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Ways of thinking senior high school student to solve geometri van hiele problem use reversible thinking ability

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Abstract

Background: Reversible thinking is a cognitive strategy that involves tracing the path from an end result back to the starting point. It is particularly useful in problem-solving.
Aim: This study aims to describe the thought process of high school students in finding solutions to van hiele geometry problems using reversible thinking ability.
Method: A case study approach was employed. The participants were two high school students, and the research tools included written tests and interviews. These instruments were used to delve into the students' written responses.
Result: The findings revealed two key aspects: firstly, the students' van Hiele geometry thinking was predominantly at the deduction stage, evidenced by their ability to model geometric shapes based on their characteristics. Secondly, their reversible thinking in geometry was demonstrated through the simplification of fractional operations to obtain whole parts.
Conclusion: The study highlights the efficacy of reversible thinking in solving geometric problems and provides insights into the cognitive processes of high school students. The ability to reverse engineer solutions from a known outcome back to the starting conditions is a valuable skill in mathematical problem-solving.

INTRODUCTION

Geometry is a discipline of mathematics that is found in the curriculum of developed countries (Acar & Serçe, 2021; Takeuchi & Shinno, 2020; Yang et al., 2017). These countries include geometry into the curriculum because through learning geometry students get a variety of interests. Some of these interests include: (a) develop problem solving skills (Febriana et al., 2020); (b) develop reasoning ability (Sundari et al., 2022); (c) related to problems in everyday life, in case the use of geometry in making batik that has high aesthetic value, using spatial imagery in creating geometry paintings, even integrated with technology to make the coding of a tool (Han et al., 2022; Khalishah & Nalim, 2023; Pérez-Fabello & Campos, 2023).

Geometry is closely related to shapes and spaces. Furthermore, Van Hiele identifies students' ability to solve problems related to shapes and spaces into five cognitive levels which are classified as follows: (1) Level 1 visualization stage, the ability of students to be able to recognize the forms of geomteri simply, not yet to understand its properties; (2) Level 2 analysis stage, the ability of students to describe the geometric forms based on its characteristics. The characteristics of the geometric shapes are obtained based on the results

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of observations, measurements or making models (Anđelković & Malinović-Jovanović, 2022); (3) Level 3 informal deduction or abstraction stage, the ability of students to be able to establish the relationship between a geometric shape with others. At this stage students are able to build classifications between geometric figures and build necessary and sufficient conditions in a figure (Arnal-Bailera & Manero, 2023); (4) Level 4 formal deduction stage, the ability of students to start building definitions, theorems and conclusions to knowledge about a geometric figure; (5) level 5 rigor stage, the ability of students to be able to reason formally in proving definitions, theorems, axioms and or consequences (Cesaria et al., 2021).

At the third and fourth levels of van hiele geometry, students are able to define a concept and integrate it with other concepts to produce a correct conclusion. Based on several studies, students' ability to solve problems at the third and fourth levels is still a minority. This is due to a lack of understanding of geometry concepts and lack of practice in solving geometry problems (Kania et al., 2022), not trying to re-check the answer or check the steps taken from the answer obtained (Ramadhania et al., 2022) and lack of basic mathematical ability to solve geometry problems (Ristanty & Pratama, 2022).

There are five basic mathematical abilities that must be improved by students to help construct mathematical problem. The five abilities are problem solving, communication, connection, reasoning, and representation. In the basic ability of problem solving, there is one of the thinking processes that can support students to think logically in two directions of solving, which is the reversible thinking ability (Pebrianti et al., 2022)

Reversible thinking ability requires students to be able to find solutions by changing the order of thinking logically (Saparwadi et al., 2020; Steffe & Olive, 2009). The mental process in reversible thinking is performed by constructing a strategy that starts from the final value, then thinks backwards until it finds the initial value. Therefore, a complete understanding of a concept is a requirement that supports the success of students in carrying out the reversible thinking process. Hackenberg (2005) defines reversible thinking as an understanding that the combination of each integrated part is a unity that forms a whole. The main idea is a person who engages in reversible thinking can begin with the result of operation and create the starting point. For example, if a student is told "this rectangle is $\frac{3}{5}$ of another rectangle," they can operate to produce a rectangle that is $\frac{5}{5}$. Being able to solve that kind of problem consistently would be evidence of a reversible fraction scheme because the student can start with a fraction of a unit and produce the unit. Based on one of these cases, the ability to think reversibly is one of the basic abilities that can be used to solve problems related to shapes and spaces.

Considering the importance of reversible thinking ability in relation to problem solving ability, both in geometry and algebra, therefore this ability must be owned by students. But in fact, based on research conducted by Maf'ulah & Juniati (2019) stated that students are still lacking in establishing reversible relationships between functions and their graphs. This research is in line with the statement Sangwin & Jones (2017) that when presented reversible thinking problems in algebra with multiple choice types, students are more prepared to do it by matching answers. Based on the research that has been done, it is stated that students' reversible thinking ability is still lacking in algebra topics. However, so far there has been no research that reveals the reversible thinking ability of students on the topic of geometry.

Based on background explanation, the general purpose of this study is to identify the thought process of high school students in finding solutions to van hiele geometry problems using reversible thinking ability. Accordingly, the research questions are as follows: (1) how is the students' process in identifying the characteristics of shapes based on Van Hiele's theory? (2) how is the reversible thinking ability of students in solving van hiele geometry problems?

METHODS

Research Design

This research uses a case study approach with the aim to explore the problem of students' reversible thinking process in finding the solution pattern of shapes or images with the provision of reversible thinking ability. We employed Bungin (2003) as a framework for conducting this research, as follow: (1) Gathering data from student responses to reversible problems and conducting interviews; (2) Streamlining the data to highlight its relevance for the research. (3) Presenting the data, revealing the outcomes of calculations and exploring answer; (4) Concluding phase involves interpreting the collected and analyzed data to generate results that address the research question.

Participant and Data Collection

A total of two high school students (15-18 years old, both male and female) studying in Bandung City, West Java, Indonesia were the subjects of this study. These two students were chosen because they have good ability in mathematical thinking based on the recap of grades during the learning process in the classroom. Furthermore, these two students were coded S1 and S2.

The instruments used to collect research data are tests, interview guidelines and documentation (audio recording). The test instrument is a number of 2 problems, aiming as the main gate to be able to identify reversible thinking style in solving van hiele geometry problems. Problems presented are selected based on several criteria, including: (a) the problem presented must be related to Van Hiele's geometry thinking process (b) the problem presented must require students to use reversible thinking rules (c) the difficulty of the problem is presented starting from the easiest problem to the difficult problem. The time given to find the solution of the problem is 15 minutes. Two problems given to students are as follows:

Level 1

You are given an equilateral pentagon. Estimate what shape is formed from $\frac{8}{5}$ parts of the equilateral triangle! Include a picture for this answer!

Level 2

An equilateral triangle is $\frac{1}{3}$ of a quadrilateral. Estimate what shape can be formed by the $\frac{5}{3}$ parts of the shape! Include a picture for this answer!

Furthermore, the interview process was conducted to the three students based on the answers obtained. In this way, the researcher conducted interviews with each student within 15-20 minutes. The interview conducted was semi-structured, with a guideline to get the main point in solving van hiele geometric problems using reversible thinking process. The general questions included: what information was obtained first? What was the first thought when getting the information? What confusion was felt when going to solve the problem?

Analyzing of Data

Data analysis was carried out based on test results and interviews. First of all, to recognize the data is done by repeatedly reading the answers written by students and repeatedly listening to the interview records obtained. This was also done to get synchronized results between written answers and oral expressions expressed by students. This initial identification also helped researchers to categorize the data appropriately. Categorization is intended to help simplify the perception or idea of the reversible ability of students in finding Van Hiele geometry solutions, so as to make the right conclusions to answer research questions.

RESULTS AND DISCUSSION

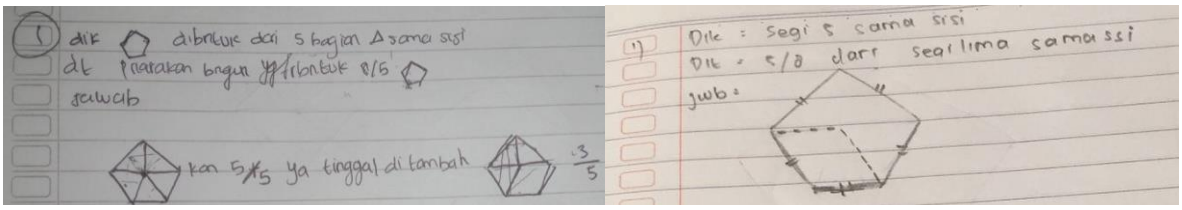
Results

The students' thinking process studied in this article is the geometry thinking ability of students based on van hiele's level of ability by identifying the characteristics of the shapes that must be constructed by students. Furthermore, students' ability to think reversibly is associated with the pattern of the shapes they obtained.

Thought Process for Identifying Geometry Characteristics

In problem no 1, students are asked to construct an equilateral pentagon which is a combination of five congruent triangles. Based on the information provided, students are asked to find the relationship between congruent triangles and the equilateral pentagon. Based on the information, the relationship obtained is that a triangle that builds a pentagon is $\frac{1}{5}$ part of the pentagon.

Based on the results of the written answers of the 2 students who became respondents, both of them had a quite good concept to define the equilateral pentagon. Furthermore, in making the relationship between the pentagon and the triangle, one student succeeded in establishing the relationship between the two figures, but the other student did not. Figure 1 below shows the two students' answers in constructing the relationship between the pentagon and the triangle, with the information that the picture on the left is S1's answer and the picture on the right is S2's answer:



Translation:
dik: Formed from 5 equilateral Δ parts
dit: Describe the shape formed by $\frac{8}{5}$
answer: Just add $\frac{5}{5}$ to $\frac{3}{5}$, maka hasilnya $\frac{8}{5}$

Translation:
dik: Equilateral pentagon
dit: $\frac{8}{5}$ dari equilateral pentagon

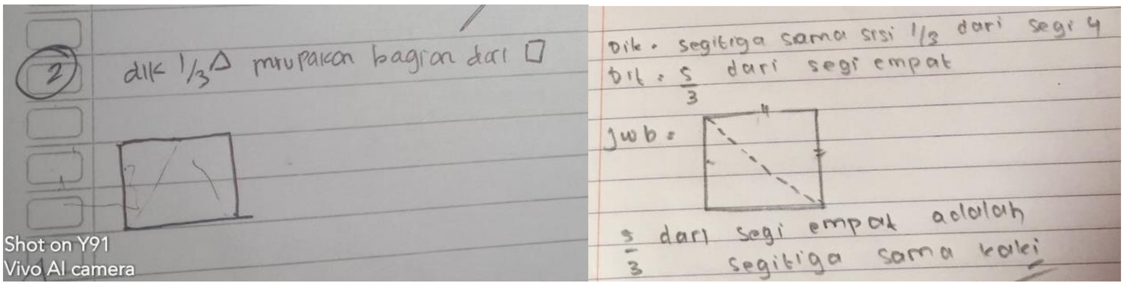
Figure 1. Students' answers to question number 1

After an in-depth interview, one of the students who made a mistake in making a solution (S2 in the right-side picture) because the student is wrong in modeling in geometry. This is because students are not careful in reading the problem. Students do not realize that the pentagon is built by five congruent triangles. So, the student took the initiative to make an assumption, that this full triangle is 100%, so that to determine the other $\frac{8}{5}$ parts, it is done by estimating. However, the weakness of the answer obtained is that there is no guarantee that the geometry formed is how many parts of the main shape. Therefore, in solving problem number 1, it can be constructed by first understanding that a triangle is $\frac{1}{5}$ part of the whole, so eight triangles can form an equilateral eighth as one of the possibilities. Table 1 below is a description of the interviews with S2 students:

Table 1. Example of Students' Interview Transcripts in Solving Problem 1

Question	Students' Answer
How did you do problem 1?	First, I drew a pentagon, then I looked for the $\frac{8}{5}$ part of 100% and the result was 62.5. Therefore, the remaining 37.5 is the flat shape you are looking for
Why the drawing becomes a parallelogram?	It's supposed to be rectangular.
How do you know that the rectangular shape is 37.5%?	At first, I divided the equilateral rhombus into 25% each, then I estimated that 37.5% of the the figure.
How do you prove that your answer is correct?	We have to know what ¼ of a pentagon looks like. Then estimate it.

Furthermore, in question number 2, information is given on a quadrilateral that is composed of three congruent equilateral triangles. Students are asked to be able to arrange these three equilateral triangles to form a quadrilateral. But before constructing the shape, students must have the right basic definition of a quadrilateral. Based on the test results, both respondents did not answer this question correctly. The interview data also supports their written statements, that students miss many basic definitions of quadrilaterals. Based on the information obtained from both respondents, a quadrilateral is a geometry consisting of 4 sides, each of which has the same length. From this erroneous perception, it has an impact on the difficulty of students to construct a new shape that has the same side length composed by three congruent equilateral triangles. Figure 2 below are two students' answers to problem number 2:



Translation:
dik: $\frac{1}{3}\Delta$ is part of □

Translation:
dik: Equilateral triangle $\frac{1}{3}$ from a quadrilateral
dit: $\frac{5}{3}$ from a quadrilateral
Answer: $\frac{5}{3}$ from a quadrilateral is isosceles triangle

Figure 2. Students' answers to question number 2

Based on the answers written by students, students assume that a quadrilateral is the same as a square. Students realize that a quadrilateral is not just a square when they are introduced to the term quadrilateral family which includes trapezoid, kite, parallelogram, rhombus, and rectangle. Furthermore, when the question was expanded to reveal the relationship between geometry in the quadrilateral family, students were also confused. One of the respondents revealed that studying the characteristics of geometry is done separately, so students cannot relate the characteristics between one geometry with another. Table 2 below is interview illustrates students' knowledge about the definition of quadrilateral.

Table 2. Example of Students' Interview Transcripts in Solving Problem 2

Question	Answer
S1	
What is a quadrilateral according to you?	The quadrilateral is a square, because it has equal sides
If a rectangular is a square, isn't it?	No, because based on what I learned from elementary school and from tutoring, what is called a quadrilateral is a square.
S2	
What is the definition of a quadrilateral?	A geometric shape that has four sides of equal length.
What does equal side length mean?	So, the left and right sides are equal in length, and the top and bottom sides are also equal in length.
If the trapezoid is a quadrilateral, isn't it?	Eh, yes, come in. Then it doesn't have to be the same length?

Problem Solving Process Using Reversible Thinking Ability

In problem number 1, one of the students (Respondent S1) was able to verbally express the basic concepts used in the context of the problem. The answers of students who can construct well the requested drawings according to the concept of fractions are as follows:

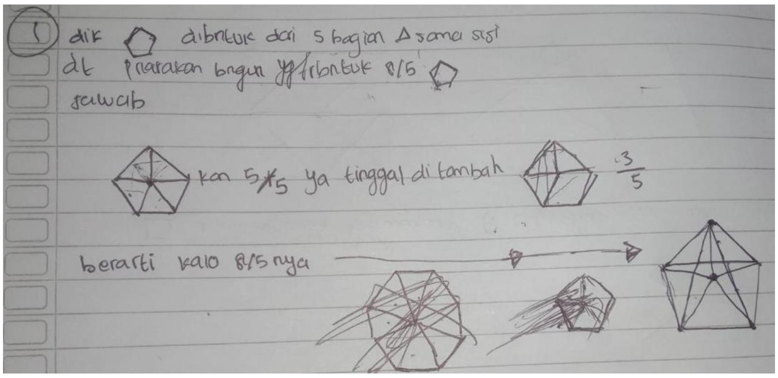


Figure 3. Students' answers use reversible thinking in fraction

Information based on the written test answers and interviews obtained that a triangle in a pentagon represents the value of $\frac{1}{5}$, so 5 triangles in a pentagon represent the value of $\frac{5}{5}$. This knowledge is in accordance with the definition of fractions that focus on reversible thinking, which is that fractions are part of the whole. That a triangle is 1 part of 5 whole triangles. Furthermore, to be able to get the value of $\frac{8}{5}$ (the context in question), fraction operations must be used, as follows:

$$\frac{8}{5} - \frac{5}{5} = \frac{3}{5}$$

So, to get the geometry that is sought, the original image must be added with $\frac{3}{5}$ parts of the original pentagon. However, what is difficult for students is to re-model the fraction form $\frac{8}{5}$ into the form of a picture. Students did not think of exploring other possible shapes that could be formed from the 8 congruent triangles that were constructed.

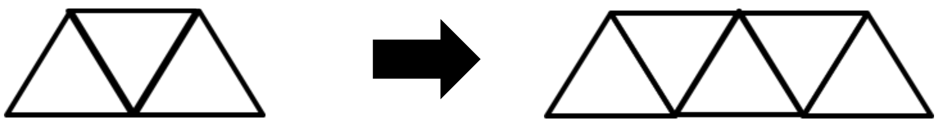
As for the other students, the reversible thinking ability cannot be identified based on the results of the written test, because they read the problem incorrectly. Based on clarification in the interview process, students actually understand that fractions are part of the whole, based on the investigation in the following Table 3:

Table 3. Example of Students' Interview Transcripts to Explore Reversible Ability

Question	Answer
How did you do problem 1?	I first drew a pentagon, then I found the 5/8part of 100% and the result was 62.5. Therefore, the remaining 37.5 is the figure you are looking for
Why is there a subtraction operation?	From the discount formula, for example, 50% discount means 100% minus the product of 50%.

From the student's thinking process above, it can be identified that students understand the concept of fractions which are part of the whole, so students can formulate an equation, that if the whole pentagon is worth 100% then the 5/8 part is worth 37.5%.

Furthermore, in problem number 2, no student succeeded in finding a solution, so it caused the students' reversible thinking process could not be identified. This is because students miss the basic definition of quadrilateral. Therefore, by fixing the definition of a quadrilateral first, students can create a solution construction by making a quadrilateral consisting of three equilateral triangles, then arranging the other two triangles as in the following figure.



$\frac{3}{3}$ part of whole $\frac{5}{3}$ part of whole

Discussion

This study aims to identify the characteristics of students' geometry thinking based on Van Hiele's levels, including visual, analysis, non-formal deduction, formal deduction, and rigor levels. The identification of students' geometry ability level is obtained based on the suitability of the object described from the exploration of the problem. Then, the object is used as the basis for determining the strategy in finding a solution. In the process, students' reversible thinking ability is measured in constructing geometry as an alternative solution.

Based on the research results obtained, students who are able to build geometry models by identifying their characteristics are categorized as students who have van hiele thinking ability at the deduction level. This corresponds to Maharani et al. (2019) research that students who have been able to classify the character of each geometric figure, understand the sequence of geometric structures even to be able to understand the relationship between one geometric figure and another are identified as having van hiele's geomtery ability at the deduction level. This is also in line with what is stated by Cesaria et al. (2021) that at the deduction level students can identify the relationship between the properties of geometric shapes and can classify them hierarchically.

After students are able to create the right geometry model and relate the shapes to each other, they must define the value of the geometry they are looking for. This process requires the use of reversible thinking ability. This is because to determine the value of a geometric figure, students start with the result of the operation and create a starting point. For example, if a student is told "this rectangle is $\frac{3}{5}$ of another rectangle," they can operate on it to produce a rectangle that measures $\frac{5}{5}$. Being able to solve such problems consistently would be evidence of a reversible fraction schema as students can start with a unit fraction and produce a unit. In his research, Hackeberg also identified students' reversible thinking ability by asking students to determine the value of an unknown quantity from a known quantity that can be repeated several times (Hackenberg, 2010).

Students who have good reversible ability will be able to identify fractions as part of the whole (Tzur, 2004). This process will be a way for students to determine the statement that a triangle is 1/3 part of the three triangles arranged. This is in line with Hackenberg's statement (Hackenberg, 2010) that negation activity is one aspect of thinking that can measure reversible thinking processes in students. This means that students can perform the process of cancelling operations in producing the intended value. In this case, to get that a triangle is $\frac{1}{3}$ part of three triangles arranged, it can be done by adding $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ so as to produce 1 part of a complete building.

Based on the results of tests and interviews, students' thinking framework in constructing solutions, tend to be passive in understanding the characteristics of geometry. Passive here can be interpreted that students recognize the characteristics of geometry but can’t build a formal connection to the characteristics of geometry. Students consider that each geometric figure has different characteristics (not overlapping). Sensitivity in identifying the characteristics of geometry is important to then be able to know the relationship between geometry shapes. Septian & Komala (2019) revealed that the ability to investigate problems, describe results, understand ideas to be developed in the next idea means the same as making mathematical connections. The passivity of students in interpreting the characteristics of geometry is preceded by concepts that are not comprehensively understood (Dewanti & Komala, 2023).

Furthermore, students who are successful in identifying geometric shapes, will have the opportunity to succeed in making proportions as solutions to equations. The use of the concept of fractions as "part of the whole" has begun to be applied in answering the questions. This is the basis for understanding students' reversible thinking process (Hackenberg, 2010). Based on the explanation of the two respondents, the reversible thinking ability of upper secondary students has indeed emerged, but it has not been optimally used. One of the causes of students' non-optimisation in performing reversible thinking strategies is the basic definition of the concept of "part of the whole" is not formally constructed. Their perception of fractions is only the comparison of two values. This statement is in accordance with the results of research conducted by Sutiarso (2020) related to reversible thinking ability in secondary school students on the topic of functions and graphs. The results of the study stated that of all students who were given reversible thinking problems, only a small proportion were able to solve them due to the incompleteness of the function concept. In line with these results, Sangwin & Jones's research (2017) also emphasises that when students are given multiple choice questions, students prefer to construct answers by verifying answers with direct methods, not by doing the reverse calculation.

Thus, the series of thought processes synthesized by students based on the results of this study that high school students have a fairly good ability to be able to construct geometry modeling with a note that the ability of its mathematical connections should be addressed, because it affects the solution strategy to be implemented. Additionally, the characteristics of reversible thinking ability of high school students have also not fully performed optimally, because students are still limited by the basic definitions that are not yet established.

This study found two characters of students' thinking in constructing answers that require reversible thinking processes. The two participants as subjects are one female and one male respectively. These two participants have different ways of thinking. Male students tend to explore answers that are in accordance with the important statements contained in the problem. This is likely influenced by gender. Based on research Langi' et al. (2023) that gender affects mindset in solving integral problems.

Based on the investigation from the interview process, the problem given had never been recognized before by the male student. Nevertheless, the basic knowledge possessed by the student can be explored to build a correct and appropriate solution. Meanwhile, female students tend to be procedural by remembering similar events to be applied in solving the given problem. As is the case in solving fraction problems which are part of the whole associated with the concept of discounting. However, the use of reversible thinking by both students is not optimal, students tend to answer by guessing, rather than thinking backwards. This is because the process of thinking backwards is more difficult to do because it requires the ability to think in both directions of completion (Flanders, 2014).

The researchers also realized that students' achievement in reversible thinking needs improvement (Balingga et al., 2016; Maf'ulah et al., 2019). When students perform reversible thinking process, they tend to actively construct their knowledge. This is because the reversible thinking process requires students to understand the basic definition well (including the theorem). Research related to reversible thinking is also one of the important aspects researched in developed countries, because it is one of the thinking strategies in problem solving (Hackenberg, 2005). This reversible thinking process is developed starting from the basic education level on multiplication material. Based on this research, it is argued that division is not a memorization but as the reverse of the multiplication operation.

Limitation and Suggestion for Further Research

This study analyses how students' optimisation in developing solution strategies when presented with a geometry problem that requires reversible thinking strategies. However, this research only focuses on reversible ability on geometry topics. Future research can expand mathematical objects in calculus at the university level. In addition, based on the findings that students' reversible thinking ability on geometry topics is still not optimal, further research can be carried out related to the development of teaching materials or media that can improve students' reversible thinking ability.

CONCLUSIONS

This study describes the thought process of students in solving geometry problems that require reversible thinking ability, so two conclusions are obtained, including: (1) the ability of high school students in expressing Van Hiele's geometry thinking ability is at the deduction stage. This is identified based on the ability of students to model geometric shapes based on their characteristics. One of the weaknesses of students to model these geometric shapes is to build connections formally to the characteristics of the geometry; (2) the ability of high school students in reversible thinking can be identified through the process of solving operations to produce fractions from a known fraction. However, this process has not worked optimally due to the knowledge of students is still limited by the basic definition that is not yet solid. This is identified from students' ability to make proportions as a solution to equations, that students are still unable to make correct statements related to the definition of fractions as part of the whole.

AUTHOR CONTRIBUTIONS STATEMENT

AP contributed to developing ideas and designing instruments. SS was responsible for developing the theory, collecting and analyzing data.

REFERENCES

Acar, F., & Serçe, F. (2021). A comparative study of secondary mathematics curricula of Turkey, Estonia, Canada, and Singapore. *Journal of Pedagogical Research*, *5*(1), 216–242. https://doi.org/10.33902/JPR.2021167798

Anđelković, S., & Malinović-Jovanović, N. (2022). Students ' achievements in primary school mathematics according to the van hiele model of the development of geometric thinking sanja Anđelković , Nela Malinović - Jovanović. *Teaching, Learning and Teacher Education*, *6*, 155–167.

Arnal-Bailera, A., & Manero, V. (2023). A characterization of van hiele's level 5 of geometric reasoning using the delphi methodology. *International Journal of Science and Mathematics Education*, *0123456789*. https://doi.org/10.1007/s10763-023-10380-z

Balingga, E., Prahmana, R. C. I., & Murniati, N. (2016). Analisis kemampuan reversibilitas siswa MTs Kelas VII dalam menyusun persamaan linier. *Jurnal Review Pembelajaran Matematika*, *1*(2), 117–131. https://doi.org/10.15642/jrpm.2016.1.2.117-131

Bungin, B. (2003). Analisa data penelitian kualitatif. In *Raja Grafindo Persada*.

Cesaria, A., Herman, T., & Dahlan, J. A. (2021). Level berpikir geometri peserta didik berdasarkan teori van hiele pada materi bangun ruang sisi datar. *Jurnal Elemen*, *7*(2), 267–279. https://doi.org/10.29408/jel.v7i2.2898

Dewanti, F., & Komala, E. (2023). Kemampuan pemahaman konsep matematika: Kajian analisis hambatan epistimologi siswa smp pada materi statistika. *Wacana Akademika: Majalah Ilmiah Kependidikan Volume*, *7*, 138–147.

Febriana, R., Yusri, R., & Delyana, H. (2020). Modul geometri ruang berbasis problem based learning terhadap kreativitas pemecahan masalah. *AKSIOMA: Jurnal Program Studi*

Pendidikan Matematika, *9*(1), 93. https://doi.org/10.24127/ajpm.v9i1.2591

Flanders, S. T. (2014). *Investigating flexibility , reversibility , and multiple representations in a calculus environment*. (Thesis, Gannon University)

Hackenberg, A. J. (2005). *Construction of algebraic reasoning and mathematical caring relations*. University of Georgia.

Hackenberg, A. J. (2010). Students' reasoning with reversible multiplicative relationships. *Cognition and Instruction*, *28*(4), 383–432.

Han, G., Botao, Z., Hanbin, L., & Weiya, C. (2022). Computational geometric approach for BIM semantic enrichment to support automated underground garage compliance checking. *Journal of Construction Engineering and Management*, *148*(1), 5021013. https://doi.org/10.1061/(ASCE)CO.1943-7862.0002230

Kania, N., Sudianto, S., & Hanipah, H. (2022). Analysis of student's geometry thinking ability based on van hiele's theory. *Journal of Mathematics and Mathematics Education*, *12*(1), 21–33. https://doi.org/10.20961/jmme.v12i1.62273

Khalishah, N., & Nalim, N. (2022). Studi etnomatematika konsep geometris dalam kearifan budaya lokal batik pekalongan. In *SANTIKA: Seminar Nasional Tadris Matematika*, *2*, 390-400.

Langi', E. L., Juniati, D., & Abadi. (2023). Students as prospective teachers' understanding of integral based on the APOS theory in terms of gender difference. *Journal of Higher Education Theory and Practice*, *23*(4), 169–185. https://doi.org/10.33423/jhetp.v23i4.5897

Maf'ulah, S., Fitriyani, H., Yudianto, E., Fiantika, F. R., & Hariastuti, R. M. (2019). Identifying the reversible thinking skill of students in solving function problems. In *Journal of Physics: Conference Series*, *1188*(1), 012033. IOP Publishing. https://doi.org/10.1088/1742-6596/1188/1/012033

Maf'ulah, S., & Juniati, D. (2019). Students' strategies to solve reversible problems of function: The part of reversible thinking. *Journal of Physics: Conference Series*, *1417*(1). https://doi.org/10.1088/1742-6596/1417/1/012051

Maharani, A., Sulaiman, H., Saifurrohman, Aminah, N., & Rosita, C. D. (2019). Analyzing the student's cognitive abilities through the thinking levels of geometry van hiele reviewed from gender perspective. *Journal of Physics: Conference Series*, *1188*(1). https://doi.org/10.1088/1742-6596/1188/1/012066

Pebrianti, A., Juandi, D., & Nurlaelah, E. (2022). Reversible thinking ability in solving mathematics problems. *Jurnal Cendekia : Jurnal Pendidikan Matematika*, *7*(1), 163–173. https://doi.org/10.31004/cendekia.v7i1.1905

Pérez-Fabello, M. J., & Campos, A. (2023). Influence of spatial imagery and imagery control on geometric form location in paintings. *Thinking Skills and Creativity*, *48*, *101298*. https://doi.org/10.1016/j.tsc.2023.101298

Ramadhania, D., Prayitno, S., Subarinah, S., & Arjudin, A. (2022). Analysis of mathematics problem-solving ability on plane figure subject based on van hieles theory at Junior High School. *Jurnal Pijar Mipa*, *17*(4), 493–498. https://doi.org/10.29303/jpm.v17i4.3413

Ristanty, D. W., & Pratama, F. W. (2022). Analisis kemampuan pemecahan masalah matematika pada materi segiempat berdasarkan teori van hiele. *Jurnal Cendekia : Jurnal Pendidikan Matematika*, *6*(2), 1648–1658.

https://doi.org/10.31004/cendekia.v6i2.1400

Sangwin, C. J., & Jones, I. (2017). Asymmetry in student achievement on multiple-choice and constructed-response items in reversible mathematics processes. *Educational Studies in Mathematics*, *94*(2), 205–222. https://doi.org/10.1007/s10649-016-9725-4

Saparwadi, L., Sa'dijah, C., As'ari, A. R., & Chandrad, T. D. (2020). The aspects and stages of reversible thinking of secondary school students in resolving the problems of fractional numbers. *Systematic Reviews in Pharmacy*, *11*(6), 1302–1310.

Septian, A., & Komala, E. (2019). Kemampuan koneksi matematik dan motivasi belajar siswa dengan mengunakan model problem-based learning (PBL) berbantuan geogebra di SMP. *Prisma*, *8*(1), 1. https://doi.org/10.35194/jp.v8i1.438

Steffe, L. P., & Olive, J. (2009). *Children's fractional knowledge*. Springer Science & Business Media. https://doi.org/10.1007/978-1-4419-0591-8

Sundari, I., Fuadiah, N. F., & Fakhrudin, A. (2022). Pengembangan modul materi geometri kelas IV berbasis masalah berorientasi pada penalaran matematis siswa. *Indonesian Research Journal on Education: Jurnal Ilmu Pendidikan*, *2*(3), 1236–1241. https://doi.org/10.31004/irje.v2i3.35

Sutiarso, S. (2020). Analysis of Student Reversible Thinking Skills on Graph Concept. *Indonesian Journal of Science and Mathematics Education*, *3*(2), 185-195. https://doi.org/10.24042/ijsme.v3i2.6768

Takeuchi, H., & Shinno, Y. (2020). Comparing the lower secondary textbooks of Japan and England: A Praxeological analysis of symmetry and transformations in geometry. *International Journal of Science and Mathematics Education*, *18*(4), 791–810. https://doi.org/10.1007/s10763-019-09982-3

Tzur, R. (2004). Teacher and students' joint production of a reversible fraction conception. *The Journal of Mathematical Behavior*, *23*, 93–114. https://doi.org/10.1016/j.jmathb.2003.12.006

Yang, D. C., Tseng, Y. K., & Wang, T. L. (2017). A comparison of geometry problems in middle-grade mathematics textbooks from Taiwan, Singapore, Finland, and the United States. *Eurasia Journal of Mathematics, Science and Technology Education*, *13*(7), 2841–2857. https://doi.org/10.12973/eurasia.2017.00721a