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Rough *u*-exact sequence of rough groups

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Abstract
The notion of a U-exact sequence is a generalization of the exact sequence. This paper introduces a rough U-exact sequence in a rough group in an approximation space. Furthermore, we provide the properties of the rough U-exact sequence in a rough group.

INTRODUCTION

In 1982, Zdzislaw Pawlak developed one of the mathematical techniques known as rough set theory (Pawlak, 1982). The fundamental concept of the rough set theory is an equivalence relation. The equivalence class is a partition used to determine universe subsets' lower and upper bounds. Assume U is a non-empty set, which we refer to as the universe set, and R is an equivalence relation on U. The pair $(U \text{ and } R)$ represents an approximation space (Miao et al., 2005). If $X \subseteq U$, the lower approximation of X, denoted by $\underline{X}$, is a union of the equivalence class contained in X. The upper approximation of X, denoted by $\bar{X}$, is a union of the equivalence class intersecting with X. If $\bar{X} - \underline{X} \neq \emptyset$, then the set X is a rough set.

In 1997, Kuroki gave the idea of the ideal rough in semigroups (Kuroki, 1997). Furthermore, Miao et al. introduce the rough groups, rough subgroups, and their properties (Miao et al., 2005). Moreover, Davvaz and Mahdavipour investigate the rough module (Davvaz & Mahdavipour, 2006). Isaac and Neelima introduce the concept of rough ideals and their properties (Isaac & Neelima, 2013). Sinha and Prakash study the exact sequence of rough modules (Sinha, 2016). Furthermore, Jesmalar gives homomorphism and rough group isomorphism (Jesmalar, 2017). Besides that, Davvaz and Parnian-Garamaleky give a concept of the *U*-exact sequence of the R-module (Davvaz & Parnian-Garamaleky, 1999). Then, Fitriani et al. introduce the notion of a sub-exact sequence of *R*-modules (Fitriani et al., 2016). Elfiyanti et al. also give an Abelian property of the category of *U*-complexes, which is motivated by the *U*-exact sequence (Elfiyanti et al., 2016). Aminizadeh et al. (Aminizadeh et al., 2017) introduce an exact sequence of S-acts. Fitriani et al. also establish the notion of an X-sub-linearly independent module (Fitriani et al., 2017). They introduce a U_V-generated module (Fitriani et al., 2018b). Furthermore, they established U-basis and U-free modules using the concept of a sub-exact sequence of modules (Fitriani et al., 2018a). Moreover, they define the rank of the U_V-generated module (Fitriani et al., 2021). Then, they apply the sub-exact sequences to determine the Noetherian property of the submodule of the generalized power series module (Faisol et al., 2021).

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Furthermore, Setyaningsih et al. introduce the sub-exact sequence in the rough groups (Setyaningsih et al., 2021). Many researchers investigate the application of rough sets to algebraic structures, such as rough groups (Nugraha et al., 2022), the properties of rough groups (Wang & Chen, 2010), rough subgroups (Bağırmaz, 2019), rough rings and rough ideals (Agusfrianto et al., 2022), rough V-coexact sequence in the rough group (Hafifullah et al., 2022), roughness in quotient group (Mahmood, 2016), roughness in module (Davvaz & Mahdavipour, 2006), rough semi prime ideals (Neelima & Isaac, 2014), and roughness in the module by using the reference points (Davvaz & Malekzadeh, 2013). In this research, we introduce the *U*-exact sequence of the rough group and its properties.

METHODS

This study was based on literature searches on rough sets, upper and lower approximation space, rough groups, exact sequence, and U-exact sequence. First, we define the rough U-exact sequence in an approximation space. Then we examine the properties of the rough group. In the following step, we use a finite set to construct an example of a U-exact sequence in a rough group. Finally, we examine the properties of the U-exact sequence in the rough group.

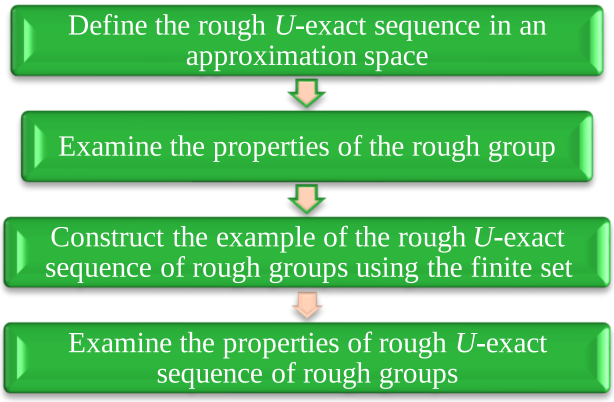

Figure 1. Research stage diagram

RESULTS AND DISCUSSION

Motivated by the definitions of the *U*-exact sequence of the *R*-module (Davvaz & Parnian-Garamaleky, 1999), we construct the definition of the rough *U*-exact sequence as follows.

Definition 1. Let (S, θ) be an approximation space, and let $K, L, and\ M$ be the rough groups in (S, θ), and U be the rough subgroup of M. A sequence

$$\overline{K} \xrightarrow{f} \overline{L} \xrightarrow{g} \overline{M}$$

is called rough U-exact in M if $\text{im}\,(f) = g^{-1}(\overline{U})$.

Before investigating the properties of the rough *U*-exact sequence, we give the construction of a rough subgroup in an approximation space.

Example 1. Let $\mathbb{Z}_9 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}, \overline{8}\}$ is a set of integers modulo 9 and $+_9$ modulo 9 summation operations. We already know that $\langle \mathbb{Z}_9\ and\ +_9 \rangle$ is a group. Then, we define a

relation R in $\mathbb{Z}_9$ as follows. For every $a, b \in \mathbb{Z}_9$, aRb if and only if $a - b = 4k$, for some $k \in \mathbb{Z}$. From this equivalence relation, we have four equivalence classes as follows:
$E_1 = \{\overline{1}, \overline{5}\}$,
$E_2 = \{\overline{2}, \overline{6}\}$,
$E_3 = \{\overline{3}, \overline{7}\}$,
$E_4 = \{\overline{0}, \overline{4}, \overline{8}\}$.

Next, we will construct three rough groups to form a rough U-exact sequence of rough groups. Let $X_1 = \{\overline{0}, \overline{4}, \overline{5}\}$. We obtain $\overline{X_1} = E_1 \cup E_4 = \{\overline{0}, \overline{1}, \overline{4}, \overline{5}, \overline{8}\}$.

Table 1. Table Cayley on X_1

$+_9$	$\overline{0}$	$\overline{4}$	$\overline{5}$
$\overline{0}$	$\overline{0}$	$\overline{4}$	$\overline{5}$
$\overline{4}$	$\overline{4}$	$\overline{8}$	$\overline{0}$
$\overline{5}$	$\overline{5}$	$\overline{0}$	$\overline{1}$

(1) Table 1 shows that for every $x, y \in X_1$, $x\ (+_9)\ y \in \overline{X_1}$;
(2) association property holds in $\overline{X_1}$;
(3) there exists $\overline{0} \in \overline{X_1}$, such that for every $\bar{x} \in \overline{X_1}$, $\bar{x}(+_9)\overline{0} = \overline{0}(+_9)\bar{x} = \bar{x}$;
(4) there exists $\bar{x} \in X_1$, there exists $\bar{y} \in X_1$ such that $\bar{x}(+_9)\bar{y} = \overline{0}$ or $\bar{y} = (\bar{x})^{-1}$, that is $(\overline{0})^{-1} = \overline{0} \in X_1$, $(\overline{4})^{-1} = \overline{5} \in X_1$, and $(\overline{5})^{-1} = \overline{4} \in X_1$.

Table 2. Rough Inverse Element on X_1

x	The rough inverse of x
$\overline{0}$	$\overline{0}$
$\overline{4}$	$\overline{5}$
$\overline{5}$	$\overline{4}$

Based on Table 2, every element of X_1 has a rough inverse in $\overline{X_1}$. Hence, X_1 is a rough group.

Now, let $X_2 = \{\overline{1}, \overline{3}, \overline{6}, \overline{8}\}$. We get $\overline{X_2} = E_1 \cup E_2 \cup E_3 \cup E_4 = \mathbb{Z}_9$. We will show that X_2 is a rough group in an approximation space $(\mathbb{Z}_9, R)$.

Table 3. Table Cayley on X_2

$+_9$	$\overline{1}$	$\overline{3}$	$\overline{6}$	$\overline{8}$
$\overline{1}$	$\overline{2}$	$\overline{4}$	$\overline{7}$	$\overline{0}$
$\overline{3}$	$\overline{4}$	$\overline{6}$	$\overline{0}$	$\overline{2}$
$\overline{6}$	$\overline{7}$	$\overline{0}$	$\overline{3}$	$\overline{5}$
$\overline{8}$	$\overline{0}$	$\overline{2}$	$\overline{5}$	$\overline{7}$

(1) Table 3 shows that for every $x, y \in X_2$, $x\ (+_9)\ y \in \overline{X_2}$;
(2) association property holds in $\overline{X_2}$;
(3) there exists $\overline{0} \in \overline{X_2}$, such that for every $\bar{x} \in \overline{X_2}$, $\bar{x}(+_9)\overline{0} = \overline{0}(+_9)\bar{x} = \bar{x}$;
(4) for every $\bar{x} \in X_2$, there exists $\bar{y} \in X_2$, such that $\bar{x}(+_9)\bar{y} = \overline{0}$ or $\bar{y} = (\bar{x})^{-1}$, that is $(\overline{1})^{-1} = \overline{8} \in X_2$, $(\overline{8})^{-1} = \overline{1} \in X_2$, $(\overline{3})^{-1} = \overline{6} \in X_2$, and $(\overline{6})^{-1} = \overline{3} \in X_2$.

Table 4. Rough Inverse Element on X_2

x	The rough inverse of x
$\bar{1}$	$\bar{8}$
$\bar{3}$	$\bar{6}$
$\bar{6}$	$\bar{3}$
$\bar{8}$	$\bar{1}$

Table 4 shows that every element of X_2 has a rough inverse in X_2. Hence, X_2 is a rough group. Let $X_3 = \{\bar{1}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{8}\}$. We obtain $\overline{X_3} = E_1 \cup E_2 \cup E_3 \cup E_4 = \mathbb{Z}_9$.

Table 5. Table Cayley on X_3

$+_9$	$\bar{1}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{6}$	$\bar{8}$
$\bar{1}$	$\bar{2}$	$\bar{4}$	$\bar{5}$	$\bar{6}$	$\bar{7}$	$\bar{0}$
$\bar{3}$	$\bar{4}$	$\bar{6}$	$\bar{7}$	$\bar{8}$	$\bar{0}$	$\bar{2}$
$\bar{4}$	$\bar{5}$	$\bar{7}$	$\bar{8}$	$\bar{0}$	$\bar{1}$	$\bar{3}$
$\bar{5}$	$\bar{6}$	$\bar{8}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{4}$
$\bar{6}$	$\bar{7}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{5}$
$\bar{8}$	$\bar{0}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{7}$

(1) Table 5 shows that for every $x, y \in X_3$, $x\ (+_9)\ y \in \overline{X_3}$;
(2) association property holds in $\overline{X_3}$;
(3) there exists $\bar{0} \in \overline{X_3}$, such that for every $x \in \overline{X_3}$, $x(+_9)\bar{0} = \bar{0}(+_9)x = x$;
(4) for every $x \in X_3$, there exists $y \in X_3$, such that $x(+_9)y = \bar{0}$ or $y = x^{-1}$, that is $(\bar{1})^{-1} = \bar{8} \in X_3$, $(\bar{8})^{-1} = \bar{1} \in X_3$, $(\bar{3})^{-1} = \bar{6} \in X_3$, $(\bar{6})^{-1} = \bar{3} \in X_3$, $(\bar{4})^{-1} = \bar{5} \in X_3$, and $(\bar{5})^{-1} = \bar{4} \in X_3$.

Table 6. Rough Inverse Element on X_3

x	The rough inverse of x
$\bar{1}$	$\bar{8}$
$\bar{3}$	$\bar{6}$
$\bar{4}$	$\bar{5}$
$\bar{5}$	$\bar{4}$
$\bar{6}$	$\bar{3}$
$\bar{8}$	$\bar{1}$

Based on Table 6, we have every element of X_3 that has a rough inverse in X_3. Hence, X_3 is a rough group.

Finally, we form a sequence $\overline{X_1} \xrightarrow{i} \overline{X_2} \xrightarrow{i} \overline{X_3}$, with i as an identity function. Then let $U_1 = \{\bar{4}, \bar{5}\} \subseteq X_3$. We have $\overline{U_1} = E_1 \cup E_4 = \{\bar{0}, \bar{1}, \bar{4}, \bar{5}, \bar{8}\}$ is a rough subgroup of X_3. We have $\bar{4}(+_9)\bar{5} = \bar{0} \in \overline{U_1}$ and $(\bar{4})^{-1} = \bar{5} \in U_1$.

We will show the sequence $\overline{X_1} \xrightarrow{i} \overline{X_2} \xrightarrow{i} \overline{X_3}$ is a rough U_1-exact in $\overline{X_3}$. We have $\mathrm{im}\,(f) = \{\bar{0}, \bar{1}, \bar{4}, \bar{5}, \bar{8}\} = g^{-1}(\overline{U_1})$. Hence the sequence $\overline{X_1} \xrightarrow{i} \overline{X_2} \xrightarrow{i} \overline{X_3}$ is a rough U_1-exact in $\overline{X_3}$.

Next, we will give the properties of the rough U-exact sequence of rough groups.

Proposition 1. Let (V,θ) be an approximation space, K a rough group in V, and $U_1, U_2,\dots U_n$ rough subgroup K. If $\overline{U_1} \cap \overline{U_2} \cap \dots \cap \overline{U_n} = \overline{U_1 \cap U_2 \cap \dots U_n}$, then $U_1 \cap U_2 \cap \dots U_n$ is a rough subgroup K in the approximation space (V,θ).

Proof. Let (V,θ) be an approximation space, K a rough group in V, and $U_1, U_2,\dots U_n$ rough subgroup K. We will prove that $U_1 \cap U_2 \cap \dots \cap U_n$ is a rough subgroup of K.

a. Since $e \in \overline{U_i}$, for every $i = 1, 2, .., n$, we have $e \in \overline{U_1 \cap U_2 \cap \dots \cap U_n}$. Hence, $\overline{U_1 \cap U_2 \cap \dots \cap U_n} \neq \emptyset$.
b. For every $a, b \in U_1 \cap U_2 \cap \dots \cap U_n$, we have $a - b \in \overline{U_i}, \forall i = 1,2,\dots,n$. So $a - b \in \overline{U_1} \cap \overline{U_2} \cap \dots \cap \overline{U_n}$. By hypothesis, $\overline{U_1} \cap \overline{U_2} \cap \dots \cap \overline{U_n} = \overline{U_1 \cap U_2 \cap \dots U_n}$ and hence $a - b \in \overline{U_1 \cap U_2 \cap \dots \cap U_n}$.

So, $U_1 \cap U_2 \cap \dots \cap U_n$ is a rough subgroup of K in the approximation space (V,θ). ■

Example 2. Let $V = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \dots, \overline{49}\}$. We define a relation R in V, where aRb if and only if $a - b = 6k$, for some $k \in \mathbb{Z}$ and $a\ b \in V$. It is easy to show that R is an equivalence relation on V. From this equivalence relation, we have six equivalence classes as follows:

$E_1 = [\bar{1}] = \{\bar{1}, \bar{7}, \overline{13}, \overline{19}, \overline{25}, \overline{31}, \overline{37}, \overline{43}, \overline{49}\}$,
$E_2 = [\bar{2}] = \{\bar{2}, \bar{8}, \overline{14}, \overline{20}, \overline{26}, \overline{32}, \overline{38}, \overline{44}\}$,
$E_3 = [\bar{3}] = \{\bar{3}, \bar{9}, \overline{15}, \overline{21}, \overline{27}, \overline{33}, \overline{39}, \overline{45}\}$,
$E_4 = [\bar{4}] = \{\bar{4}, \overline{10}, \overline{16}, \overline{22}, \overline{28}, \overline{34}, \overline{40}, \overline{46}\}$,
$E_5 = [\bar{5}] = \{\bar{5}, \overline{11}, \overline{17}, \overline{23}, \overline{29}, \overline{35}, \overline{41}, \overline{47}\}$,
$E_6 = [\bar{6}] = \{\bar{0}, \bar{6}, \overline{12}, \overline{18}, \overline{24}, \overline{30}, \overline{36}, \overline{42}, \overline{48}\}$.

Let $Y = \{\bar{2}, \bar{4}, \bar{5}, \bar{8}, \overline{10}, \overline{14}, \overline{15}, \overline{16}, \overline{20}, \overline{22}, \overline{23}, \overline{25}, \overline{27}, \overline{28}, \overline{30}, \overline{34}, \overline{35}, \overline{36}, \overline{40}, \overline{42}, \overline{45}, \overline{46}, \overline{48}\} \subseteq V$.

Therefore, the lower approximation and the upper approximation of Y are as follows:
$\underline{Y} = E_4 = \{\bar{4}, \overline{10}, \overline{16}, \overline{22}, \overline{28}, \overline{34}, \overline{40}, \overline{46}\}$
$\overline{Y} = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5 \cup E_6 = V$
The rough set Y is the ordered pair of the lower approximation and the upper approximation written as $Apr(Y) = (\{\bar{4}, \overline{10}, \overline{16}, \overline{22}, \overline{28}, \overline{34}, \overline{40}, \overline{46}\}, V)$.

Next, we define the binary operation $+_{50}$ on rough set Y. We will show that $\langle Y, +_{50}\rangle$ is a rough group.

1. $a +_{50} b \in \overline{Y}$, for every $a, b \in Y$.
2. Association property holds in $\overline{Y}$, i.e., $(a +_{50} b) +_{50} c = a +_{50} (b +_{50} c)$, for every $a, b \in \overline{Y}$.
3. There exists the rough identity element $0 \in \overline{Y}$, such that for every $y \in Y, y(+_{50})0 = 0(+_{50})y = y$.
4. For every $y \in Y$, there is a rough inverse element of y, i.e., $y^{-1} \in Y$ such that $y +_{50} y^{-1} = y^{-1} +_{50} y = 0$.

Hence, $\langle Y, +_{50}\rangle$ is a rough group on the approximation space (S,θ).

Next, let $I = \{\overline{2}, \overline{4}, \overline{20}, \overline{22}, \overline{28}, \overline{30}, \overline{46}, \overline{48}\}$. We have $\overline{I} = E_2 \cup E_4 \cup E_6$. Since $I \subseteq Y$ and each element of I have a rough inverse in I, then I is a rough subgroup of Y. Now, let $J = \{\overline{2}, \overline{5}, \overline{23}, \overline{27}, \overline{45}, \overline{48}\}$ and $\overline{J} = E_2 \cup E_3 \cup E_5 \cup E_6$. We will show that J is a rough subgroup *of* Y. Since $J \subseteq Y$ and each element of J has a rough inverse in J, then J is a rough subgroup of Y.

The two rough subgroups constructed in the previous section can be obtained.

$I \cap J = \{\overline{2}, \overline{48}\}$

$\overline{I \cap J} = \{\overline{0}, \overline{2}, \overline{6}, \overline{8}, \overline{12}, \overline{14}, \overline{18}, \overline{20}, \overline{24}, \overline{26}, \overline{30}, \overline{32}, \overline{36}, \overline{38}, \overline{42}, \overline{44}, \overline{48}\}$

The same can be obtained.

$\overline{I} \cap \overline{J} = \{\overline{0}, \overline{2}, \overline{6}, \overline{8}, \overline{12}, \overline{14}, \overline{18}, \overline{20}, \overline{24}, \overline{26}, \overline{30}, \overline{32}, \overline{36}, \overline{38}, \overline{42}, \overline{44}, \overline{48}\}$

Consequently $\overline{I} \cap \overline{J} = \overline{I \cap J} = E_2 \cup E_6$

$= \{\overline{0}, \overline{2}, \overline{6}, \overline{8}, \overline{12}, \overline{14}, \overline{18}, \overline{20}, \overline{24}, \overline{26}, \overline{30}, \overline{32}, \overline{36}, \overline{38}, \overline{42}, \overline{44}, \overline{48}\}$

Next, it will be indicated that $I \cap J = \{\overline{2}, \overline{48}\}$ is a rough subgroup of Y.

i. $\overline{2}(+_{50})\overline{48} = \overline{0} \in \overline{I \cap J}$,

ii. $(\overline{2})^{-1} = \overline{48} \in I \cap J$.

Hence $\{\overline{2}, \overline{48}\}$ is a subgroup rough of Y.

Proposition 2. Let (S, θ) be an approximation space, and $K, L, and\ M$ be rough groups. U_1 and U_2 are a rough subgroup of M and $U_1 \neq U_2$ where $\overline{U_1} = \overline{U_2}$. If a sequence

$$\overline{K} \xrightarrow{f} \overline{L} \xrightarrow{g} \overline{M}$$

is a rough $\overline{U_1}$-exact sequence, then $\overline{K} \xrightarrow{f} \overline{L} \xrightarrow{g} \overline{M}$ is a rough $\overline{U_2}$-exact sequence.

Proof. We assume that the sequence $\overline{K} \xrightarrow{f} \overline{L} \xrightarrow{g} \overline{M}$ is a rough U_1-exact sequence. Based on Definition 1, we have $\text{im}\,(f) = g^{-1}(\overline{U_1})$. Since $\overline{U_1} = \overline{U_2}$, we have $\text{im}\,(f) = g^{-1}(\overline{U_2})$. In other words, the sequence $\overline{K} \xrightarrow{f} \overline{L} \xrightarrow{g} \overline{M}$ is U_2-exact rough in M. ∎

Example 3. Let $\mathbb{Z}_9 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}, \overline{8}\}$ is a set of integers modulo 9 and $+_9$ modulo 9 summation operations. We define a relation R on $\mathbb{Z}_9$ as follows. For every a, $b \in \mathbb{Z}_9$, aRb if and only if $a - b = 4k$, for some $k \in \mathbb{Z}$. From this equivalence relation, we have four equivalence classes as follows:

$E_1 = \{\overline{1}, \overline{5}\}$,

$E_2 = \{\overline{2}, \overline{6}\}$,

$E_3 = \{\overline{3}, \overline{7}\}$,

$E_4 = \{\overline{0}, \overline{4}, \overline{8}\}$.

Next, we construct three rough groups to form a rough U-exact sequence of rough groups. Let $X_1 = \{\overline{0}, \overline{4}, \overline{5}\}$. We have $\overline{X_1} = E_1 \cup E_4 = \{\overline{0}, \overline{1}, \overline{4}, \overline{5}, \overline{8}\}$.

(1) for every $x, y \in X_1$, $x\ (+_9)\ y \in \overline{X_1}$;

(2) association property holds in $\overline{X_1}$;

(3) there exists $\overline{0} \in \overline{X_1}$, such that for every $\bar{x} \in \overline{X_1}$, $\bar{x}(+_9)\overline{0} = \overline{0}(+_9)\bar{x} = \bar{x}$;

(4) for every $x \in X_1$, there exists $y \in X_1$ such that $x(+_9)y = \bar{0}$ or $y = x^{-1}$, that is $(\bar{0})^{-1} = \bar{0} \in X_1$, $(\bar{4})^{-1} = \bar{5} \in X_1$, and $(\bar{5})^{-1} = \bar{4} \in X_1$.
Hence, X_1 is a rough group.

Now, let $X_2 = \{\bar{1}, \bar{3}, \bar{6}, \bar{8}\}$. We have $\overline{X_2} = E_1 \cup E_2 \cup E_3 \cup E_4 = \mathbb{Z}_9$.
(1) for every $x, y \in X_2$, $x\ (+_9)\ y \in \overline{X_2}$;
(2) association property holds in $\overline{X_2}$;
(3) there exists $\bar{0} \in \overline{X_2}$, such that for every $\bar{x} \in \overline{X_2}$, $\bar{x}(+_9)\bar{0} = \bar{0}(+_9)\bar{x} = \bar{x}$;
(4) for every $\bar{x} \in X_2$, there exists $\bar{y} \in X_2$ such that $\bar{x}(+_9)\bar{y} = \bar{0}$ or $\bar{y} = (\bar{x})^{-1}$, that is $(\bar{1})^{-1} = \bar{8} \in X_2$, $(\bar{8})^{-1} = \bar{1} \in X_2$, $(\bar{3})^{-1} = \bar{6} \in X_2$, and $(\bar{6})^{-1} = \bar{3} \in X_2$.
Hence, X_2 is a rough group.

Let $X_3 = \{\bar{1}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{8}\}$. We obtain $\overline{X_3} = E_1 \cup E_2 \cup E_3 \cup E_4 = \mathbb{Z}_9$.
(1) for every $x, y \in X_3$, $x\ (+_9)\ y \in \overline{X_3}$;
(2) operation $(+_9)$ association property holds in $\overline{X_3}$;
(3) there exists $\bar{0} \in \overline{X_3}$, such that for every $\bar{x} \in \overline{X_3}$, $\bar{x}(+_9)\bar{0} = \bar{0}(+_9)\bar{x} = \bar{x}$;
(4) for every $\bar{x} \in X_3$, there exists $\bar{y} \in X_3$ such that $\bar{x}(+_9)\bar{y} = \bar{0}$ or $\bar{y} = (\bar{x})^{-1}$, that is $(\bar{1})^{-1} = \bar{8} \in X_3$, $(\bar{8})^{-1} = \bar{1} \in X_3$, $(\bar{3})^{-1} = \bar{6} \in X_3$, $(\bar{6})^{-1} = \bar{3} \in X_3$, $(\bar{4})^{-1} = \bar{5} \in X_3$, and $(\bar{5})^{-1} = \bar{4} \in X_3$.
Hence, X_3 is a rough group.

Next, we form a sequence $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$. Let $U_1 = \{\bar{4}, \bar{5}\} \subseteq X_3$. We obtain $\overline{U_1} = E_1 \cup E_4 = \{\bar{0}, \bar{1}, \bar{4}, \bar{5}, \bar{8}\}$ is a rough subgroup of X_3. We get $\bar{4}(+_9)\bar{5} = \bar{0} \in \overline{U_1}$ and $(\bar{4})^{-1} = \bar{5} \in U_1$.
We will show the sequence $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$ is U_1-exact in $\overline{X_3}$.
Since $\overline{X_1} \xrightarrow{f} \overline{X_2}$, $f: a \bmod 9$, and $\overline{X_2} \xrightarrow{g} \overline{X_3}$, g identity function, we obtain

$$\begin{aligned} \text{im}\,(f) &= \{x \in \overline{X_2} \mid x = f(\overline{X_1})\} \\ &= \{x \in \overline{X_2} \mid g(x) = \overline{U_1}\} \\ &= g^{-1}(\overline{U_1}) \\ &= \{\bar{0}, \bar{1}, \bar{4}, \bar{5}, \bar{8}\} \end{aligned}$$

Hence $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$ is U_1-exact in $\overline{X_3}$.
Next, we form the second sequence: $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$ with f homomorphism rough group $f: a \bmod 9$ and g identity function. Then let $U_2 = \{\bar{1}, \bar{8}\} \subseteq X_3$. We obtain $\overline{U_2} = E_1 \cup E_4 = \{\bar{0}, \bar{1}, \bar{4}, \bar{5}, \bar{8}\}$ is a rough subgroup of X_3, and $\bar{1}(+_9)\bar{8} = \bar{0} \in \overline{U_2}$ and $(\bar{1})^{-1} = \bar{8} \in U_2$.
We will show the sequence $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$ is U_2- exact in $\overline{X_3}$.
Since $\overline{X_1} \xrightarrow{f} \overline{X_2}$, $f: a \bmod 9$ dan $\overline{X_2} \xrightarrow{g} \overline{X_3}$, g identity function, we have:

$$\begin{aligned} \text{im}\,(f) &= \{x \in \overline{X_2} \mid x = f(\overline{X_1})\} \\ &= \{x \in \overline{X_2} \mid g(x) = \overline{U_2}\} \\ &= g^{-1}(\overline{U_2}) \\ &= \{\bar{0}, \bar{1}, \bar{4}, \bar{5}, \bar{8}\} \end{aligned}$$

Hence $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$ is U_2-exact in $\overline{X_3}$.

From Example 3, we can conclude that if the sequence $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$ is a rough U_1-exact sequence and U_2 is a subgroup of rough $\overline{X_3}$ where $U_1 \neq U_2$ and $\overline{U_1} = \overline{U_2}$, the sequence $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$ is a rough U_2-exact rough in $\overline{X_3}$.

CONCLUSIONS

The rough U-exact sequence is a generalization of the rough exact sequence in the rough groups. If $K, L, and\ M$ are rough groups, U_1 and U_2 are rough subgroups of M and $U_1 \neq U_2$ where $\overline{U_1} = \overline{U_2}$ and the sequence $\overline{K} \xrightarrow{f} \overline{L} \xrightarrow{g} \overline{M}$ is a rough $\overline{U_1}$-exact sequence, then the sequence $\overline{K} \xrightarrow{f} \overline{L} \xrightarrow{g} \overline{M}$ is a rough $\overline{U_2}$-exact sequence. Furthermore, if we have an approximation space (V, θ), a rough group K in V, and rough subgroups $U_1, U_2, \ldots U_n$ in K, and $\overline{U_1} \cap \overline{U_2} \cap \ldots \cap \overline{U_n} = \overline{U_1 \cap U_2 \cap \ldots U_n}$, then $U_1 \cap U_2 \cap \ldots U_n$ is a rough subgroup K of the approximation space (V, θ).

AUTHOR CONTRIBUTIONS STATEMENT

FA conducted a study of the design. AF and FF participate in preparing the research design, coordinate and help organize manuscript, and data analysis. All authors read and approved the final manuscript.

REFERENCES

Agusfrianto, F. A., Fitriani, & Mahatma, Y. (2022). Rough rings, rough subrings, and rough ideals. *Journal of Fundamental Mathematics and Applications*, *5*(2), 1–8.

Aminizadeh, R., Rasouli, H., & Tehranian, A. (2019). Quasi-exact sequences of S-acts. *Bulletin of the Malaysian Mathematical Sciences Society*, *42*(5), 2225-2235.

Bağırmaz, N. (2019). Direct products of rough subgroups. *Eskişehir Technical University Journal of Science and Technology A - Applied Sciences and Engineering*, *20*(3), 307–316.

Davvaz, B., & Mahdavipour, M. (2006). Roughness in modules. *Information Sciences*, *176*, 3658–3674.

Davvaz, B., & Malekzadeh, A. (2013). Roughness in modules by using the notion of reference points. *Iranian Journal of Fuzzy Systems*, *10*(6), 109-124.

Davvaz, B., & Parnian-Garamaleky, Y. A. (1999). A note on exact sequences. *Bulletin of the Malaysian Mathematical Society*, *22*, 53–56.

Elfiyanti, G., Muchtadi, I., Nasution, D., & Amartiwi, U. (2016). Abelian property of the category of U-Complexes chain of U-Complexes. *International Journal of Mathematical Analysis*, *10*(17), 849–853.

Faisol, A., Ftriani, & Sifriyani. (2021). Determining the noetherian property of generalized power series modules by using X-sub-exact sequence. *Journal of Physics: Conference Series*, *1751*(1).

Fitriani, F., Wijayanti, I. E., Surodjo, B., & Faisol, A. (2021). The rank of U_V-generated modules. *Journal of Mathematics Research*, *13*(4), 81.

Fitriani, Surodjo, B., & Wijayanti, I. E. (2016). On sub-exact sequences. *Far East Journal of Mathematical Sciences (FJMS)*, *100*(7), 1055–1065.

Fitriani, Surodjo, B., & Wijayanti, I. E. (2017). On X-sub-linearly independent modules. *Journal of Physics : Conferences Series*, *893*.

Fitriani, Wijayanti, I. E., & Surodjo, B. (2018a). A generalization of basis and free modules relatives to a family of r-modules. *Journal of Physics: Conference Series*, *1097*(1).

Fitriani, Wijayanti, I. E., & Surodjo, B. (2018b). Generalization of U-generator and M-subgenerator related to category σ [M]. *Journal of Mathematics Research*, *10*(4), 101–106.

Hafifullah, D., Fitriani, F., & Faisol, A. (2022). The properties of rough v-coexact sequence in rough group. *BAREKENG: Jurnal Ilmu Matematika Dan Terapan*, *16*(3), 1069–1078.

Isaac, P., & Neelima, C. (2013). Rough ideals and their properties. *J Global Research in Mathematical Archives*, *1*(6), 90-98.

Jesmalar, L. (2017). Homomorphism and isomorphism of rough group. *International Journal of Advance Research, Ideas and Innovations in Technology*, *3*(3), 1382–1387.

Kuroki, N. (1997). Rough ideals in semigroups. *Information Sciences*, *100*(1–4), 139–163.

Mahmood, W. (2016). Roughness in quotient groups. *arXiv*, 1–13.

Miao, D., Han, S., Li, D., & Sun, L. (2005). Rough group, rough subgroup and their properties. In *International Workshop on Rough Sets, Fuzzy Sets, Data Mining, and Granular-Soft Computing* . Springer, Berlin, Heidelberg.

Neelima, C. A., & Isaac, P. (2014). Rough semi prime ideals and rough bi-ideals in rings. *Int. J. Math. Sci. Appl*, *4*, 29-36.

Nugraha,A, A., Fitriani, F., Ansori, M., & Faisol, A. (2022). The implementation of rough set on a group structure. *Jurnal Matematika MANTIK*, *8*(1), 45-52.

Pawlak, Z. (1982). Rough sets. *International Journal of Computer & Information Sciences*, *11*(5), 341–356.

Setyaningsih, N., Fitriani, F., & Faisol, A. (2021). Sub-exact sequence of rough groups. *Al-Jabar: Jurnal Pendidikan Matematika*, *12*(2), 267-272.

Sinha, A. K., & Prakash, A. (2016). Rough exact sequences of modules. *International Journal of Applied Engineering Research*, *11*(4), 2513-2517.

Wang, C., & Chen, D. (2010). A short note on some properties of rough groups. *Computers and Mathematics with Applications*, *59*(1), 431–436.