



Exploring algebraic change and relationship: A qualitative study for aspiring mathematics teachers

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Abstract

Background: This study stems from the necessity to explore the proficiency in algebraic change and relationship among sixth-semester pre-service mathematics teachers, considering they have comprehensively encountered most mathematics content subjects.

Aim: The primary aim is to delineate the ability of change and relationship in algebraic material and identify common errors, thereby contributing insights to enhance the overall quality of mathematics education.

Method: Employing a qualitative descriptive approach, this research involved eight students selected based on their responses to instrument questions. The instrument utilized high-level essay questions, validated with a PISA orientation and developed in alignment with the assimilation of new knowledge specific to local students. Data were analyzed descriptively using Miles and Huberman's model.

Result: Findings revealed that when confronted with PISA-level questions equivalent to junior high school level, participants exhibited difficulties, making errors in calculations, concepts, information accuracy, and in connecting or converting the given information.

Conclusion: The study underscores the imperative for holistic guidance on high-level questions like PISA for aspiring mathematics teachers, serving as a catalyst for educational innovation, particularly in mathematics education.

INTRODUCTION

Grasping the ability of change and relationship within algebra is a fundamental cornerstone for prospective mathematics teachers. Algebra, being a branch of mathematics, plays a pivotal role in shaping conceptual understanding and problem-solving skills (Genareo et al., 2019; Gravemeijer et al., 2016). Referring to the latest Program for International Student Assessment (PISA) results, Indonesia's ranking remains below the average of OECD countries in terms of mathematical abilities (OECD, 2022). This indicates an urgent need to improve the quality of math instruction in Indonesia, particularly in algebra, which is a key component of the PISA assessment. Mastery of change and relationship in algebra is crucial in aiding the development of effective curriculum and teaching methods (Rustika & Rohaeti, 2020; Sáez-Benito & Arnal-Bailera, 2022). Ability of change refers to an individual's capacity to recognize, analyze, and respond to alterations or variations in a mathematical situation or pattern (Aßmus & Fritzlar, 2022; Subekti et al., 2022; Uzun & Öğretmen, 2021). A deep understanding of this concept allows prospective teachers to design learning strategies that are more adaptive and responsive to student needs. The skill to comprehend and explain the relationship between variables and constants in an expression or equation is a core aspect of algebra (Ayala-Altamirano et al., 2022; Kumru & Özdemir, 2021). According to Kristidhika et al. (2020), mastering this concept can help teachers in developing instructional materials that encourage conceptual understanding and critical thinking skills in students. The ability to integrate an understanding of change and relationship within algebra creates a holistic framework. Recent research by Kim & Bastani

(2017) suggests that these skills are vital for facilitating problem-oriented math learning and the development of logical thinking.

For aspiring math teachers, mastering the ability of change and relationship not only prepares them to deliver subject matter but also helps in formulating innovative and effective teaching strategies (Algani, 2019; Dong-Joong et al., 2019). Additionally, this skill supports the development of student-centered curriculum and assessments. Although crucial, building a strong understanding of the ability of change and relationship presents its own set of challenges. However, with the right approach, prospective teachers can overcome these obstacles and seize opportunities to enhance the quality of math education. Given the significance and existing challenges, this research aims to delve deeper qualitatively into the ability of change and relationship among aspiring math teachers. The findings from this study are expected to make a meaningful contribution to the field of math education and the development of competent and innovative future teachers.

Several qualitative studies in the field of Math Education, such as those conducted by Alexander & Hermann (2016), Büscher (2020), Lipovec et al. (2015) and Planas (2019) have provided valuable insights. However, there has been no research focusing specifically on the ability of change and relationship in algebra and the identification of common mistakes that often occur. Therefore, we intend to investigate these abilities more closely and identify prevalent errors that are frequently encountered. The hope is that the results of this study can offer fresh perspectives in efforts to improve the quality of math education.

METHODS

Design

Providing prospective teacher students with PISA-oriented questions focused on algebra topics and then analyzing them can yield insights into literacy and numeracy skills (Khoirudin et al., 2017; Nissa & Lestari, 2015). This research will scrutinize the answers given to essay questions on the subject of change and relations. These questions, based on PISA-oriented algebra material, will be adapted to fit the local customs and practices of the research area, incorporating high-level thinking questions specifically tailored for aspiring teacher students (Kurniati et al., 2016). The study is a qualitative descriptive research effort aimed at detailing the capabilities of prospective math teacher students in tackling PISA-oriented questions on change and relationship at levels 4, 5, and 6, customized to the circumstances of the research locale.

Participants

This research was conducted during the even semester of the 2022/2023 academic year, spanning from September to December 2022. The study involved prospective mathematics teacher students in their sixth semester at the Faculty of Teacher Training and Education (FKIP) at Tanjungpura University. The participants were aged between 19 and 22 years and had completed courses that equipped them with knowledge in mathematical problem-solving and mathematical literacy. Additionally, the participants selected for the study were students enrolled in a mathematics program that aligned with the topic of the algebra test. The participants for this research were chosen based on the study's objectives to address specific research questions, resulting in a purposive sampling approach. The sample consisted of 9

students, categorically spanning those with complete, fairly complete, and incomplete mastery at levels 4, 5, and 6 for PISA questions on the topic of change and relationship in algebra.

Instrument

The research instrument consisted of six questions, two each from PISA levels 4, 5, and 6, focusing on the topics of change and relationship. These questions were validated by experts in the field of mathematics education. The questions were crafted based on the attributes of PISA-oriented algebra content at levels 4, 5, and 6. Each question featured different content and levels of difficulty, tailored to reflect the local customs and practices of the research area.

Data collection and analysis

The question instrument utilized in this study was developed to be oriented towards local customs and practices in the content areas of change and relationship, and it has been validated by expert validators. The research instrument comprises six questions, two from each of PISA levels 4, 5, and 6, focusing on change and relationship topics. The student responses provide data that describe answers, which will be analyzed in depth. The categorization of student work is based on their answer procedures for each question, and it is divided into three categories: complete, quite complete, and incomplete. The technique used for data collection is documentation via a written test containing essay questions. Data analysis for this study will involve data reduction.

Table 1. Content Change and Relationship Indicators and HOTS Levels in PISA

| Content | PISA levels |
|--|--|
| Change and Relationships | Level 4 |
| Create, interpret, and translate between symbolic and graphic relationship representations | Work effectively with models in concrete but complex situations. They can select and integrate different representations and relate them to real situations. |
| Mathematical content, functions and algebra are central to describing, modeling and interpreting change phenomena. | Level 5 |
| Defining and interpreting changes, perhaps related to changes in area, or relationships between the lengths of sides | Work with models for complex situations, knowing the constraints encountered and making guesses. They can select, compare and evaluate strategies to solve problems related to the model |
| Estimates the distance traveled in various amounts of time when traveling at a given speed. | Level 6 |
| | Carrying out conceptualization and generalization using information based on modeling and research in complex situations. Then you can connect different sources of information flexibly and translate them. |

RESULTS AND DISCUSSION

Based on student responses to the essay questions focusing on the topic of change and relationship in PISA-oriented algebra material, an analysis of the collected data revealed insights into the students' problem-solving abilities. These abilities were then categorized into high, medium, and low levels.

A. Question number 1

The section that follows showcases student solutions that fall into the high category, featuring complete and correct answers. The PISA content featured in question 1 involves mathematical concepts, functions, and core algebraic principles in describing, modeling, and interpreting phenomena of change. It also emphasizes effective work with models in concrete yet complex situations. Students at this level are capable of selecting and integrating different

representations and linking them to real-world situations (content 2, level 4). As per the indicator, students are able to correctly relate the scenarios involving three different bead bracelet arrangements using the concept of Simultaneous Linear Equations (SPLTV).

Handwritten student work for question 1. The student lists three equations for bracelets A, B, and C. They then use elimination to solve for the variables. The final result is that the length of bracelet C is 60 mm.

$$\begin{aligned} \text{gelang A} &: 40 + 3y + 4z = 100 \text{ mm} \\ \text{gelang B} &: 20 + y + 2z = 44 \text{ mm} \\ \text{gelang C} &: 30 + y + 3z = 60 \text{ mm} \end{aligned}$$

Elimination steps shown:

$$\begin{aligned} & 40 + 3y + 4z = 100 \\ & 20 + y + 2z = 44 \quad \times 2 \\ \hline & 40 + 3y + 4z = 100 \\ & 40 + 2y + 4z = 88 \\ \hline & 0 + y = 12 \end{aligned}$$

Substitution steps shown:

$$\begin{aligned} & 20 + 12 + 2z = 44 \\ & 32 + 2z = 44 \\ & 2z = 12 \\ & z = 6 \end{aligned}$$

Final calculation for bracelet C:

$$30 + 12 + 3(6) = 60 \text{ mm}$$

Picture 1. high-level student completion of question no 1

Based on the analysis of high-level student A1's solutions to the questions with the above-mentioned content, it is evident that student A1 is highly proficient in problem-solving. Their answers predominantly employ Simultaneous Linear Equations (SPLTV) to determine the value of each variable. In addition to this, student A1 skillfully manipulated the models and linked the three different bracelet arrangement scenarios, which served as the focal points for questions about change and relationship. Skills in modeling, elimination, and substitution are evident in Figure 1. The precision in their calculations is also noteworthy, as can be observed from their procedural work.

Handwritten student work for question 1. The student lists three equations for bracelets A, B, and C. They then use elimination to solve for the variables. The final result is that the length of bracelet C is 60 mm.

$$\begin{aligned} \text{gelang A} &: 40 + 3y + 4z = 100 \text{ mm} \\ \text{gelang B} &: 20 + y + 2z = 44 \text{ mm} \\ \text{gelang C} &: 30 + y + 3z = 60 \text{ mm} \end{aligned}$$

Elimination steps shown:

$$\begin{aligned} & 40 + 3y + 4z = 100 \\ & 20 + y + 2z = 44 \quad \times 2 \\ \hline & 40 + 3y + 4z = 100 \\ & 40 + 2y + 4z = 88 \\ \hline & 0 + y = 12 \end{aligned}$$

Substitution steps shown:

$$\begin{aligned} & 20 + 12 + 2z = 44 \\ & 32 + 2z = 44 \\ & 2z = 12 \\ & z = 6 \end{aligned}$$

Final calculation for bracelet C:

$$30 + 12 + 3(6) = 60 \text{ mm}$$

Picture 2. Intermediate level student completion of question no 1

Based on the analysis of medium-level student A2's solutions to questions with the above content, it's clear that student A2 is adept at problem-solving. They successfully connect the three real-world scenarios related to bracelet arrangement, which are central to questions on change and relationship. However, there is an error in their execution. Specifically, the mistake occurs in the step where they substitute the value of 'y' into the second equation during the process of solving the Simultaneous Linear Equations (SPLTV). This calculation error triggers a chain reaction, causing subsequent calculations to also be incorrect. Despite this, upon reviewing their thought process and subsequent steps, it appears that student A2 generally has a correct understanding and displays skills in the relevant areas.

Langkah penyelesaian:

- Permisalan; $\square = a$ $\square = b$ $\bigcirc = c$.

sehingga untuk

gelang I, $4a + 4b + 3c = 100 \text{ mm} \dots (1)$

gelang II, $2a + b + 2c = 44 \text{ mm} \dots (2)$

gelang III, $3a + b + 3c = x \text{ mm} \dots (3)$

- mencari nilai panjang ^{segar} manik.

eliminasi

$$\begin{array}{r} 4a + 4b + 3c = 100 \\ 4a + 2b + 4c = 88 \\ \hline 2b - c = 12 \dots (4) \end{array}$$
$$\begin{array}{r} 4a + 4b + 3c = 100 \\ 3a + b + 3c = x \\ \hline a + 3b = 100 - x \dots (5) \end{array}$$
$$\begin{array}{r} 3b - c = 144 - 2x \\ 2b - c = 12 \\ \hline b = 132 - 2x \dots (6) \end{array}$$
$$\begin{array}{r} a - c = 44 - x \\ -296 + 15x - c = 44 - x \\ \hline 896 - 5x - c = 44 - x \\ \hline c = 252 + 4x \\ c = 252 - 4x \dots (7) \end{array}$$
$$\begin{array}{r} 2a + b + 2c = 44 \\ 2(-296 + 15x) + (132 - 2x) + 2(252 - 4x) = 44 \\ \hline 4x = 44 \\ \hline x = 11 \end{array}$$

↳ $2b - c = 12$

$$2b - (252 - 4x) = 12$$
$$2b - 252 + 4x = 12$$
$$2b = 264 - 4x$$
$$b = 132 - 2x$$

↳ $3a + b + 3c = x$

$$3(-296 + 15x) + 132 - 2x + 3(252 - 4x) = x$$
$$-888 + 15x + 132 - 2x + 756 - 12x = x$$
$$-100 + 2x = x$$
$$-100 = x - 2x$$
$$-100 = -x$$
$$x = 100$$

Picture 3. low-level student completion of question no 1

Based on the analysis of low-level student A3's solutions to the questions with the above-mentioned content, it is evident that student A3 struggles with problem-solving. They have not been able to successfully connect the various bracelet arrangements that are central to the questions about change and relationship. Student A3's work shows signs of confusion; they generate a fair amount of mathematical modeling but struggle to link these models effectively. As a result, they are unable to fully and correctly solve the problem at hand.

B. Question Number 2

The section that follows highlights student solutions that are both complete and correct, falling into the high category. The PISA content in question 2 focuses on estimating the distance traveled over varying time periods at a given speed. This involves effective work with models in concrete yet intricate situations. Students in this category can adeptly select and integrate different representations, linking them to real-world scenarios (content 2, level 4). In this particular case, students successfully connect the scenario of bus rental costs to distances traveled, using the concept of Simultaneous Linear Equations (SPL) accurately.

| | Bus A | Bus B | Bus C |
|------------------------------|--|--|---|
| Harga | 2.100.000,- | 1.900.000,- | 2.000.000,- |
| Jarak tempuh (km) | 110 | 105 | 100 |
| Jarak yg belum di cover (km) | $152 - 110 = 42$ | $152 - 105 = 47$ | $152 - 100 = 52$ |
| Pembayaran Biaya | $\frac{42}{45} \times 8.000 = 672.000$ | $\frac{47}{45} \times 35.000 = 1092.000$ | $\frac{52}{42} \times 16.000 = 693.333$ |
| Total Biaya | 2.772.000,- | 2.992.000,- | 2.693.333,- |

Soda Bus yang akan dipilih adalah Bus Perusahaan C dengan total pengeluaran Rp 2.693.333,-

Picture 4. high-level student completion of question no 2

Based on the analysis of high-level student B1's solutions to the questions with the specified content, it's clear that student B1 is adept at problem-solving. They effectively link three real-world scenarios related to choosing a cost-effective bus service for the same distance traveled. Student B1 demonstrates skill in understanding the problem through the lens of

comparison. Moreover, the calculations executed in the procedural steps are quite accurate, aligning closely with the correct answers. Student B1 presents their solutions in a tabular format, indicating a strong ability to effectively use different forms of representation.

Carilah :
 Bus A : 110 km = 2,1 jt
 Setiap penambahan 0,5 km harganya 8rb, 1 km = 16 rb.
 Penambahan Jarak : 152 - 110 km = 42 km
 $42 \times \text{harga per km} = 42 \times 16 \text{ rb} = 672 \text{ rb}$
 Jadi harga untuk pengewaan bus perusahaan A yaitu $2.100.000 + 672.000 = 2.772.000$

Bus : B = 105 km = 1,7 jt
 Setiap penambahan 0,5 km harga ditambah 8rb
 penambahan Jarak : 152 - 105 km = 47 km
 $47 \times \text{harga per km} = 47 \times 11.67 = 548,49$
 Jadi harga untuk pengewaan bus perusahaan B yaitu $1.700.000 + 548.490 = 2.248.490$

Bus : C = 100 km = 2 jt
 Setiap penambahan 0,5 km harganya 16 rb
 Penambahan Jarak : 152 - 100 = 52 km
 $52 \times \text{harga per km} = 52 \times 2.67 = 138,84$
 Jadi harga untuk pengewaan bus perusahaan C yaitu $2.000.000 + 138.840 = 2.138.840$
 Jadi harga bus dari perusahaan C yang termurah dan bus digewakan.

Picture 5. completion of mid-level students question no 2

Based on the analysis of medium-level student B2's solutions for the questions with the specified content, it's evident that their final answer is correct. However, some calculation errors were made along the way. Specifically, in determining the cost for bus 2, the student incorrectly calculated the additional price per kilometer. It should have been Rp. 23,333, but the student wrote down the price for a 0.5-kilometer distance as 11.67. Although the final conclusion—that bus B is more expensive than bus C—was accurate, the intermediate calculation was incorrect. A similar mistake occurred in the calculations for bus C. The additional price per kilometer should have been Rp. 13,333, but the student wrote down 2.67, which appears to be the price for 2 liters, and didn't specify it in rupiah. Both of these errors seem to originate from difficulties with equivalent comparisons involving decimal numbers. Despite these errors, the overall conceptual understanding appears clear, except for some inconsistencies in the use of symbols and mistakes in comparative calculations.

Bus A apabila untuk jarak 152 km
 maka biayanya adalah
 $2,1 \text{ juta} + 42 \times 16 \text{ ribu} = \text{Rp} 2.100.000 + \text{Rp} 672.000$
 $= \text{Rp} 2.772.000$

Bus B apabila untuk jarak 152 km
 maka biayanya adalah
 $1,7 \text{ juta} + 47 \times 11,67 \text{ ribu} = \text{Rp} 1.700.000 + \text{Rp} 548.490$
 $= \text{Rp} 2.248.490$
 Bulatkan aja

Bus C apabila untuk jarak 152 km
 maka biayanya adalah
 $2 \text{ juta} + 52 \times 2,67 \text{ ribu} = \text{Rp} 2.000.000 + \text{Rp} 138.840$
 $= \text{Rp} 2.138.840$
 Bulatkan aja

maka pilihlah bus A karena paling murah :)

Picture 6. Completion of low-level students problem number 2

Based on the analysis of the solutions provided by low-level student B3, it's evident that they show some level of skill in solving the problem by connecting real-world scenarios about choosing an economically viable bus for the same distance. However, several calculation errors were made, affecting the accuracy of the answers. Specifically, for bus A, the student made an error in multiplying the additional distance by the additional cost per kilometer. For buses B and C, the student used approximations for the additional cost per kilometer, and these approximations were substantially rounded. For example, the actual additional cost per kilometer for bus B should be Rp. 23,333, but the student rounded it to 24,000, which can have a significant impact on the overall calculation.

Similarly, for bus C, the student made two types of errors: first, rounding the additional cost per kilometer too aggressively, and second, making a multiplication error similar to the one made for bus A. These errors indicate that while the student has a basic grasp of the problem's real-world context and the necessity to make comparisons between different options, they struggle with the mathematical rigor needed to accurately solve the problem. This highlights areas where further instruction and practice may be beneficial.

C. Question Number 3

The text provided outlines exemplary student responses in the high-performing category. Specifically, the third question in the PISA exam deals with calculating distances over varying time intervals at specific speeds, understanding complex scenarios through models, recognizing limitations, and formulating educated guesses. Students excel at this by choosing, assessing, and applying problem-solving strategies related to these models. In addition, students effectively link the three seafood package scenarios by accurately applying the SPLTV (Simultaneous Linear Equations in Two Variables) concept to the given statements.

Model matematika:

Paket A: $3x + y + 2z = 615.000$
 Paket B: $2x + 2y + 3z = 710.000$
 Paket C: $3x + 2y + 3z = 820.000$

• Eliminasi C dan B

$$\begin{array}{r} 3x + 2y + 3z = 820.000 \\ 2x + 2y + 3z = 710.000 \\ \hline x = 110.000 \end{array}$$

• Eliminasi C dan A

$$\begin{array}{r} 3x + 2y + 3z = 820.000 \\ 3x + y + 2z = 615.000 \\ \hline y + z = 205.000 \end{array}$$

• Substitusi nilai $x = 110.000$ dan $y + z = 205.000$ ke persamaan paket A.

$$\begin{array}{r} 3x + y + 2z = 615.000 \\ 3(110.000) + y + 2z = 615.000 \\ 330.000 + y + 2z = 615.000 \\ y + 2z = 285.000 \end{array}$$

• Maka, substitusikan $z = 80.000$ ke persamaan $y + z = 205.000$

$$\begin{array}{r} y + z = 205.000 \\ y + 80.000 = 205.000 \\ y = 125.000 \end{array}$$

• Persamaan $3x + y = 410.000$

Jawab: $3(110.000) + 125.000 = 455.000$
 Maka, persamaannya di atas SALAH

• Persamaan $2x + 2z = 470.000$

Jawab: $2(110.000) + 2(80.000) = 380.000$
 Maka, persamaannya di atas SALAH

• Persamaan $2y + 3z = 490.000$

Jawab: $2(125.000) + 3(80.000) = 490.000$
 Maka, persamaannya di atas BENAR

Picture 7 high level student completion number 3

Based on the displayed performance of C1 high-level students, they exhibit proficiency in problem-solving tasks that align with the course material. These students adeptly analyze multiple pieces of information presented in different formats. Specifically, C1 students demonstrate a strong capability to employ the SPLTV (Simultaneous Linear Equations in Two Variables) concept in accurately assessing the given statements. This is evident from their skill

in correlating information regarding seafood pricing and package options, as well as their precision in calculations, ensuring that all steps and answers are correct.

Pernyataan a) 3kg udang + 1kg kepiting = 410.000.
 Bukti kan:

$$3\text{kg udang} + 1\text{kg kepiting} = 3(120.000) + 130.000$$

$$= 360.000 + 130.000$$

$$= 490.000 \text{ (Salah)}$$

Pernyataan b) 2kg udang + 2kg kakap = 470.000
 Bukti kan:

$$2\text{kg udang} + 2\text{kg kakap} = 2(120.000) + 2(85.000)$$

$$= 240.000 + 170.000$$

$$= 410.000 \text{ (Salah)}$$

Pernyataan c) 2kg kepiting + 3kg kakap = 490.000.
 Bukti kan:

$$2\text{kg kepiting} + 3\text{kg kakap} = 2(130.000) + 3(85.000)$$

$$= 260.000 + 255.000$$

$$= 515.000 \text{ (Salah)}$$

Picture 8 completion of mid-level students question number 3

Based on the observed performance of intermediate level C2 students, they appear to struggle with tasks that require focused attention on relevant details. Specifically, instead of using SPLTV (Simultaneous Linear Equations in Two Variables) for calculations and evaluating the presented statements, these students prematurely rely on the standard prices listed, neglecting the key information about package pricing. This oversight suggests a lack of attentiveness to the tabular data related to seafood package prices, leading to errors in their responses. However, it's worth noting that their procedural steps were executed systematically and their calculations were mathematically sound; the issue lay primarily in their incorrect focus on nominal prices rather than package costs.

paket a) salah karena seharusnya 490.000
 paket b) salah karena seharusnya 410.000
 paket c) salah karena seharusnya 515.000

Picture 9. completion of low level student number 3

Based on the performance displayed by C3 low-level students, their problem-solving attempts are notably incorrect. This is evident from the absence of a structured problem-solving procedure; instead, these students hastily confirm each statement using incorrect nominal prices. A likely contributing factor to these errors is a deficiency in literacy skills, particularly in understanding problem statements and mathematical representations. This suggests that the students' inadequate responses may stem from a fundamental misunderstanding of the questions and the mathematical concepts involved.

D. Question Number 4

The provided text highlights high-quality student responses, focusing on specific areas outlined in the PISA framework. In question 3, students concentrate on calculating the distance covered over different time periods when given a set speed. They also deal with modeling complex

situations, identifying the limitations, and formulating educated guesses. Furthermore, they can choose, assess, and employ various strategies to tackle problems related to these models. This aligns with Content 2 at Level 5 of the PISA scale. In practical applications, students successfully link scenarios involving three seafood packages by making accurate assumptions based on the statements provided, using the concept of SPLTV effectively.

Handwritten student work for problem number 4. The student lists four statements and evaluates them:

- a)** Nilai x , y , dan z berdasarkan nomor 3. dengan $x = 110.000$, $y = 125.000$, $z = 80.000$ (harga paket paket). Dit: Benar atau Salah.
 Harga 1kg udang wangi pada paket lebih murah Rp 500,- dari harga normal.
 Harga 1kg kepiting pada paket lebih murah Rp 5.000,- dari harga normalnya.
 Harga 1kg ikan kakap putih pada paket lebih murah Rp 5.000,- dari harga normalnya.
 Perbedaan harga antara 1kg udang wangi dan 1kg ikan kakap putih pada paket adalah Rp 10.000,-.
 Men cari tahu selisih asli harga paket dan harga normal.
 Membandingkan selisih harga yang telah dihitung dengan selisih harga pada pernyataan.
 a) Selisih 1kg udang wangi = $120.000 - 110.000 = 10.000$.
 Maka, pernyataan a) SALAH.
- b)** Selisih 1kg kepiting = $130.000 - 125.000 = 5.000$.
 Maka, pernyataan b) BENAR.
- c)** Selisih 1kg ikan kakap putih = $85.000 - 80.000 = 5.000$.
 Maka, pernyataan c) BENAR.
- d)** Harga 1kg udang wangi pada paket = 110.000 .
 Harga 1kg ikan kakap putih pada paket = 80.000 .
 Selisih harganya = 30.000 .
 Maka, pernyataan d) SALAH.

Picture 10. high-level student completion of problem number 4

The image indicates that D1-level students have excelled in solving the problems, especially given that question 4 builds on the previous, question 3. Since they've already answered question 3 correctly, the assessment for question 4 can proceed without delay. Among the four statements given, the D1 students responded accurately. They provided counterexamples to disprove incorrect statements and furnished proofs to validate the correct ones.

Handwritten student work for question number 4, showing a system of linear equations in three variables (SPLTV) and its solution:

$$\begin{aligned} \text{dik: } A &: 3x + y + 2z = 615 \\ B &: 2x + 2y + 3z = 710 \\ C &: 3x + 2y + 3z = 820 \end{aligned}$$

$$\begin{aligned} 3x + 2y + 3z &= 820 \\ 2x + 2y + 3z &= 710 \\ \hline x &= 110 \end{aligned}$$

$$\begin{aligned} 2x + 2y + 3z &= 710 \\ 2(110) + 2y + 3z &= 710 \\ 220 + 2y + 3z &= 710 \\ -220 & \\ 2y + 3z &= 490 \end{aligned}$$

$$\begin{aligned} 3x + y + 2z &= 615 \\ 3(110) + y + 2z &= 615 \\ 330 + y + 2z &= 615 \\ -330 & \\ y + 2z &= 285 \end{aligned}$$

$$\begin{aligned} y + 2z &= 285 \\ 2y + 3z &= 490 \\ \hline -y - z &= -205 \\ y + z &= 205 \end{aligned}$$

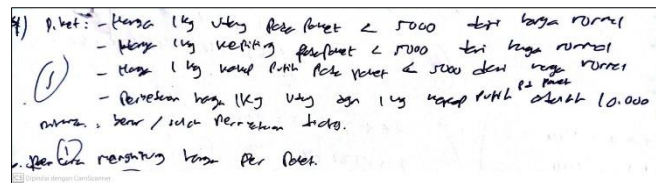
$$\begin{aligned} y + z &= 205 \\ y + 2z &= 285 \\ \hline -z &= -80 \\ z &= 80 \end{aligned}$$

$$\begin{aligned} y + z &= 205 \\ y + 80 &= 205 \\ y &= 125 \end{aligned}$$

Penyelesaian:
 a) Pernyataan salah karena harga paket lebih murah 10.000 dari harga normal.
 b) Pernyataan benar.
 c) Pernyataan benar.
 d) Pernyataan salah. untuk harga normal 205.000 sedangkan untuk harga paket 190.000 sehingga berbeda 15.000 dari harga paket.

Picture 11. Completion of intermediate level students question number 4

Based on the visual evidence, D1-level students at the intermediate stage correctly concluded the questions but lacked detailed explanations. They did not provide counterexamples to challenge incorrect statements nor did they clearly show the calculation proofs for the correct answers. Additionally, the SPLTV mixed-method calculation, which should have been the solution approach for the preceding question, was not presented. This further supports the hypothesis stated in the answer description for question 3 for C2 intermediate-level students: it suggests that they weren't thorough in gathering and processing all the relevant information for the problem at hand.

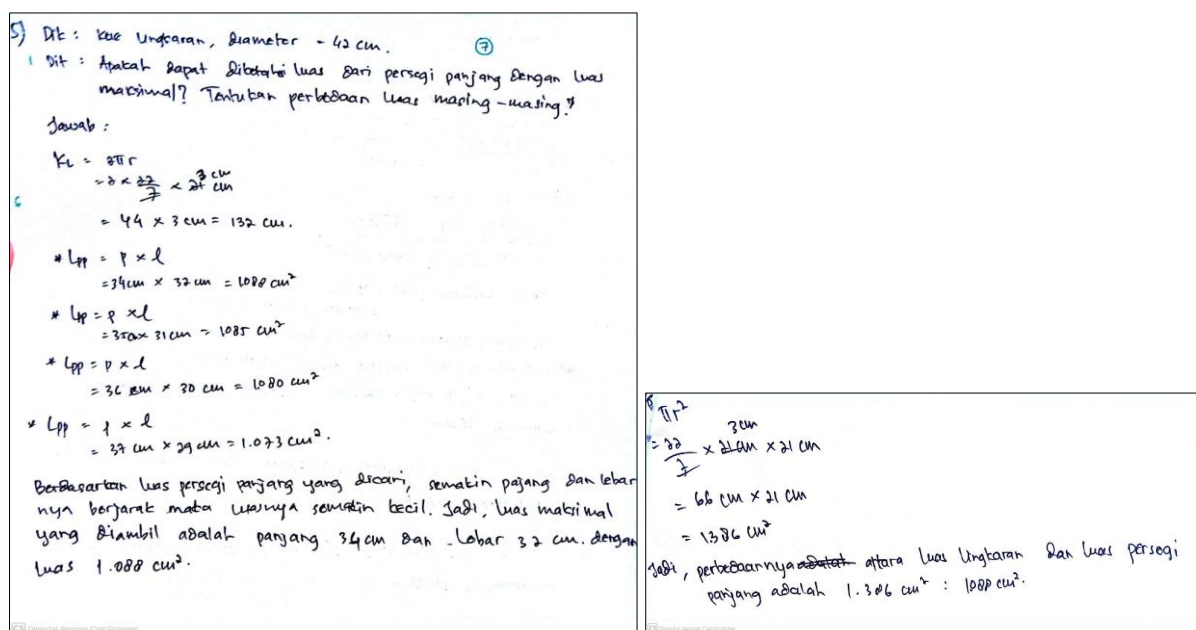


Picture 12. low-level student completion of problem number 4

Based on the visual data for D1 students at a low skill level, the only discernible work involves listing known information and identifying the questions asked. Given that question 4 is a continuation of question 3 and covers the same subject matter, it's understandable that these students would struggle with the problem. This suggests that the D1 students at this level have difficulty comprehending the information and representations in both questions 3 and 4, leading to their suboptimal performance.

E. Question Number 5

The text describes high-quality student responses that align with the PISA framework. Specifically, question 3 delves into understanding and interpreting changes, which may be related to alterations in area or the relationships between the lengths of sides. Students are able to conceptualize and generalize information, applying modeling and analysis to complex situations. They are also skilled at synthesizing information from a variety of flexible sources. This falls under Content 3 and Level 6 on the PISA scale. When presented with real-world problems involving the surface and circumference of a cake, the students successfully apply the principles of area and circumference to transform flat shapes while maintaining the same perimeter.



Picture 13. Completion of high-level students problem number 5

The visual evidence concerning the high-level E1 students indicates a coherent thought process leading to correct answers. The E1 students' approach starts with calculating the circumference of the circle. They then proceed to list all possible circumferences for a flat

rectangular shape in order to compute the maximum area. While they compare sizes of various rectangles, they initially do not find the rectangle with the maximum area. Once they identify this rectangle, they go on to calculate the area of the circle. To address the question about the difference in areas between the two shapes, they conduct a comparative analysis of both areas. This performance suggests that E1 students are highly proficient in interpreting changes, possibly related to shifts in area or the relationships between side lengths.

Dik: diameter $O \cdot d = 42 \text{ cm}$
 Dit: $|L_O - L_{\square}| = ?$
 b) Rengama penyelesaian: 1
 Mencari keliling lingkaran, luas lingkaran dan luas persegi panjang
 c) $K.O = K_{\square} = \pi d$
 $= \frac{22}{7} \times 42$
 $= 132 \text{ cm}$
 $2 \cdot (p+l) = 132 \text{ cm}$
 $p+l = 66 \text{ cm}$
 Persegi panjang akan memiliki luas maksimal jika $p=l$
 $p+l = 66 \text{ cm}$
 $2l = 66 \text{ cm}$
 $l = 33 \text{ cm}$
 $L_{\square} = l^2 = 33^2$
 $= 1089 \text{ cm}^2$
 $L_O = \pi r^2$
 $= \frac{22}{7} \times 21 \times 21$
 $= 1386$
 Maka: $|L_O - L_{\square}| = |1386 - 1089|$
 $= 297$

Picture 14. completion of mid-level students question number 5

The visual representation for intermediate-level E2 students indicates a mostly logical thought process, leading to answers that are nearly accurate. However, a lapse in attention regarding the specified flat shape, which should be a rectangle, leads to an incorrect determination of the maximum area, which they mistakenly identify as a square. While E2's work procedure appears systematic and generally correct, this oversight in recognizing the intended shape causes subsequent errors, affecting the final answer.

$Kel = \frac{22}{7} \times 42 \text{ cm} = 132 \text{ cm}$
 maka persegnya adalah

 $L = 88 \text{ cm} \times 44 \text{ cm}$
 $= 3872 \text{ cm}^2$
 (luas) lingkaran $= \pi \cdot r^2 = \frac{22}{7} \times 21^2$
 $= 66 \times 21$
 $= 1386 \text{ cm}^2$

Picture 15 low-level student completion of problem number 5

Based on the visual evidence for the low-level E3 students, there's a mostly logical thought process that leads to nearly accurate answers. Similar to E2 students, E3 students also make an error, but their mistake lies in the concept of circumference. This can be observed in the students' visual representation, where they incorrectly equate the complete circumference of a circle with the sum of just one side's width and one side's length. Such misunderstandings in the concept of circumference are the main source of error for E3 students. However, when looking at their overall procedural approach, a logical flow of thinking is evident, despite the

conceptual mistake midway through their work. Additionally, they do not address the difference in the areas of the two shapes as posed in the problem.

F. Question Number 6

The text provided showcases student answers that fall under the high-quality category, covering topics from the PISA framework. Specifically, question 3 focuses on key mathematical concepts such as functions and algebra, which are applied to understand, model, and interpret changes in various scenarios. The students are able to conceptually generalize and analyze information for complex situations. Additionally, they are adept at linking information from diverse and adaptable sources. In terms of skill level, this corresponds to Content 2, Level 6. The students successfully demonstrate the ability to apply social arithmetic in algebraic terms to make sense of real-world issues like reducing electricity costs.

Dit: Beban biaya listrik = Rp120.000.000,00/bulan
 Beban pembayaran = 35 %
 Biaya keseluruhan pemasangan = Rp52.000.000,00
 Dit: Berapa bulan penggunaan PLTS dilatukan oleh pihak rumah
 sabit untuk menutupi biaya pemasangan PLTS?
 Jawab:
 * biaya hemat = 35 % x Rp120.000.000,00

$$= \frac{35}{100} \times Rp120.000.000,00$$

$$= Rp42.000.000,00$$

 * ~~Berapa~~ Bulan biaya penutupi pemasangan PLTS = $\frac{\text{Biaya keseluruhan}}{\text{Biaya hemat}}$

$$= \frac{Rp52.000.000,00}{Rp13.000.000,00}$$

$$= 4 \text{ bulan}$$

 Jadi, untuk menutupi biaya pemasangan PLTS yang dilatukan oleh pihak rumah sabit adalah selama 4 bulan.

Picture 16 Completion of all students who answered question number 6

Based on the observations above, students at both the high and medium levels produced answers that were strong from logical, conceptual, and procedural standpoints. In contrast, low-level students did not provide any answers, which could be attributed to either insufficient time for processing or difficulty in tackling the questions. The most plausible explanation leans towards time constraints, as evidenced by the absence of partially completed answers for question number 6; responses were either correct or completely missing. For those who answered correctly, they exhibited an ability to interpret contextual phenomena and flexibly integrate various sources of information. This suggests that they possess the necessary skills to address complex, real-world problems effectively.

However, it's crucial to note that despite these encouraging results, not all aspiring math teachers answered correctly or were classified as high-level performers. This finding aligns with the research by Sumardi & Herawanto (2021), which indicates a generally low capability among students to solve PISA-oriented questions. Therefore, the data underscores the need for targeted educational interventions to elevate the skill sets of prospective math teachers, particularly in areas that PISA questions aim to assess. This is vital for ensuring that future educators are well-equipped to foster a higher level of mathematical understanding among their students.

Implications

This research identified three distinct skill levels among prospective teacher students when tackling PISA-oriented questions. According to the study by Syawahid (2023), high-ability students excel at modeling and interpreting phenomena based on specific changes. These students are adept at connecting models and effectively working with concrete yet complex situations. They are skilled at selecting and integrating different representations, relating them to real-world contexts. Although some inaccuracies in calculations were noted, these were considered minor and did not detract from their overall high performance.

In contrast, students with medium abilities generally possess a good understanding of the concepts involved but are prone to calculation errors that can adversely affect their overall work. Despite having correct conclusions, a closer look at their problem-solving procedures reveals conceptual errors, specifically in the comparative methods used in algebraic calculations. This suggests that medium-ability students struggle with effectiveness in relatively intricate situations. Such inaccuracies can lead to misunderstandings, even if their conceptualization and generalization are logically sound (Aini, 2014; Farida et al., 2021). Low-ability students demonstrated difficulty in establishing relationships between situations and displayed limited skills in model construction. Their attempts to use approximation in contextual situations were largely unsuccessful, often leading to pervasive calculation errors. Overall, this study underscores the need for a targeted educational approach to enhance the competencies of future teachers, particularly in the nuanced areas of problem-solving, modeling, and conceptual understanding, to better prepare them for the global educational stage.

Limitations and Suggestions

This study focuses specifically on the content related to change and relationships within the PISA framework. Researchers suggest conducting analogous analyses for different subject matter also covered in PISA. Beyond that, the primary goal is to assess the proficiencies of future teacher candidates in literacy, numeracy, and PISA-oriented Higher-Order Thinking Skills (HOTS). If the results are found to be below the desired standards, corrective actions should be taken to ensure that the skill levels of prospective educators in Indonesia are competitive on a global scale. This is crucial, as these future math teachers will be responsible for shaping and implementing educational practices in schools, setting the stage for the next generation of learners.

CONCLUSIONS

Based on the analysis of student responses across different levels in the context of PISA-oriented questions focused on change and relationships in a localized setting, it's clear that students continue to face challenges despite the questions being designed for middle school level (SMP/MTS/equivalent). The difficulties encountered by students fall into various categories, such as calculation errors, conceptual misunderstandings, inaccuracies in processing information, and a lack of skill in linking or transforming the given data. Given these observations, one way to drive educational innovation, particularly in the field of mathematics, is to offer comprehensive training on tackling high-level questions like those found in PISA to

aspiring math teachers. This holistic approach aims to improve teaching methods and learning experiences, thereby enhancing overall educational quality.

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