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Implementation of interpolation method in reconstructing damaged satellite image caused by impulse noise

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Abstract
Background: Images are crucial in various fields like engineering, health, and defense. However, during transmission, they often lose information due to noise, particularly impulse noise.
Aim: This study aims to address the issue of noise in images, specifically salt and pepper noise, which frequently occurs during image transmission.
Method: To combat this, we propose a numerical approach using an interpolation method to restore images to their original state.
Result: Experimental results on several images show that this method improves the peak signal-to-noise ratio and the structural similarity index, particularly for images with low to medium density noise.
Conclusion: The interpolation method proves more effective than other methods in reducing salt and pepper noise in images, enhancing both signal-to-noise ratio and structural similarity index.

INTRODUCTION

Image is a 2D visual form of an object that can be represented numerically through the value of each pixel that composes the image arranged in rows and columns. Based on the value of each constituent pixel, images can be divided into 3, namely binary images, grayscale images, and color images (Azzeh et al., 2018). Binary image consists of pixels that have a value of 0 representing black and 1 representing white. A grayscale image is an image that consists of pixels that have integer values between 0 to 255, where each value represents a change in color from black to white. Each pixel that makes up a color image is a combination of three primary colors namely red, green, and blue, where each color has its own intensity value (Abdurrazzaq, Junoh, et al., 2020; Azzeh et al., 2018; Erkan et al., 2018). Most computers today can display colors in an intensity value format with a value of 0 representing the lowest color intensity and 255 representing the highest color intensity (Azzeh et al., 2018; Shi et al., 2022).

At present, images have been widely used in various fields such as engineering, health, defense, etc. In the field of defense, images have played an important role in the process of information gathering and visual communication for military defense and non-military defense (Mustafa et al., 2020; Wang et al., 2022). Images used in defense systems usually have information lost due to the transmission process carried out, the information lost in the image is in the form of damaged pixel values in the digital image caused by noise that appears. The noise that often appears in the transmitted image is impulse noise (Erkan et al., 2018; Fu et al., 2019; Thanh et al., 2020a). Impulse noise causes the appearance of very sharp and randomly

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distributed pixel value differences in the image, this occurs due to the lack of light. There are two types of impulse noise, namely random value noise and salt and pepper noise, where the random value noise will cause the pixel value to be damaged, with damage values ranging from 0 to 255, and scattered randomly in the image. Meanwhile, an image containing salt-and-pepper noise will have pixels with a value of 0, dark, in bright areas and vice versa which are scattered randomly in the image pixels, so this noise is often known as impulse noise (Azzeh et al., 2018; Thanh et al., 2020b). Salt and pepper noise can be caused by analog to digital converter errors, bit errors in transmission, synchronization errors in digitizing or transmitting images (Azzeh et al., 2018).

Noise in digital images is an unavoidable side effect of camera capture, this is also exacerbated by the image transmission process which can add to the effect of noise in the image. Images that have noise such as salt and pepper noise are very disturbing because they can result in the failure of the image recognition process (Azzeh et al., 2018; Erkan et al., 2018; Ganjeh-Alamdari et al., 2022). Therefore, it is necessary to process the images through image filter to restore the image to match the actual condition. Currently, many methods of repairing damage to digital images have been proposed to restore information of the image (Azzeh et al., 2018; González-Hidalgo et al., 2018; Shi et al., 2022; Thanh et al., 2020b). The most widely developed methods usually use statistical approach such as median filter, directed weighted median filter, switching median filter, Based on Pixel Density Filter (BPDF), dan Modified Decision Based Unsymmetric Trimmed Median Filter (MDBUTMF) (Abdurrazzaq et al., 2019b; Charmouti et al., 2022; Erkan et al., 2018; Lu et al., 2016; Thanh et al., 2020a, 2020b). However, apart from using statistical approach, there are other development of methods with other mathematical concepts such as fuzzy logic, and linear algebra, with several methods including: Noise Adaptive Fuzzy Switching Median (NAFSM), Adaptive Type-2 Fuzzy Filter (AT2FF) dan Tropical singular value decomposition (TSVD) (Abdurrazzaq et al., 2019b, 2019a; Abdurrazzaq, Mohd, et al., 2020; Ganjeh-Alamdari et al., 2022; González-Hidalgo et al., 2018; Schuster & Sussner, 2017; Singh et al., 2018; Vijaya & Nagaraju, 2019). However, the filtering method with a statistical approach can reduce the image quality because the selection of a new pixel value is an estimate of the pixel value from the surrounding pixels even though we do not know whether the surrounding pixels have noise or not (Azzeh et al., 2018; Yan et al., 2022). Therefore, this research aims to overcome the problem of image noise, especially salt and pepper noise, which often occurs during image transmission.

METHODS

The proposed method in this study is using the interpolation approach where this method uses a numerical approach that is often used to estimate missing data. The interpolation method estimates the value of point data (x, y) using a mathematical equation obtained from the formulation of x and y values. An image consisting of rows and columns can be assumed as a matrix form with column values as x and rows as y (Aziz, 2018; Chakrabarty, 2016). The proposed interpolation method is a formulation of linear algebra as follows.

$$f(x, y) = \alpha xy + \beta x + \gamma y + \delta$$

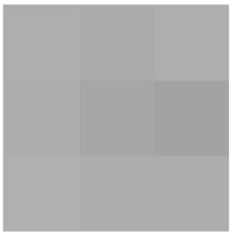
Here are the steps to remove noise in the image using the proposed method:

1. Detection

Firstly, we must detect the damaged pixels, marked with a value of 0 or 255, by starting from the first row of the first column to the last row of the last column. After that we start by taking the undamaged pixels in its surroundings, then calculate the distance between the damaged pixels and the undamaged pixels that will be used as a reference. Choose 4 undamaged pixels that have the smallest distance value, with the distance formula as follows:

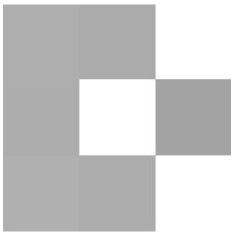
$$l(x,y) = \sqrt{(x' - x)^2 + (y' - y)^2}$$

where (x, y) is the location of the damaged pixels and (x', y') is the location of the surrounding undamaged pixels. For example, in the plane image with set of point (14:16,3:5) as follows:



174	171	175
173	167	163
176	173	173

Figure 1. Original Pixel



174	171	255
173	255	163
176	173	255

Figure 2. Noisy Pixel

Pixels (15.4), (14.5) and (16.5) are noise in digital images caused by salt and pepper noise, because they have a maximum value of (255) and a minimum value of (0). Therefore, it is necessary to perform image filtering to improve the pixel value. Interpolation method is used to estimate the value of the two pixels. The value that will be estimated is the value for pixels (15,4). We need to find the surrounding pixels that do not experience alteration, namely at pixels (14,3), (14,4), (15,3), (15,5), (16,3), and (16,4). Now, we will use 4 free-noise pixel values by calculating the distance to each pixel that we specified earlier with the pixel to be estimated. We choose the pixels at (14,4), (15,3), (15,5), and (16,4) because they have the shortest distance, which is 1 unit each.

2. Pixel Restoration

After the noise is detected, the next process is the restoration of the damaged pixels. The pixel restoration procedure is as follows.

- After we determine the pixels to be recovered, and determine the surrounding pixels that will be used as a reference, the next step is to find the estimated parameter value from the formula we use which is α, β, γ, and δ by using matrix approach as follows.

$$F(x,y)=\begin{bmatrix}14\cdot 4 & 16 & 4 & 1\\ 15\cdot 3 & 15 & 3 & 1\\ 15\cdot 5 & 15 & 5 & 1\\ 16\cdot 4 & 16 & 4 & 1\end{bmatrix}\cdot\begin{bmatrix}\alpha\\ \beta\\ \gamma\\ \delta\end{bmatrix}$$

$$\begin{bmatrix}56 & 16 & 4 & 1\\ 45 & 15 & 3 & 1\\ 75 & 15 & 5 & 1\\ 64 & 16 & 4 & 1\end{bmatrix}\cdot\begin{bmatrix}\alpha\\ \beta\\ \gamma\\ \delta\end{bmatrix}=\begin{bmatrix}171\\ 173\\ 163\\ 173\end{bmatrix}$$

such that

$$\begin{bmatrix}\alpha\\ \beta\\ \gamma\\ \delta\end{bmatrix}=\begin{bmatrix}0.2500\\ 4.0000\\ -8.7500\\ 128.0000\end{bmatrix}$$

- After that, choose the set of point (x, y) values for the restored pixels. The result of the formulation will be the estimated value of the noise pixel, then the formulation for the estimated pixel (15,4) is

  $$f(x,y)=0.25xy+4x-8.75y+128$$

  such that the estimated value of pixel (15,4) is

  $$f(15,4)=0.25\cdot 15\cdot 4+4\cdot 15-8.75\cdot 4+128$$
  $$f(15,4)=16$$

- The interpolation method used in this article is a linear approach with the solution obtained through a matrix approach, so that an estimated pixel value equal to 0 or 255 can be produced. If the pixel estimation result is equal to 0 or 255, the image filtering algorithm is repeated by using the interpolation method 5 times.

The steps of the proposed method can be seen more clearly in the following flowchart.

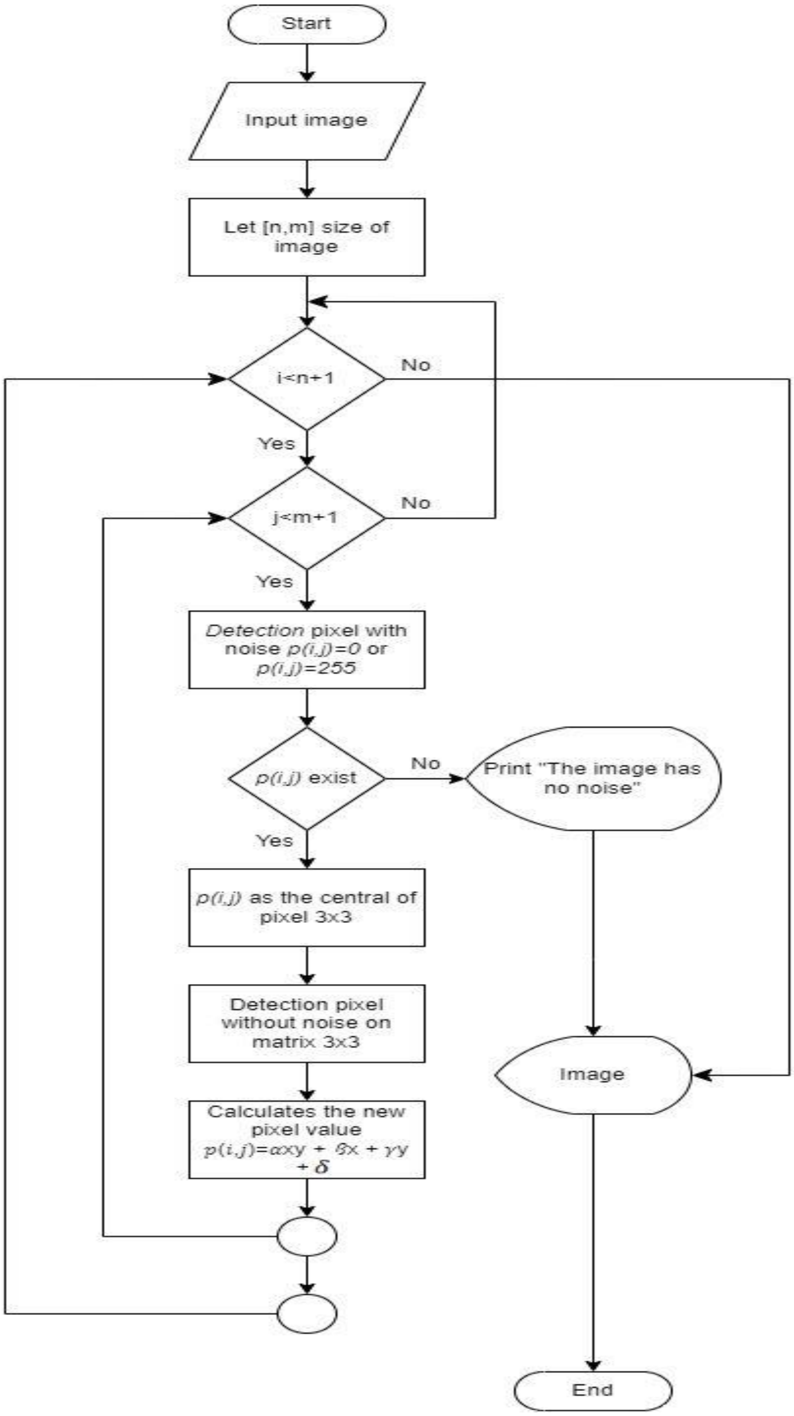


Figure 3. Flowchart of the proposed method

Here is the pseudocode of the proposed method.

Filtering process: ***Input***
Let $[n, m]$, as the size of the image.
Detection pixel with noise, $p(i, j)$.
Let *p(i, j)* as the central pixel of 3 × 3 template.
Do
Detection pixel without noise on 3 × 3 template.

While the image have noise.
If $p(i,j) = 0$ or $p(i,j) = 255$; $i = 1,2,\dots,n; j = 1,2,\dots,m$.
New pixel obtained from
$p(i,j) = \alpha xy + \beta x + \gamma y + \delta$
end
end
Output

RESULTS AND DISCUSSION

In this section, we present the results of image filtering using the proposed method. To measure the performance of the proposed method, several existing methods are used as a comparison such as Median filter, Based on Pixel Density Filter (BPDF), Modified Decision Based Unsymmetric Trimmed Median Filter (MDBUTMF), Noise Adaptive Fuzzy Switching Median (NAFSM), Adaptive Type-2 Fuzzy Filter (AT2FF) dan Tropical Singular Value Decomposition (TSVD). The image data used in this study is a grayscale digital image with a resolution of 512×512 namely Mandril, Lena, Fingerprint, Lichtenstein, Cameramen, Plane, dan Satelite.

1. Image Quality Assessment

To quantitatively measure the performance of the proposed method quantitatively, there are two types of digital image quality assessment, namely the peak signal-to-noise ratio (PSNR) and the structural similarity index (SSIM). The PSNR equation is as follows;

$$PSNR(dB) = 10 \log_{10}\left(\frac{MAX}{MSE}\right)$$

Where MAX is the maximum pixel value in a grayscale image, which is 255. MSE is the Mean-Square-Error denoted as;

$$MSE = \frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\left|q_{ij} - \bar{q}_{ij}\right|^2$$

Where q_{ij} and $\bar{q}_{ij}$ are the pixel values of the original image and filtered image. M and N represent the size of the image.

Furthermore, SSIM is used to measure the similarity between the original image and the filtered image and denoted as;

$$SSIM = \frac{(2\mu_x\mu_y + C_1) + (2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1) + (\sigma_x^2 + \sigma_y^2 + C_2)}$$

Where $\mu_x, \mu_y, \sigma_x\ and\ \sigma_y$ are the standard deviation and covariance of the original image and the filtered image. C_1 and C_2 two variables to stabilize the division with weak denominator.

2. Experimental Result

Six sample grayscale images with resolution 512 × 512, namely Mandril, Lena, Fingerprint, Lichtenstein, Cameramen, and Plane, are used for experiment of the filtering methods. Six sample images will be used in filter test on noisy images with low – medium density noise (10 – 50 %). Quantitatively, the image enhancement method using the interpolation approach shows better results than other methods with the same image as shown in Table 1.

Table 1. Average PSNR and SSIM values of the sample images

ND		SMF	MDBUTMF	NAFSM	T2FF	BPDF	TSVD	TMF	Proposed
10	PSNR	27,8015	29,3263	29,4536	31,4183	30,8364	32,3642	32,7146	33,0854
	SSIM	0,7792	0,8064	0,8134	0,8202	0,8184	0,8218	0,8223	0,8236
20	PSNR	26,3325	26,7594	26,8329	28,3943	27,6400	29,0076	29,1487	29,7015
	SSIM	0,7646	0,7763	0,7911	0,8050	0,8001	0,8070	0,8078	0,8102
30	PSNR	24,8714	25,7240	25,3928	26,2774	25,4418	26,6303	26,9189	27,4533
	SSIM	0,7440	0,7700	0,7741	0,7865	0,7773	0,7869	0,7895	0,7915
40	PSNR	23,4155	24,6182	24,2048	24,3846	23,6205	24,8021	25,1102	25,5462
	SSIM	0,7156	0,7508	0,7529	0,7613	0,7470	0,7610	0,7658	0,7659
50	PSNR	21,6462	23,5360	23,1962	22,6678	21,7917	23,1461	23,5527	23,7515
	SSIM	0,6738	0,7254	0,7275	0,7266	0,7066	0,7253	0,7352	0,7297

In Figure 4. and Figure 5. it can be seen that the PSNR and SSIM values of the proposed method have higher performance than the existing methods.

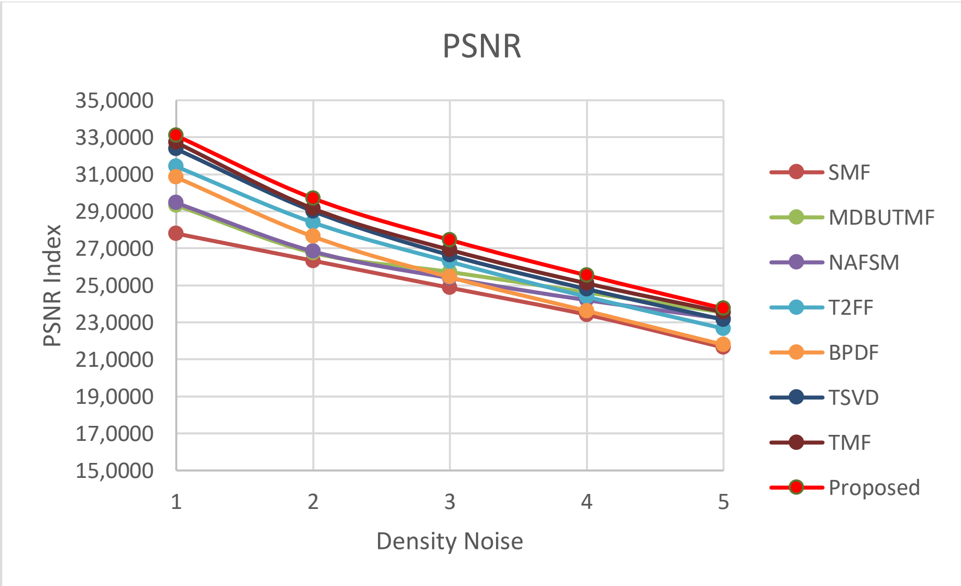


Figure 4. Graph of the average PSNR value of the sample images

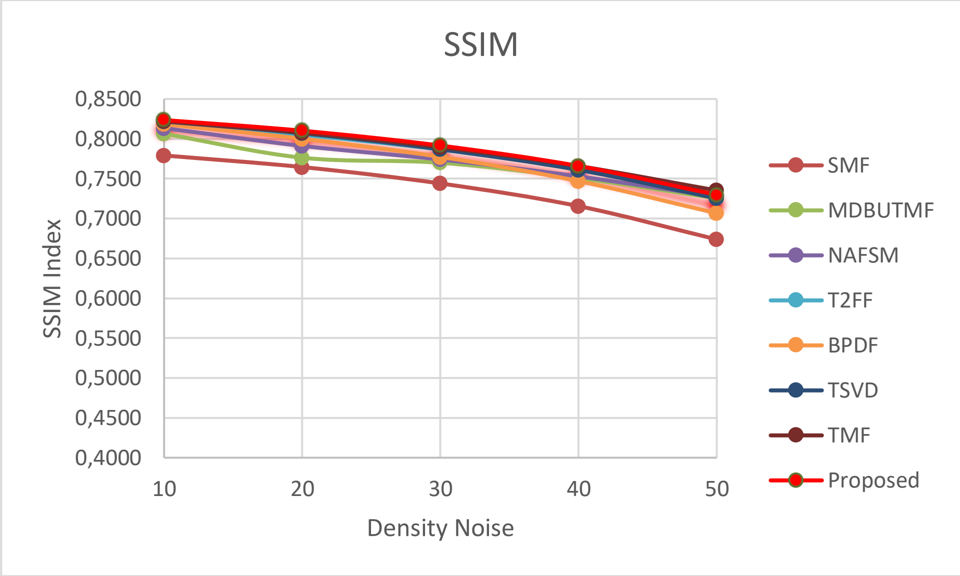


Figure 5. Graph of the average SSIM value of the sample images

Visually, Figure 6 shows a satellite image with 50% salt and pepper noise that has been recovered using the proposed method and the existing methods. The proposed method uses simple numerical approach that are easy to understand and the algorithm can work quickly. The results given by other methods that use statistical approaches produce blurry images, such as MDBUTMF which can maintain the edges of the image, but the resulting texture is blurred, or BPDF which gives visual results that are at first glance better than MDBUTMF but the resulting image is blurry.

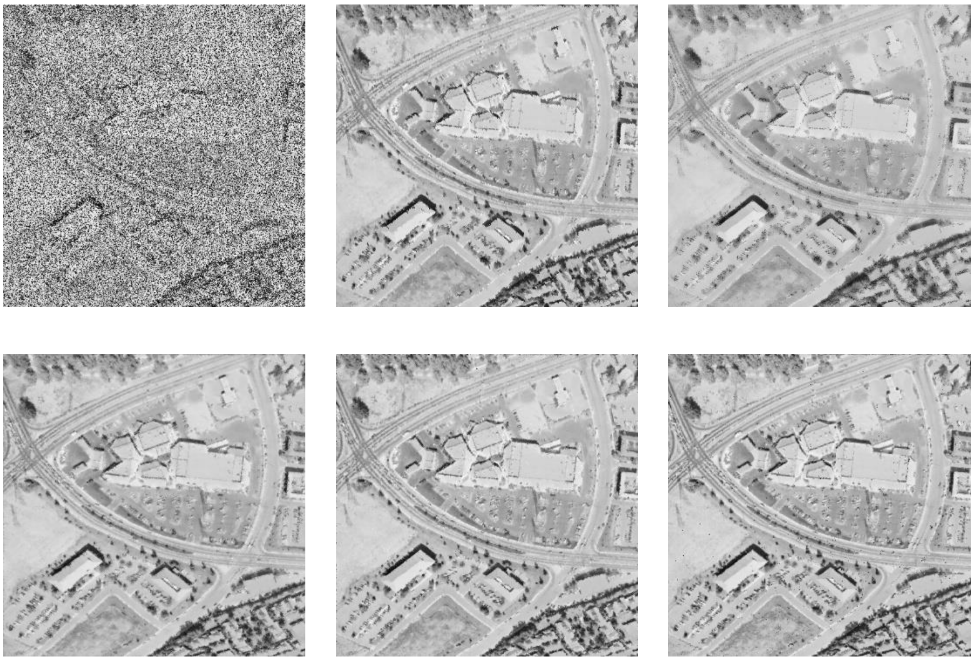


Figure 6. Experimental results on satellite images with 50% salt and pepper noise: Noisy image, BPDF, MDBUTMF, AT2FF, TSVD, and Proposed method.

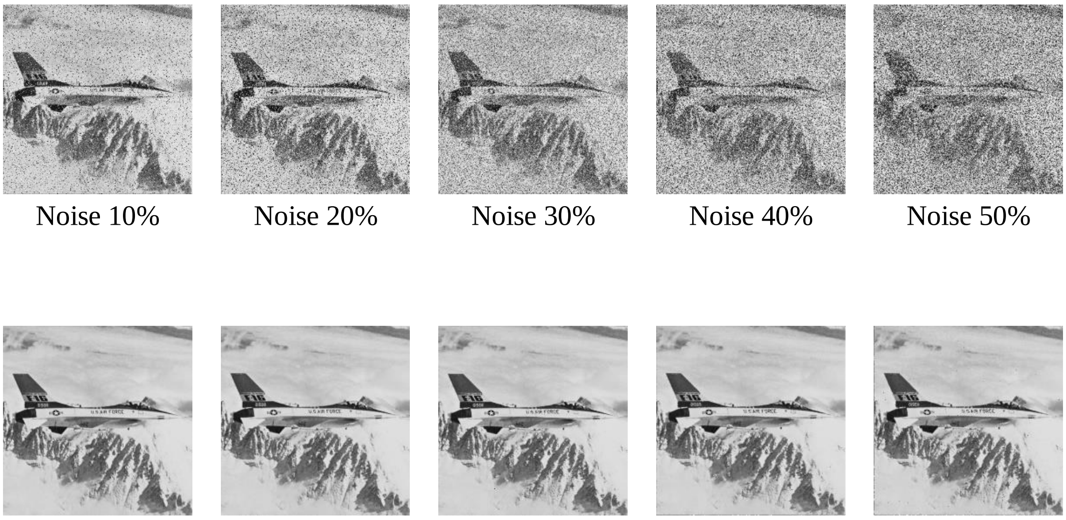


Figure 7. Experimental results of image filtering by using the proposed method on plane images.

CONCLUSIONS

Image enhancement using the proposed method has two steps, namely noise detection and pixel restoration. The noise detection stage is performed by detecting noise on the 3×3 template, and using the surrounding pixels as reference values. While at the pixel restoration stage, pixels that have noise will be estimated using the interpolation method with reference values based on its surrounding that have been obtained from the detection stage. The experimental results using the proposed method show better performance compared to other methods. Quantitatively, the graph shows that the proposed method produces the average PSNR and SSIM values above other filtering methods in the experiment using six sample images with low - medium density noise. Visually, the proposed method also gives better results than other image filtering methods, while maintaining image edges and image textures according to the original condition. For further research, it is necessary to develop the concept used to obtain a better new pixel value where the linear interpolation concept allows the neighborhood pixels to be singular so that the estimation of pixel value becomes less accurate.

AUTHOR CONTRIBUTIONS STATEMENT

HRS acted as the research coordinator, playing a pivotal role in developing the core ideas and methods for this study. AA and AKJ were instrumental in formulating the theoretical framework, designing the necessary instruments, and undertaking the collection. MS performed data analysis to validate the proposed methodology.

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