



# The distance between students' concept image and quadrilateral object definition based on students' mathematical ability

Idris Fadhillah<sup>1\*</sup>, Kusnandi Kusnandi<sup>1</sup>, Dadang Juandi<sup>1</sup>, Suparman Suparman<sup>1</sup>

<sup>1</sup> Department of Mathematics Education, Universitas Pendidikan Indonesia.

✉ [idris.fadhillah151194@upi.edu](mailto:idris.fadhillah151194@upi.edu)

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## Abstract

Students learn mathematics through practical applications without applying it. Consequently, the concept images and definitions that students offer do not match. This study examines the gap in mathematical ability between the concept images of professionals in mathematics education and students' concept images of content, including quadrilaterals. This study employed a qualitative approach with a hermeneutic phenomenology method. Sixty-two seventh-grade students were involved in conducting this study. Some instruments, such as quadrilateral-related tests and semi-structured interview questions, were used to collect the data. The results of quadrilateral-related tests and interviews revealed that most students with high mathematical ability, some with medium mathematical ability, and a small number with low mathematical ability have a concept image that matches the definition but cannot produce proof of the properties of a quadrilateral. In addition, a small number of students with high mathematical talents, some with medium mathematical abilities, and a large number of students with low mathematical abilities were unable to completely explain each rectangle's formal definition and properties. This indicates that there are some students whose concept image is low. So, several alternatives and effective mathematics learning should be implemented to facilitate students in enhancing students concept image.

## INTRODUCTION

Geometry is a part of mathematics that is studied in secondary schools. It is required in communication, for instance, demonstrating a certain area that is surely simple to identify if you supply communication tools such as images, problem-solving in everyday life, and mathematics (Hwang et al., 2019; Novitasari et al., 2021). It is also an important aspect of aesthetics (Glaeser, 2020). One example is three-dimensional learning, which needs knowing various geometric shapes of flat planes. Geometry is also used in everyday life to find the volume of objects and surface areas and look for specific locations on a map. Geometry is the study of forms, which includes a description of objects starting from basic elements like lines and angles to an object's shape (Yavuzsoy-Köse et al., 2019).

In learning geometry, especially quadrilaterals, students will have no trouble recognizing correct conclusions from quadrilateral categorization problems if they already understand the formal definition (Musla et al., 2018). However, suppose there is a gap between students' comprehension and the formal description of the figure being studied; they cannot establish a connection between one property and another of a given geometry object (Seah, 2015).

The term concept image describes the complete cognitive structure connected with a concept, which comprises mental representations related to the attributes and processes and definition of concepts, notably the form of words used to characterize a concept (Tall, 1988). A concept's definition is a verbal definition that describes the idea accurately, simply, and unambiguously and does not produce conflicts (Vinner, 1983). In contrast, a concept image is a visual representation, and the nature of a concept is a concept's nature. So, a concept image is an abstract image of a person that incorporates a concept's description, qualities, and processes. In contrast, a concept definition is a verbal explanation used to specify a simple concept. As a result, a definition is a word used to specify something clearly and straightforwardly, which is the ideal image of a mathematics education expert in the context of this research.

Each individual has their non-verbal association in mind, and whenever one hears a concept's name, what is usually evoked in one's memory is not its definition (Dahl, 2016). Students develop concept images after studying particular concepts, which do not always have to be the same as definitions and are not always consistent. This is because a person can translate visual representations and experiences into verbal forms they have learned by remembering a part of what they are studying (Dahl, 2016).

Revealing student concept images is important for teachers in the learning process because a critical component of students' understanding is sometimes missing (Dahl, 2016). Geometry learning begins with introducing object properties through the senses, but students should progressively be invited to define geometric objects verbally (Tall, 2013; Yavuzsoy et al., 2019). Additionally, concept image is generated by experience, and students' concept image may be predicted using the teaching materials offered and the teacher's concept image (Ulku et al., 2017).

In Israel, where the concept image of the prospective early-year teacher still has a significant gap with the formal definition, research on the concept images of teachers and students has been conducted. Teachers find it difficult to classify what includes a circle and what a circle has been. Furthermore, the teacher ignores important triangle and circle characteristics, which affect the written definition (Tsamir et al., 2015). The study's findings suggest a concept picture gap with formal definitions in several sections of the quadrilateral material. It employed 57 potential first-year student mathematics teachers in Turkey and was done on the quadrilateral content. All study participants accurately define parallelograms, rectangular, and rectangles, but they cannot define rhombuses and trapezoids, and only half of the participants correctly classify quadrilaterals (Erdogan & Dur, 2014). The next finding is that students fail to write definitions even though they already know the characteristics of a quadrilateral, and only 4 out of 28 students can correctly describe a parallelogram (Musla et al., 2018). The concept image of future math teachers' students has been examined from some perspectives and findings. Following the executed strategies, it is plain that understanding the gaps with a formal definition and analyzing students' concept images are important. So, this research analyzes students' concept images and analyzes the concepts gaps between students' concept images and formal definitions in the seventh grade with quadrilateral substance.

## METHODS

This study used a qualitative approach supported by a hermeneutic phenomenology method. Phenomenology is a qualitative technique deemed appropriate to study participants' past experiences, including re-experience, to collect detailed data (Cohen et al., 2018; Creswell, 2014; Ramsook, 2018). Meanwhile, hermeneutics is viewed as the core philosophy of science for qualitative research to deepen understanding and enrich interpretation (Ramsook, 2018). From this perspective, hermeneutic phenomenology was a method for examining how meaning was interpreted based on personal understanding gained through lived experience. Students' concept images and definitions on quadrilateral material were examined using hermeneutic phenomenology, which was then used to investigate the experiences the students had, evaluate the gap between student concept image and definition, and determine if the cause of the student's concept image was accurate or whether it was inaccurate.

Sixty-two seventh-grade students were involved in this study, consisting of sixteen students with high mathematical ability, thirty students with medium mathematical ability, and sixteen with low mathematical ability. The group division was derived from an exam on students' mathematical abilities, with 27% of students at the top having high mathematical abilities, 27% having low mathematical abilities, and the remaining students having moderate mathematical abilities (Sugiyono, 2015). They were selected purposively as research participants. Some instruments, such as quadrilateral-related tests and semi-structured questions for interviews, were used to collect the data. Valid and reliable instruments were administered to the research participants. The obtained data were analyzed by using four steps that were: (1) explanation, a technique for interpreting qualitative research data acquired from tests and interviews, (2) naive understanding, (3) in-depth understanding; and (4) appropriation (R. Dungal & Joshi, 2020; Ramsook, 2018).

## RESULTS AND DISCUSSION

### Results of Quadrangle Definition and Rectangle Types

The test results were analyzed using the students' mathematical ability, as shown in Table 1. The test results from the research data obtained will be used as a rating scale, which is more flexible and can be made into a scale for the measurement of knowledge so that it can be categorized. The data that was previously in the form of quantitative numbers can now be used as qualitative data (Jaya & Ardat, 2013; Sugiyono, 2015), for example:

- 0 = Don't know at all
- 1 = If something is not very good
- 2 = If something is not good
- 3 = If something is good
- 4 = If something is very good

The adaptation of the gap comparison towards concept definition follows, progressing from very far to very close based on categorization. Resulting in the following categorization:

- 0 = If the student does not answer
- 1 = If the student's concept image gap is very far from the concept definition
- 2 = If the students' concept image gap is far from the concept definition
- 3 = If the student's concept image gap is close to the concept definition
- 4 = If the student's concept image gap is very close to the concept definition

The student's concept image that is inappropriate in test answers has a very large gap with the concept definition in this study. When a new student writes down the properties of a quadrilateral and the types of quadrilaterals, there is a gap between their concept image and concept definition. Furthermore, in this study, the concept image close to the concept definition is the concept image of students who write in a different language inside test answers but adhere to the concept definition. Besides that, if students have a concept image that is very close to the concept definition but has several words that are different from the concept definition.

**Table 1.** Analysis of the Definition of Quadrilateral and Quadrilateral Types

Mathematical Ability	Concept Image Score	Student Count for Each Question					Total
		Quadrilateral	Parallelogram	Rectangular	Rhombus	Square	
High	0	0	0	0	0	0	0
	1	1	2	0	0	1	4
	2	0	0	8	8	7	23
	3	15	14	8	1	0	38
	4	0	0	0	7	8	15
Medium	0	0	1	0	0	1	2
	1	7	1	5	9	2	24
	2	3	5	13	15	9	45
	3	20	19	12	6	18	75
	4	0	0	0	0	0	0
Low	0	0	0	2	0	1	3
	1	10	8	1	6	2	27
	2	2	0	7	4	9	22
	3	4	8	6	6	4	28
	4	0	0	0	0	0	0

Besides that, samples of student answers that are less accurate at each level of mathematical competence in response to the test questions will be presented and discussed.

### Quadrilateral Material Definition

Concept definition is a verbal definition that expresses the concept accurately, simply, and unambiguously without causing contradictions and is used to specify a concept simply. In the context of this study, a quadrilateral is a concept image of a mathematics education expert, essentially the form of a word used to explain something unambiguously.

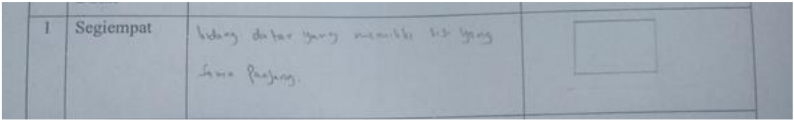

**Table 2.** The Definitions of Mathematics Education Experts in the Quadrilateral.

Part of Quadrilateral	Definition
Quadrilateral	Closed curvature (Polygon) made up of four straight line segments
Parallelogram	A quadrilateral with two parallel pairs of sides
Rectangular	A quadrilateral with four equal right angles
Rhombus	A quadrilateral with the same length sides
Square	A quadrilateral with the same length sides and all four angles are right angles.

**Student Concept Image**

1. Concept Image of Students with High Mathematical Ability

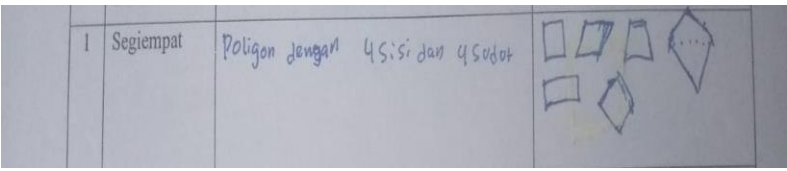
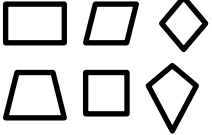
a. Quadrilateral Definition

			
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q1	Quadrilateral	A plane that has equal sides	

**Figure 1.** Samples of Solutions by Students with High Mathematical Abilities that are Less Accurate in Their Quadrilateral Definition.

Figure 1 shows student errors in which students write down the definition of a quadrilateral as a flat shape with only sides of the same length, and the angle formed is  $90^\circ$  because they believe the quadrilateral is the same as a square. The following are the outcomes of the interviews with the students mentioned above.

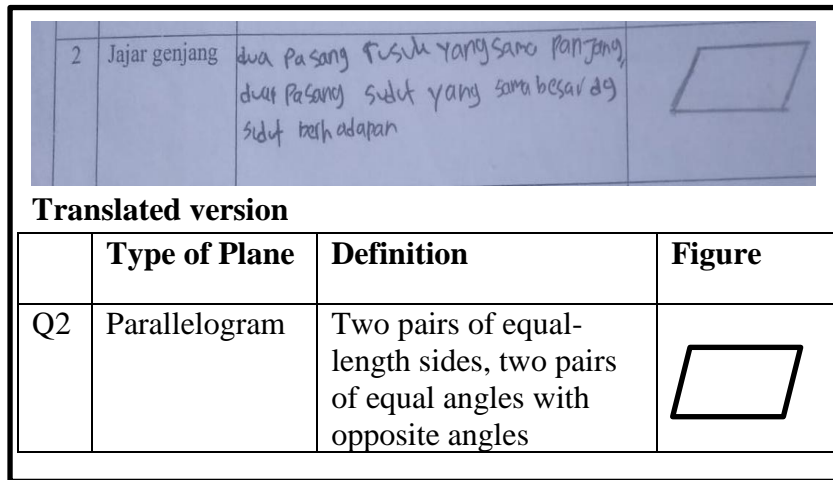
- Q : I have a question about your answer. Is it important for quadrilaterals to have  $90^\circ$  angles and equal sides?
- S1 : Yes, sir, because it's a square.
- Q : So, it's the same for squares and rectangles?
- S1 : Yes, sir, that's part of it.
- Q : What if the side lengths change from the four sides and four angles? Is there a square in it?
- S1 : No, sir, they're not the same length.
- Q : Is there a rectangle included?
- S1 : Of course, sir.

			
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q1	Quadrilateral	A polygon with four sides and four angle	

**Figure 2.** Examples of Correct Answers from High Mathematical Ability Students to Quadrilateral Definition.

Figure 2 shows that the student's answers are correct based on the definition. Students who properly answered used textbooks from the 2013 curriculum, but when learning outside of school, they asked adults to re-explain the quadrilateral subject with tutoring.

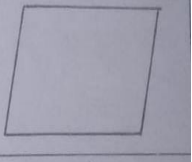

b. Parallelogram Definition



**Figure 3.** Examples of Answers from Students with High Mathematical Abilities that are Inaccurate in the Definition of a Parallelogram.

According to Figure 3, the students' inaccuracies in expressing the definition of a parallelogram are two pairs of ribs that are the same length and two pairs of angles that are the same size as the angle in front of them. The following are the outcomes of the interviews with the students mentioned above.



- Q : Next, enter the definition of a parallelogram. What does a parallelogram mean?
- S2 : two pairs of ribs of equal length, two pairs of angles equal to the angle opposite them.
- Q : What are the components of a quadrilateral?
- S2 : Sure, sir.
- Q : Why may it be obtained in this manner?
- S2 : Because the angles in the picture are the same size and the sides are the same length, sir.
- Q : Is it part of its character or not?
- S2 : Sure, sir.
- Q : What is the relationship between the two pairs of sides?
- S2 : Sir, parallel.
- Q : Because they are parallel, the sides are the same length, and the angles opposite each other are equal. So, a parallelogram is a quadrilateral with two pairs of parallel sides.
- S2 : Ok, sir.

2	Jajar genjang	Bangun segi 4 dimana masing 2 pasang sisinya berhadapan sama panjang dan sejajar.	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q2	Parallelogram	A quadrilateral with each opposite side equal and parallel.	

**Figure 4.** Example of the Correct Answer of High Mathematical Ability Students on the Definition of a Parallelogram.

Figure 4 shows that the student's answers are consistent with the definition. Even though the student's concept image is not yet in its simplest form, students who answered correctly could categorize a parallelogram as a quadrilateral with two pairs of parallel sides and the same length.

c. Rectangular Definition

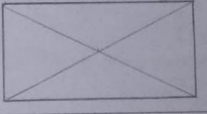

5	Persegi Panjang	bidang yang memiliki 2 pasang sisi yang sama panjang	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q3	Rectangular	A plane that has two pairs of equal sides	

**Figure 5.** Examples of Answers from Students with High Mathematical Abilities but Less Accurate in Their Definitions of Rectangular.

Figure 5 shows the student's inaccuracy, where the student typed down the definition of a rectangular as a field with two pairs of sides of the same length. The student's concept image is unacceptable in this section since they have not written a rectangle yet, it is not a quadrilateral, and the most significant aspect is that every edge is at a right angle. The following are the results of the interviews with the developing sense above.

- Q : What is rectangular?
- S1 : A plane with two pairs of sides of equal length.
- Q : Why may the definition be written in this form?
- S1 : Yes, sir, because the short and long sides are the same length.
- Q : Do you include the quadrilateral?
- S1 : Sir, yes.
- Q : What about the other side?
- S1 : Face to face, sir, with the same length
- Q : Parallel or not parallel?


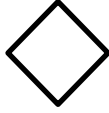
- S1 : Sir, parallel.  
 Q : Is the angle correct?  
 S1 : Sir, yes.

5	Persegi Panjang	Bangun datar yg ke 4 sudutnya siku <sup>2</sup> serta sisi <sup>2</sup> yg berhadapan sama panjang dan sejajar.	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q3	Rectangular	A plane whose four angles are right angles, and each opposite side is equal and parallel.	

**Figure 6.** Examples of Correct High Mathematical Ability Students' Answers on the Definition of a Rectangular

Figure 6 shows that the student's responses match the definition. Students who accurately answered could write the four corners of a right-angled rectangular. The concept image in the image is not yet in its most basic form since it is still writing down the properties of a rectangular, which is the outcome of a quadrilateral with a right angle at each corner.

d. Rhombus Definition

6	Belah Ketupat	Bangun datar yg dibentuk oleh empat buah setiga siku-siku masing <sup>3</sup> sama besar	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q4	Rhombus	A shape formed by four right triangles, each of the same size	

**Figure 7.** Examples of students' answers with high mathematical abilities that are less precise on the definition of a rhombus

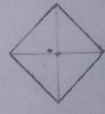

According to Figure 7, student inaccuracy is where students write down the description of a rhombus as a flat shape created by four right-angled triangles of the same size. The following are the outcomes of the interviews with the students mentioned above.

- Q : What is a rhombus?  
 S2 : It is a flat shape of four right triangles of the same size.  
 Q : Why may the definition be written in this direction?  
 S2 : Yes, sir, if it's divided into four right-angled triangles.  
 Q : Isn't it already diagonal?  
 S2 : Sure, sir.



Q : How is the picture put together?

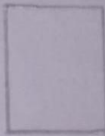

S2 : It's the same length, sir.

6	Belah Ketupat	Bangun datar yg dibentuk oleh 4 buah rusuk yg memiliki panjang yang sama.	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q4	Rhombus	. Each Four sides form a plane side has the same size.	

**Figure 8.** Examples of Students' Answers with High Mathematical Abilities that are Correct on the Definition of a Rhombus

Figure 8 shows that the student's answers are correct. Even if some words were improper, students who answered correctly could write a simple form of the concept image of a rhombus.

e. Square Definition

7	Persegi	bangun datar yg dibentuk oleh empat buah rusuk yg sama panjang	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q5	Square	A flat shape made of four equal-length edges	

**Figure 9.** Examples of Student Answers that are Not Correct in the Definition of a Square.

According to Figure 9, students' inaccuracies when students put down the definition of a square is a flat shape made of four equal-length edges. The following are the outcomes of the interviews with the students mentioned above.

Q : What exactly is a square?


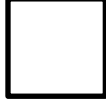
S3 : It has a flat shape made up of four equal segments.

Q : Doesn't that look like a rhombus?

S3 : But, sir, the squares' corners are right angles.

Q : So, the definition of a square is simple, but does it include a quadrilateral?

S3 : Yes, sir.

7	Persegi	Bangun segiempat yang memiliki 4 sisi sama panjang dan 4 sudut siku-siku	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q5	Square	A quadrilateral with each side the same length and every corner a right angle	

**Figure 10.** Example of the Correct Answer of High Mathematical Ability Students on the Definition of a Square.

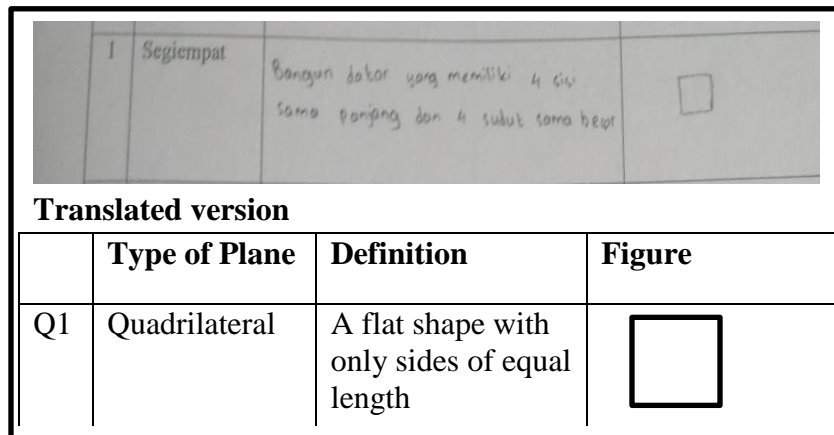
Figure 10 shows that the student's answers are correct. Students who accurately answer can already write that a square is a quadrilateral with each side the same length and every corner a right angle, even though this is not the same as the definition.

Students with high mathematical ability tests show the difference between the concept images of mathematics education experts. Their definition is not too far apart in explaining the meaning of a quadrilateral because the concept image of students with high mathematical abilities is consistent with the definition even though students use different words. In contrast to other studies like the study of Fujita (2012) and Erdogan & Dur (2014), which did not explain quadrilaterals, this one does. At the same time, the definition of the many types of quadrilaterals will be related to the quadrilateral's definition. Students with high mathematical abilities explain that a quadrilateral is a polygon with four sides and that a parallelogram is a quadrilateral with two parallel sides. Similar results were reported by Fujita (2012), Erdogan & Dur (2014), who found preservice mathematics teachers defined a parallelogram as "a quadrilateral with the opposite parallel sides" (Fujita, 2012).

Furthermore, the difference between the concept image between the experts and students with high mathematical ability in terms of rectangles and definitions of rhombuses is quite large. The students' conceptual image of rectangles, rhombuses, and squares is only at the level of understanding the characteristics of rectangular, rhombi, and squares. Similar results were reported by Fujita (2012), who found that the participants had insufficient knowledge of rectangular, rhombus, and square (Fujita, 2012).

2. Concept Image of Students with Medium Mathematical Ability.

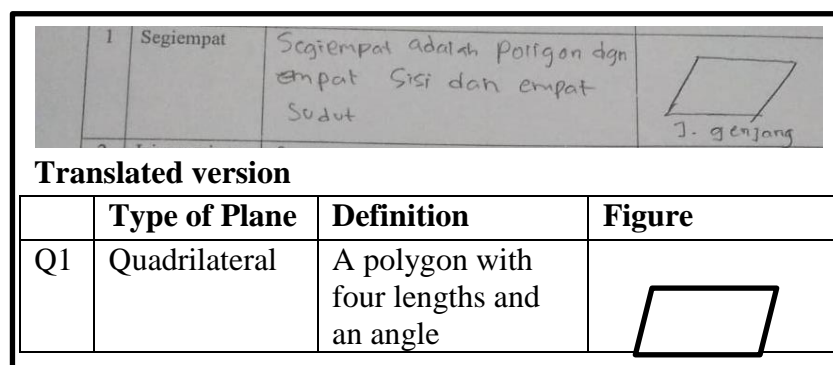
a. Quadrilateral Definition



**Figure 11.** Examples of Students' Answers with Medium Mathematical Abilities that are Less Precise on the Definition of a Quadrilateral

According to Figure 11, the student's inaccuracy is where students write down the definition of a quadrilateral, which is a flat shape with only sides of equal length. The outcomes of the interviews with the students mentioned above are as follows.

- Q : Before I ask about your solution to the definition of a quadrilateral, can you name some rectangular items around?
- S1 : Doors, Books, and Marble
- Q : According to you, are all the sides the same length?
- S1 : No way, sir.
- Q : Then why can it be written that a quadrilateral is a flat shape with only four equal-length sides and four equal-size
- S1 : Because, sir, a quadrilateral is a square.
- Q : If four sides have various side lengths. Is there a quadrilateral in it?
- S1 : Of course, sir.
- Q : A quadrilateral is a polygon with four corners and four sides, whereas a polygon is a flat shape made up of straight lines connected.
- S1 : I see, sir, and thank you.



**Figure 12.** Examples of Students' Answers with Medium Mathematical Abilities that Correct the Definition of a Quadrilateral.

Figure 12 shows that the student's answers are correct. Students who correctly answer can already write that a quadrilateral is a polygon with four lengths and angles.

b. Parallelogram Definition

2	Jajar genjang	Benda yang miring, yang memiliki 2 sisi yang sejajar	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q2	Parallelogram	an inclined object with two parallel	

**Figure 13.** Examples of Students' Answers with Medium Mathematical Abilities that are not Quite Right on the Definition of a Parallelogram

According to Figure 13, when students wrote the definition of a parallelogram for an inclined object with two parallel sides, they did not accurately write the definition of a parallelogram because they only focused on the hypotenuse and the parallel side. The following are the outcomes of the interviews with the students mentioned above.

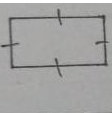
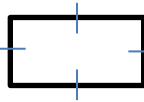
- Q : What is the definition of a parallelogram?  
 S2 : A flat object with two parallel sides.  
 Q : Why may it be obtained in this manner?  
 S2 : Right, it's tilted, sir, and the tilted one is parallel, sir.  
 Q : What is included?  
 S2 : Sir, a flat thing.  
 Q : Do you include the rectangle?  
 S2 : Yes, sir.

2	Jajar genjang	Segi empat dgn sisi yang berhadapan sejajar dan sama panjang serta sudut yang berhadapan sama besar.	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q2	Parallelogram	A quadrilateral with opposite sides that are parallel	

**Figure 14.** Examples of Students' Answers with Medium Mathematical Abilities that are Correct on the Definition of a Parallelogram.

Figure 14 shows that the student's answers are correct. Students who properly answered were able to write that a parallelogram is a quadrilateral with opposite sides that are parallel.

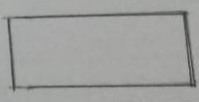

c. Rectangular Definition

5	Persegi Panjang	yaitu benda yang memiliki dua sisi sejajar	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q3	Rectangular	A plane that has two pairs of parallel sides	

**Figure 15.** Examples of Students' Answers with Medium Mathematical Abilities that are Less Precise on the Definition of a Rectangle

Figure 15 shows that student mistakes happen when students write the definition of a rectangular while only paying attention to the sides and not the angles formed. The following are the outcomes of the interviews with the students mentioned above.


- Q : What exactly is a rectangle?  
 S2 : Rectangle is a flat shape with two parallel sides.  
 Q : Didn't that look like a parallelogram?  
 S2 : Sure, sir.  
 Q : Does that indicate you only pay attention to the side?  
 S2 : Sure, sir.  
 P : What is the shape of the angle when you look at it?  
 S2 : right angle, sir.

5	Persegi Panjang	merupakan bangun segi empat yg keempat sudutnya siku-siku dan sisi-sisi yg berhadapan sama panjang dan sejajar	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q3	Rectangular	A quadrilateral with two sets of parallel sides and four right angles	

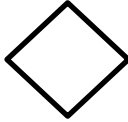
**Figure 16.** Correct Definition of Rectangular Answers from Students with Medium Mathematics Abilities.

Figure 16 shows that students properly answered where they could write that a rectangular is a quadrilateral with two sets of parallel sides and four right angles.

d. Rhombus Definition

6	Belah Ketupat	Bangun datar 2 dimensi yg dibentuk oleh 4 buah segitiga siku-siku masing-masing sama besar dgn sudut dihadapannya	
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**Translated version**

	Type of Plane	Definition	Figure
Q4	Rhombus	A flat shape created by four right triangles with equal length sides in front of the triangle	


**Figure 17.** Examples of Students' Answers with Medium Mathematical Abilities that are Less Precise on the Definition of a Rhombus.

According to Figure 17, student inaccuracy is where students put down the definition of a rhombus based on what they view after specifying two diagonals where four right triangles will be seen. The outcomes of the interviews with the student mentioned above are as follows.

- Q : What is a rhombus?  
 S5 : A flat shape created by four right triangles with equal-length sides in front of the triangle.
- Q : Why is it the claim?  
 S5 : Because dividing it diagonally gives four right triangles, sir.
- Q : What if there is no diagonal, for example?  
 S5 : Sir, the sides are the same length.
- Q : Is there a Quadrilateral included?  
 S5 : Yes, sir.

6	Belah Ketupat	Bangun datar yang memiliki 4 sisi sama panjang dan sudut yang berhadapan sama besar	
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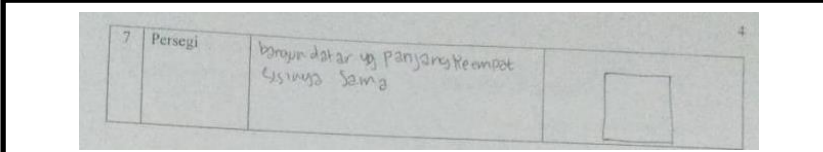
**Translated version**

	Type of Plane	Definition	Figure
Q4	Rhombus	A quadrilateral and all four sides have the same length.	


**Figure 18.** Examples of Correct Medium Mathematical Ability Students' Answers on the Definition of a Rhombus

Figure 18 shows that the student's answer was correct, as the student who correctly answered was able to write that a rhombus is a flat shape with four sides, which includes a quadrilateral and all four sides have the same length.

e. Square Definition



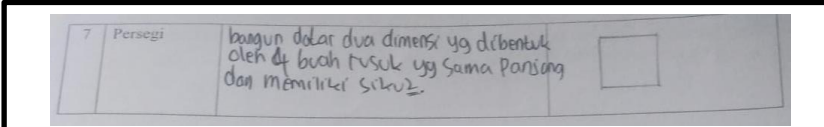
**Translated version**

	Type of Plane	Definition	Figure
Q5	Square	A shape with all sides the same length	


**Figure 19.** Examples of Students' Answers with Medium Mathematical Abilities that are Less Precise on the Definition of a Square

According to Figure 19, the student's inaccuracy is when students write down the definition of a square based on what they observe rather than paying attention to the geometry of a right angle. The following are the outcomes of the interviews with the students mentioned above.

- Q : What exactly is a square?  
 S6 : Make a quadrilateral with all sides the same length.  
 Q : Why is it the case?  
 S6 : Because a quadrilateral has the same side length as a square, sir.  
 Q : How about a rhombus?  
 S6 : Same, sir.  
 Q : Is there a square included?  
 S6 : Sure, sir.  
 Q : A square is a quadrilateral with equal-length sides and equal angles.  
 S6 : Okay, sir.



**Translated version**

	Type of Plane	Definition	Figure
Q5	Square	A plane and all four sides have the same length and have right angles.	

**Figure 20.** Examples of correct medium mathematical ability students' answers on the definition of a square.

Figure 20 shows that the student's answers are correct. Students who correctly replied might write that a square is a two-dimensional flat form with four edges. Even though they


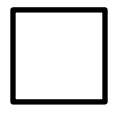
have not written down the four right angles, the intended side is a side, which implies it includes a quadrilateral, and all four sides have the same length and have right angles.

A student with medium mathematical ability test shows the difference between the concept image of mathematics education experts and the concept image of students with medium mathematical abilities in Table 1 is mostly quite large in explaining the meaning of a quadrilateral because students have only reached the level of understanding expressing the characteristics of a rectangular shape because students are not precise. They mainly focus on the square's sides without regard for the angles. Furthermore, when writing the definition of a parallelogram, students pay attention to the sides of the same length without noting that the sides of the same length are caused by the presence of two sets of parallel sides, indicating that students' level of comprehension remains the same. Students did not pay attention to the right angles that are characteristic of rectangular and rectangle in rectangular and rectangle parts because students only notice the qualities of a geometric object and not its individual properties. Most study participants did not pay attention to the angles in rectangular and rectangles, which is the same finding as this one (Erdogan & Dur, 2014).

Meanwhile, students are still focused on the fact that a rhombus can be produced by four right-angled triangles cut from the rhombus by its diagonals when formulating the definition of a rhombus. Contrary to what the participants in the same study believed, a rhombus is defined as a quadrilateral with four equal sides (Erdogan & Dur, 2014). However, some students have been able to explain their understanding of the types of quadrilaterals that are consistent with the concept image of experts in the field of mathematics education, even though they are not the same, such as a parallelogram, which is a quadrilateral with two pairs of parallel sides and two pairs of equal sides. Previous studies also concluded that a parallelogram is a quadrilateral with two sets of parallel sides of the same length (Erdogan & Dur, 2014; Fujita, 2012).

### 3. Concept Image of Students with Low Mathematical Ability

#### a. Quadrilateral Definition

1	Segiempat	Bangun datar yang memiliki 4 sisi sama panjang dan 4 sudut sama besar	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q1	Quadrilateral	A flat shape with sides of equal length and four	

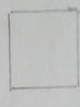

**Figure 21.** Examples of Students' Answers with Low Mathematical Abilities that are not Quite Right on the Definition of a Quadrilateral

According to Figure 21, the student's inaccuracy is where students put down the definition of a quadrilateral as a flat shape with sides of equal length and four angles of equal size



because they believe a quadrilateral is a rectangle. The outcomes of the interviews with the students mentioned above are as follows.

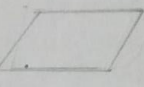

- Q : How is it possible to write that a quadrilateral is a flat shape with only equal sides and four equal angles?  
 S1 : Because we believe it to be a rectangle.  
 Q : If four sides have various side lengths. Is there a rectangle in it?  
 S1 : Of course, sir.  
 Q : A quadrilateral is a polygon with four corners and four sides, whereas a polygon is a flat shape made up of connected straight lines.  
 S1 : I see, sir, and thank you.

1	Segiempat	adalah sosok bidang yg memiliki 4 sisi sehingga memiliki empat sudut / simpul.	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q1	Quadrilateral	A plane with four sides and make four angles	

**Figure 22.** Examples of Correct Answers of Students with Low Mathematical Abilities on the Quadrilateral Definition.

Figure 22 shows that the student's answers were correct even though they were in different languages. Students who correctly answer can already write in different languages that a quadrilateral has four sides.

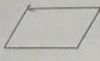

b. Parallelogram Definition

2	Jajar genjang	adalah Poligon terdiri dari 4 buah garis lurus yg saling terhubung. sedangkan sudut yg beraturan bersumlah 180°	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q2	Parallelogram	A Polygon with a connection of four lines and a sequential angle is 180°	

**Figure 23.** Examples of Students' Answers with Low Mathematical Abilities that are Less Precise on the Definition of a Parallelogram

Figure 23 shows that student mistakes are made when students write the definition of a parallelogram that only represents the number of sides and the properties of adjoining angles equal to 180 or opposite angles equal. The next are the results of the interviews with the students mentioned above.



- Q : I want to know more about parallelograms. What does that mean?  
 S2 : We only know that a parallelogram is the opposite angle of the same size and the opposite side of the same length.  
 Q : A parallelogram is a quadrilateral, did you know?  
 S2 : Not quite yet, sir.  
 Q : Why?  
 S2 : We believe the sides must be the same length.  
 Q : A parallelogram is a quadrilateral with two pairs of parallel sides.  
 S2 : I see, sir, and thank you.

2		Jajar genjang	BANGUN DATAR DUA DIMENSI yg Di bentuk oleh dua PASANG BUSUK yg masing2 sama panjang dan sejajar dan pasangannya memiliki SUDUT SAMA BESAR	
<b>Translated version</b>				
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>	
Q2	Parallelogram	A Polygon is made up of two pairs of parallel edges that have the same length.		

**Figure 24.** Examples of Correct Answers of Students with Low Mathematical Abilities on the Definition of a Parallelogram.

Figure 24 shows that the student's answers were correct even though they were in different languages. Students can already write that a parallelogram comprises two pairs of parallel edges with the same length.



c. Rectangular Definition

5		Persegi Panjang	memiliki sepasang sisi sama panjang	
<b>Translated version</b>				
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>	
Q3	Rectangular	A pair of parallel sides		

**Figure 25.** Examples of Student Answers that are Not Correct in the Definition of a Rectangular.

Figure 25 shows that students' failures happen when they write the definition of a rectangular with only a pair of parallel sides which they'd written pairs of right angles. The results of the interviews with the students mentioned above are as follows.


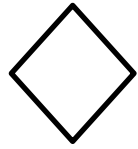
- Q : What does the rectangular next mean?  
 S3 : Makes a two-dimensional flat shape with pairs of sides that are the same length and parallel to the pair of right angles.  
 Q : How did you arrive at that explanation?  
 S3 : A pair of parallel sides, sir, and each right angle.  
 Q : So, how do you tell if a rectangle is a rectangle?  
 S3 : Sure, sir.  
 Q : Are the two parallel pairs of sides? And four angle is the right angle?  
 S3 : Yes, sir.

5	Persegi Panjang	Bangun Datar 2 dimensi yg di bentuk oleh 2 pasang sisi yg masing2 sama panjang dan seajar dg pasangannya memiliki sudut siku-siku	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q3	Rectangular	A plane with two pairs of equal sides with right angles	

**Figure 26.** Examples of Correct Answers of Students with Low Mathematical Abilities on the Definition of a Rectangular.

Figure 26 shows that the student's answers were correct because they were in different languages. Even if the word pair represents the opposite sides, students can already write that a rectangle has two equal sides with right angles.


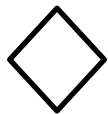
d. Rhombus Definition

6	Belah Ketupat	Bangun Datar dua dimensi yg dibentuk oleh 4 buah segi siku-siku masing2 sama besar yg dihadapannya	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q4	Rhombus	A flat shape created by four right triangles with equal length sides in front of the triangle	

**Figure 27.** Examples of Students' Answers with Low Mathematical Abilities that are Less Precise on the Definition of a Rhombus.

According to Figure 27, student inaccuracy is when students write down the definition of a rhombus, a shape created by four right-angled triangles. The following are the results of the interviews with the two students mentioned above.



- Q : What is a rhombus?  
 S2 : A two-dimensional flat shape produced in front of it by four right-angled triangles of equal size.  
 Q : How did you come up with that perception?  
 S2 : I spotted the diagonal, sir.  
 Q : What happens if you delete the diagonal?  
 S2 : Sir, the sides are the same length.

6	Belah Ketupat	Bangun datar yang memiliki 4 sisi sama panjang dan sudut yang berhadapan sama besar	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q4	Rhombus	A shape has four equal-length sides, and the facing angles are equal.	

**Figure 28.** Examples of Correct Low Mathematical Ability Students' Answers on the Definition of a Rhombus.

Figure 28 shows that the student's answers are correct despite being in a different language or having attributes written down. Students can write that a rhombus has four equal-length sides.

e. Square Definition

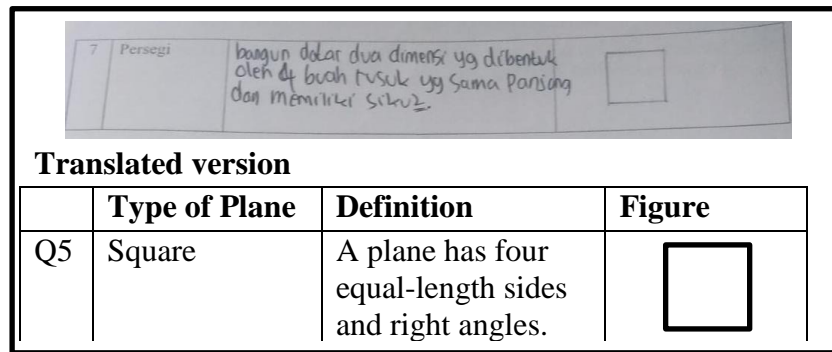
7	Pesegi	memiliki sisi sama panjang	
<b>Translated version</b>			
	<b>Type of Plane</b>	<b>Definition</b>	<b>Figure</b>
Q5	Square	All sides have the same length	

**Figure 29.** Examples of Students' Answers with Low Mathematical Abilities that are Less Precise on the Definition of a Square.

Based on Figure 29, students' inaccuracies can be seen when students write down the definition of a square with the same length as a rhombus without looking at the other detailed things a square has. The following are the outcomes of the interviews with the students mentioned above.

- Q : What exactly is a square?  
 S4 : A square is a shape with equal sides.  
 Q : How did you come up with that perception?  
 S4 : I notice the sides are both the same length, sir.

- Q : How many sides are there?  
 S4 : consists of four packs.  
 Q : Then, is it a quadrilateral?  
 S4 : Yes, sir.  
 Q : Did you take note of the angle?  
 S4 : When saying no, sir  
 Q : What is the angle?  
 S4 : Sir, right angle sir



**Translated version**

	Type of Plane	Definition	Figure
Q5	Square	A plane has four equal-length sides and right angles.	

**Figure 30.** Examples of Correct Answers of Students with Low Mathematical Abilities on the Definition of a Square.

Figure 30 shows that the student's answers are correct, even though they are in a different language or have not written right angles for all angles. Students who correctly answer can already write that a square has four equal-length sides and right angles.

According to Table 1, the difference in explaining the meaning of quadrilaterals between concept images of experts of mathematics education and concept images of students with low mathematical abilities is mostly quite large because students with low mathematical abilities assume that a quadrilateral is the same as a rectangle, so they write that a quadrilateral is a rectangle. Some students have low mathematical abilities while writing the definition of a parallelogram, focusing exclusively on nearby angles summing 180, even though this results from a line crossing two parallel lines. Contrary to the results of Erdogan & Dur's (2014) study, which showed that participants could accurately define a parallelogram, Furthermore, in determining the definition of a rhombus, some students with low mathematical abilities are still based on the triangle that forms a rhombus if diagonals are made, which means that some students still spend attention to the characteristics of the rhombus as well as rectangular and rectangle, where some students do not notice that rectangular and rectangle are formed by angles that must be right-angle. Contrary to the results of Erdogan & Dur's (2014) study, which showed that participants could accurately define a rectangle.

## CONCLUSIONS

### Conclusion

Based on the findings and discussion of this research, several major points are derived from the observations and study. Firstly, mathematical education experts have an accurate, simple, and unambiguous concept image. Secondly based on Table 1, the majority of students with

high mathematical abilities, some students with medium mathematical abilities, and a small portion of students with low mathematical abilities could define each aspect of a quadrilateral based on a formal definition (concept image of an expert in mathematics education) but in a different language. Also, they provided an overview of the relationship between the properties of a quadrilateral. Furthermore, a small number of students with strong mathematical abilities, some with medium mathematical abilities, and the majority with low mathematical abilities could not fully explain the formal definition.

### **Implication**

The difference between the concept images of mathematics education experts and students with high mathematical abilities shown in Table 1 is not too far apart in explaining the meaning of a quadrilateral because the concept image of students with high mathematical abilities is consistent with the definition even though students use different words. Furthermore, a student with high mathematical ability in terms of rectangular, rhombuses, and squares is only at the level of understanding the characteristics of rectangles, rhombuses, and squares. Furthermore, the difference between the concept image of mathematics education experts and the concept image of students with medium mathematical abilities is mostly quite large in explaining the meaning of a quadrilateral. The difference in explaining the meaning of quadrilaterals between concept images of mathematics education experts and concept images of students with low mathematical abilities is mostly quite large. Errors are influenced by various factors, including the book used and the revised 2013 curriculum book, which does not explicitly explain quadrilaterals and their properties. Furthermore, there are less reliable online learning sources, so adult help is needed in sorting out what is appropriate and not following the applicable content.

### **Limitations and Suggestions for Further Research**

Although the results of this study indicate that students who have good mathematical ability will be able to have good concept images, these findings are only available on quadrilateral content and limited to some research samples during the Covid-19 pandemic. Therefore, future research on various research materials and samples is proposed to be supported by optimal learning.

### **ACKNOWLEDGMENT**

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### **AUTHOR CONTRIBUTIONS STATEMENT**

Sup designed the instruments. In addition, IF collected the data. Furthermore, KN and DJ analyzed the data. All authors collectively wrote this manuscript.

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