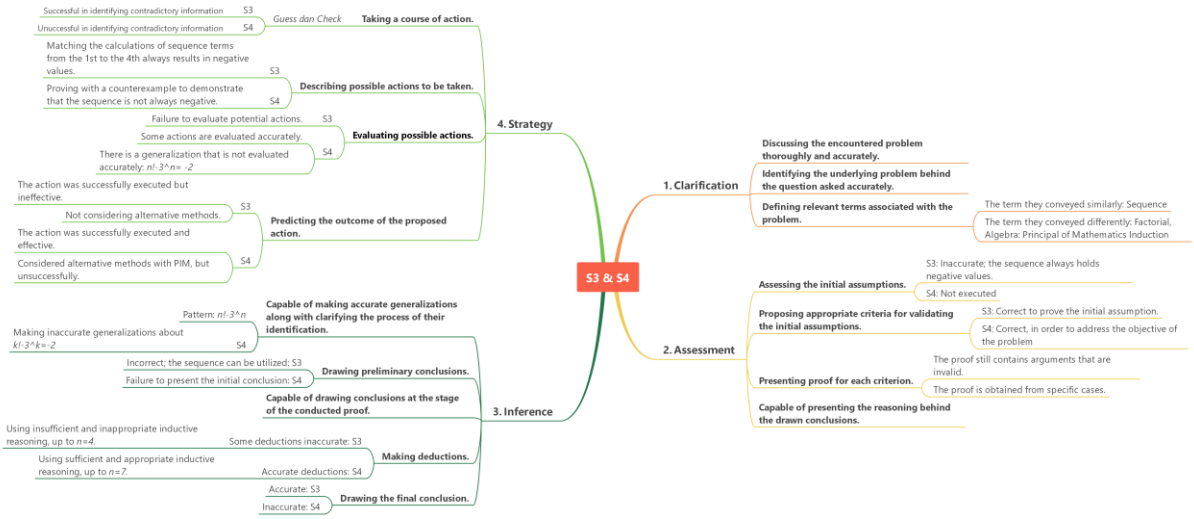

Figure 8. Critical thinking of students on PWCI with moderate prior knowledge

S3's responses reveal the ability to record information in the *Clarification* stage, albeit not in a comprehensive manner. Furthermore, S3 can document the results of their identification regarding the problem being addressed. In contrast, S4 only transcribes the information presented in the question but fails to specify the nature of the problem at hand. The interview results indicate that both are capable of articulating information comprehensively. They were given a sequence of four terms and tasked with verifying whether the sequence was consistently negative. This insight is derived from S3 and S4's analyses of the problem, indicating their capacity to identify the underlying issues within the posed question. S3 and S4 define relevant mathematical concepts for the problem, albeit conveying distinct concepts.

Concerning S3's response, it is apparent that S3 can justify their proofs and illustrate how a step-by-step examination can lead to conclusions. A similar observation is found in S4's response, albeit with differing conclusions, and S4 can similarly provide justifications for the evidence they present. These observations were validated during the interviews. Moreover, it was noted that S3 has yet to establish suitable criteria for their assumptions, specifically, that the pattern in question should yield negative values based on the known values of n. Conversely, S4 exhibits proficiency in defining the appropriate criteria. S4 posits that if the sequence yields negative values for several different values of n, it is a viable pattern; however, if it yields

positive values, it cannot be used. Both S3 and S4 provide evidence based on the established criteria. They assess whether the calculated results are negative or positive and subsequently formulate their conclusions.

Based on the responses from S3 and S4, it's evident that they both possess the capability to make accurate generalizations regarding the sequence pattern, which is $n! - 3^n$. They can also elucidate the rationale behind this pattern. Nevertheless, S3 struggles to articulate statements related to the pattern. Conversely, S4 incorporates the pattern into statements that lack relevance, such as $k! - 3^k = -2$, primarily focusing on the initial statement $1! - 3^1 = -2$. These generalisations are formed due to S4's emphasis on the initial statement, a conclusion supported by the interview findings. Additional interview results indicate that S3 initially concludes that the sequence can be applied, whereas S4 refrains from making an initial conclusion. Furthermore, S3 generates conclusions, either positive or negative, based on their calculations, although they fail to derive a final correct conclusion. This shortfall can be attributed to their utilization of inductive reasoning, which relies on an insufficient number of cases for conclusions. In contrast, S4 initiates their conclusion process with inductive reasoning based on a sufficient number of cases, up to $n = 7$. Nevertheless, neither S3 nor S4 can successfully execute a formal proof through mathematical induction.

Critical thinking of prospective mathematics teachers on PWCI with low prior knowledge

Low prior knowledge skills are represented by S5 and S6. In general, both of them provide answers with the same conclusion. They are unable to recognize the contradictory information presented. This is indicated by the conclusions provided by S5 and S6 in the subject's answers, as depicted in Figure 9. A comparison of the critical thinking skills of S5 and S6 can be observed in a chart resembling Figure 10.

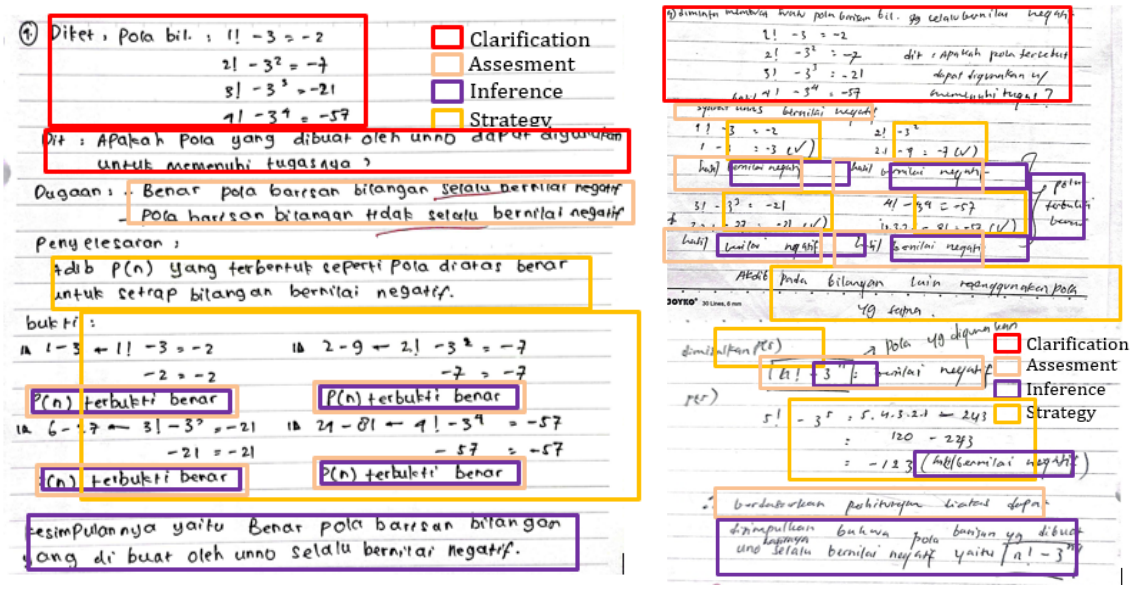

Figure 9. The Responses of Critical Thinking Tests for S5 and S6 Respectively

The responses of S5 and S6 reveal that they can document the information they know and articulate the objectives of the problem. While S5 does not express this clearly, they are capable of interpreting the problem, particularly after an interview, where he was able to scrutinise

whether the sequence created by Unno consistently yields negative values. Confirming the results of the interviews, both S5 and S6 demonstrate their capacity to articulate information comprehensively. They were provided with a sequence of four terms and asked to verify whether the sequence consistently produced negative values. This insight is derived from the analyses conducted by S5 and S6 regarding the problem, signifying their ability to identify the underlying issues within the posed questions. Despite the disparity in.

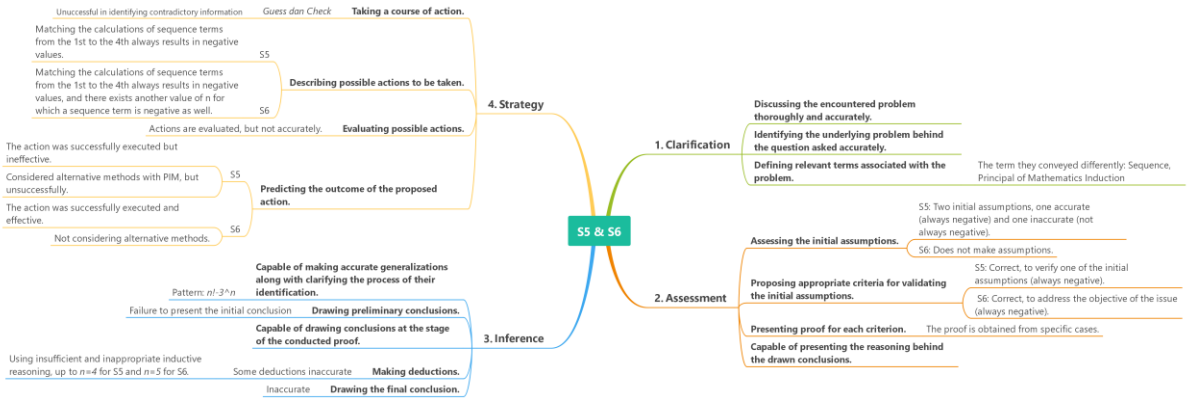

Figure 10. Critical thinking of students on PWCI with low prior knowledge

Referring to the responses from S5 and S6, it is apparent that they possess the capability to provide justifications by meticulously analyzing each case to derive conclusions. S5 even puts forth two potential hypotheses based on the presented problem, which were substantiated by the findings. Moreover, S5 demonstrates an aptitude for establishing precise criteria or prerequisites for their conjectures. Specifically, they require the pattern to consistently yield negative values, irrespective of the value of n, for it to be considered valid. Conversely, if it results in positive values, the sequence is deemed unsuitable. S6 exhibits a similar ability by stipulating appropriate criteria, namely, that the pattern should consistently yield negative values when tested with varying n values. However, both S5 and S6 support their claims through inductive reasoning. S5 attempts to provide a mathematical induction proof but solely addresses the base case. In each case, they meticulously assess the calculation outcomes, determining whether they are negative or positive and concluding accordingly.

S5's response suggests that they drew several conclusions from the calculation results, namely, that $P(n)$ is proven true for n values of $1, 2, 3$, and 4. However, the conclusion was reached through insufficient inductive reasoning, determining that the sequence created by Unno consistently yields negative values through a case-by-case analysis. S6's response similarly reflects this pattern, but they were able to accurately generalize the sequence pattern, $n! - 3^n$, and the interview results confirmed their approach. S6 introduced a conclusion related to $P(5)$ before arriving at a conclusion, which was also established through insufficient inductive reasoning. Interview transcripts supported the fact that both subjects, S5 and S6, did not have initial conclusions before answering the questions. All findings from their responses were validated during the interviews.

In the *Strategy* phase, both S5 and S6 reported utilizing a trial-and-error approach (guess and check). However, they both failed to identify the contradiction within the given information. This failure was attributed to S5's omission of verifying the term values of the

sequence for natural numbers greater than 4, while S6 didn't extend their evaluation of the sequence's term values beyond $n = 5$. An analysis of their responses reveals that both individuals could outline potential steps or methods they might employ.

S5's goal was to establish the negativity of the formed $P(n)$. Their approach involved a less efficient process of matching computed term values for $n = 1$ to $n = 4$, leading them to conclude that the sequence consistently produced negative values. Similarly, S6 sought to demonstrate the negativity of other numbers sharing the same pattern. However, they focused solely on the case when $n = 5$ and assessed the term value for that specific instance.

The interview results revealed that both subjects could anticipate the outcomes of their actions and set strategic objectives. Nevertheless, the strategies used by S5 and S6 were deemed less effective. S5 indicated that their strategy was grounded in the base case of the Principle of Mathematical Induction. They recognized the necessity of an inductive step for concluding but did not explore alternative approaches. Likewise, S6 didn't contemplate alternative methods. They believed that their trial-and-error strategy, supplemented by calculations to ensure negativity, sufficed to address the problem's requirements.

Discussion

The results of the first phase using hypothesis testing, indicate a significant influence of prior knowledge on the critical thinking abilities of prospective mathematics teachers at IAIN Kediri when solving mathematical induction in the PWCI context. This finding aligns with prior research (Razak, 2017). This suggests that prior knowledge plays a crucial role in students' logical thinking abilities (Pamungkas et al., 2017). However, the strength of the relationship between the prior knowledge skills of students and their critical thinking abilities falls into the moderate category, which is 21.5%. The variation in critical thinking abilities can be attributed to 79.5% being influenced by other variables. For instance, logical intelligence, encompassing skills such as pattern recognition, relationship analysis, precise calculations, and logical reasoning, also impacts students' mathematical problem-solving abilities. Both prior knowledge and logical intelligence contribute to various stages of problem-solving (Irawan et al., 2016).

While this study does not establish logical intelligence as a predictor of critical thinking ability, solving problems with contradictory information requires the capacity to analyze patterns or relationships, as demonstrated by S1. This is essential for recognizing that a problem contains contradictions (Mutmainah et al., 2021). Individuals faced with problems featuring contradictory information tend to assume that the problem presents information that is always correct and should be answered based on the available information (Kurniati, 2018).

From the research findings of the second phase, it was found that in each subject, the clarification phase can be successfully met by posing the problem accurately and identifying alternative interpretations of the problem with the contradictory information provided. In this phase, students analyze the information and then restate the problem. Through analysis, individuals distinguish what is relevant to the problem, determine appropriate concepts, and decide how the problem is presented (Rosyadi et al., 2022). The findings align with the notion that in the process of comprehending PWCI, it is necessary to describe or write down the known information and the questions posed in the problem, intending to recognize that the problem contains contradictory information (Mutmainah et al., 2021).

The assessment phase can be fulfilled by evaluating initial conjectures in the form of mathematical symbols. However, a duality of right and wrong appears in high-level subjects. In the case of moderate-level subjects, they assess conjectures that are inappropriate or do not make conjectures. Initial conjectures are used to assess the truth of truth-seeking problems. This stage also marks the beginning of the process of verifying the accuracy of the information in the question before solving the problem (Kurniati et al., 2019). Critical thinking leads individuals to check the accuracy of questions and classify elements in the question before solving it (Ardiansyah et al., 2022). Students establish criteria according to their objectives and provide evidence using their predefined criteria, whether by counterexamples or the Principle of Mathematical Induction, or by providing evidence from specific cases. Making decisions by setting conditions or criteria indicates the need for experience in critical thinking (Rosyadi, 2021). Through evaluation, an individual can select essential information, establish assumptions about the information in the problem, connect essential information based on assumptions, and identify possible strategies to solve the problem (Sutini et al., 2017). Each subject can provide reasons for each conclusion drawn. This aligns with the idea that retesting conclusions are influenced by mastery of the relevant mathematical material or concepts, allowing each decision to be justified (Ismail & Bempah, 2018). The assessment phase is part of the evaluation, aimed at assessing the credibility of statements or other representations and evaluating the logical strength of actual or intended inferential relationships between statements, descriptions, questions, or other forms of representation (ŽivkoviĿ, 2016).

The inference phase can be effectively met by making accurate generalizations and reinforcing the identification process through the pattern of the formed sequence. However, high and moderate-level subjects sometimes make generalizations that are not suitable, for example: *(1)* $3 > k + 1$, *(2) Because* $P(1), P(2), P(3), P(4)$ *is true, then* $P(n)$ *is true for all natural numbers*. Low-level subjects provide incomplete and irrelevant generalizations. High-level andmoderate-level subjects establish initial conclusions but exhibit a duality. They are capable of concluding each stage of proof, making deductions through both inductive and deductive reasoning, often utilizing the Principle of Mathematical Induction. On the other hand, low-level subjects draw deductions mainly through inductive reasoning, and some deductions are inappropriate. Sub-skills of critical thinking, such as deduction and assumption identification, are lacking in low-level thinking skills (Aktaş & Ünlü, 2013). A teaching approach is required to motivate students to be more sceptical about the truth of statements, to be more aware of various ways to view the world, and to be better at deciding what needs to be done or considered in the face of diversity (Dekker, 2020). Furthermore, posing questions explicitly encourages students to discuss problems while distinguishing between findings and conclusions, drawing valid conclusions from data, and identifying and evaluating controls (Cheng & Wan, 2017).

In the strategy phase, individuals employ a “trial and error” approach, leading to the identification of inconsistencies among some high and moderate-level participants but not among those with lower-level proficiency. Nevertheless, low-level participants generally struggle to discern these inconsistencies. High-level participants propose potential actions, relying on counterexamples or the Principle of Mathematical Induction, whereas one moderate-level participant and all low-level participants rely on specific yet insufficient examples. This strategy allows individuals to present their reasoning persuasively and coherently (Facione,

1990; ŽivkoviĽ, 2016). This is closely associated with their capacity to employ information, concepts, principles, or other pertinent elements for effective problem-solving (Marzuki et al., 2022; Mastuti et al., 2022; Setiana, 2018; Yuwono et al., 2019). Some high-level participants exhibit inadequate assessment of their actions, such as using inappropriate properties or applying unsuitable constraints to the problem. Moderate and low-level participants evaluate actions that are unfit for aligning calculations, involving insufficient special cases and misguided generalizations. To accurately represent one's reasoning process, it is imperative to engage in analysis, evaluation, conclusion, or results monitoring (Facione, 1990). Participants make predictions based on the description of their actions but frequently fail to explore alternative methods or solutions. In each proficiency level, one participant refrains from considering alternative solutions, whereas another contemplates utilizing the Principle of Mathematical Induction but struggles with its execution. Consequently, a degree of flexibility in exploring alternative problem-solving methods is crucial for success (Mutmainah et al., 2021).

The research results indicate that the contribution of prior knowledge to critical thinking abilities is 21.5%, signifying that the influence of prior knowledge on critical thinking abilities is not substantial. Sharma and Hannafin (2004) suggest that prior knowledge plays a role in either facilitating or hindering the development of critical thinking skills. In this case, we used PWCI. It contains contradictive information that can deceive students into drawing conclusions. When prerequisite knowledge, such as mathematical induction, is mastered, it does not necessarily mean that students can recognize the information. As a result, they may fail to engage in critical thinking, including drawing conclusions, and may be hindered from considering other more effective strategies. Prerequisite knowledge can be a double-edged sword for critical thinking. It is recognized that prior knowledge is a necessary but not sufficient condition for critical thinking (Ennis, 1989; Sharma & Hannafin, 2004). Consistent with Sumarna and colleagues' findings, there is no apparent interaction between the impact of educational factors and prior mathematical knowledge on the enhancement of critical thinking skills (Sumarna et al., 2017). The effect of prior knowledge on critical thinking is closely linked to the distinction between knowledge and pre-existing biases. Occasionally, prior knowledge can give rise to counterproductive strategies (Sharma & Hannafin, 2004).

The limitation of this study lies in the quantitative data, which is linearly constrained only to the aspect of Linearity, but not to the Deviation from Linearity. Another limitation in the elaboration of critical thinking stages is the continued use of the four stages: clarification, assessment, inference, and strategy. However, in PWCI, truth-seeking behavior is required, ideally falling between the clarification and assessment stages of critical thinking. This presents further opportunities for advanced research.

This research contributes in several ways. Reinforcing prerequisite knowledge for students is still necessary as a foundation to support individuals through the critical thinking stages. An infusion approach is required, involving explicit learning of critical thinking principles or components. Integration is achieved as instructors implement an infusion learning model that incorporates mathematical problems containing contradictory information or other problems that can serve as predictors of critical thinking, such as investigative and truth-seeking problems. Furthermore, instructors should regularly develop questions that require students to practice verifying the accuracy of all the information in the questions. Learning is designed to

focus on questions that acquaint students with truth-seeking behaviors so that they develop a disposition for critical thinking. Instructors should also engage with students in the process of checking the information in the problems. This will result in habituation for students to behave critically and develop their critical thinking processes.

CONCLUSIONS

The research results and discussions lead to the conclusion that prior knowledge significantly influences the critical thinking abilities of prospective mathematics teachers at IAIN Kediri when solving problems involving mathematical induction with contradictory information. This is supported by the significance value (0,000)$<$ 0,05 (α) and the result of $F_{score} = 18.054$ compared to$F_{table} = 3.986$ on $df_1 = 1$, $df_2 = 66$, and $\alpha = 0.05$. As $F_{score} > F_{table}$, it indicates the rejection of the null hypothesis. The strength of the relationship between the prior knowledge skills of students and their critical thinking abilities falls into the moderate category which is 21.5%.

The clarification phase at each level of students' prior knowledge can be successfully met by posing the problem accurately and identifying alternative interpretations of the problem with the contradictory information provided. The assessment phase can be fulfilled by evaluating initial conjectures in the form of mathematical symbols, but a duality of right and wrong emerges among high prior knowledge students. Students with moderate prior knowledge evaluate incorrect conjectures or do not make conjectures at all. They set criteria in line with the objective and provide evidence according to those criteria. The inference phase can be met by making accurate generalizations and reaffirming the identification process, typically in the form of the sequence pattern formed. However, there are inappropriate generalizations among students with high and moderate prior knowledge. Students with low prior knowledge offer incomplete and irrelevant generalizations. The strategy phase involves taking actions, often in the form of "guess and check." This results in some recognizing contradictions while others do not among high and moderate prior knowledge students. However, students with low prior knowledge fail to recognize the existing contradictions. High prior knowledge students describe possible actions through counterexamples or the Principle of Mathematical Induction. Moderate and low prior knowledge students use specific examples that are insufficient. Some actions taken by high prior knowledge students are evaluated inaccurately, such as the use of inappropriate properties or constraints that do not match the problem.

For future research, further exploration can be conducted to identify where truth-seeking behavior emerges within the stages of critical thinking, when solving PWCI. Additionally, opportunities to examine dispositional critical thinking or stages of critical thinking from other theories can still be pursued to enrich grounded theories related to students' critical thinking when solving PWCI problems. Besides prerequisite abilities, many different factors are suspected to have a strong relationship with critical thinking, such as logical reasoning, decision-making, and agility in strategizing.

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