Volume 13. Number 2. 147-154 DECEMBER 2024

Jurnal ilmiah pendidikan fisika Al-Biruni
https://ejournal.radenintan.ac.id/index.php/al-biruni/index
DOI: 10.24042/jipfalbiruni.v13i2.24576

e-ISSN: 2503-023X

Fast Forward Method on Landau-Majorana-Zener System

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Article Info

Article history:

Received: September 11, 2024
Accepted: December 20, 224
Published: December 30, 2024

Keywords:

adiabatic dynamics;
fast forward;
landau-majorana-zener;
quantum computers;
qubit.

ABSTRACT

Quantum computers have data in the form of quantum bits (qubits), which are 0 and 1 simultaneously. This 0 and 1 state is the electron spin state on the quantum computer, namely spin up and spin down. Qubits are very sensitive to environmental disturbances, so it is necessary to accelerate the adiabatic dynamics so that the system's coherence is not disturbed by environmental interactions. These spin dynamics can be accelerated using the fast forward method, which uses an additional phase to drive the spin adiabatically. The adiabatic state is a state where the dynamics of a system have the same characteristics as the initial state after the dynamics take place. This research was conducted by qualitatively reviewing physical theory and conducting literature studies on previous research on the fast-forward method and the Landau-Majorana-Zener system. The results of this study obtained wave function solutions, regularization terms, and additional Hamiltonians in the Landau-Majorana-Zener system to accelerate adiabatic spin dynamics so that spin dynamics on quantum computers are not disturbed by the environment. After adding additional Hamiltonian adiabatic dynamics in the Landau-Majorana Zener system can be maintained but requires considerable energy. The magnetic field needed to create the same graph as the wave function graph before adding Hamiltonian, B_0 = 100 Tesla. The energy required to accelerate the adiabatic spin dynamics is large, so the cost is also quite large. Therefore, efforts are needed to minimize the energy required to accelerate the adiabatic spin dynamics.

INTRODUCTION

One example of the progress of science and technology is the discovery of quantum computers. Quantum computers process their data using the principle of quantum mechanics with quantum bits (qubits) as the data unit (Dewi & Lubis, 2021). Using the superposition principle in quantum mechanics, the spin on the quantum computer can be spun up and down simultaneously (Yuan et al., 2019). Qubits are very sensitive to environmental disturbances, so it is necessary to accelerate the adiabatic dynamics so that environmental interactions do not disturb the system's coherence (Setiawan et al., 2019). If the spin dynamics are accelerated, not in an adiabatic state, the system will experience interference due to interactions with the environment, which will cause decoherence. Decoherence in the spin dynamics will disrupt the data processing on the quantum computer, causing its performance to decrease and data processing to take longer.

Time is one of the important factors in making products or work (Syafitri et al., 2023). From a microscopic point of view, electron spins also need acceleration to change from spin up to spin down or vice versa. This can be done by optimizing the time to move the electron spin (Aszhar et al., 2024). With this, efficiency will be obtained in moving the spin. However, this time, efficiency should not change the shape or

content of a system (Rohayati et al., 2024). This state is called the adiabatic state. In quantum systems, this state is called quantum adiabatic (Hutagalung et al., 2023). The fast-forward method and shortcuts to adiabaticity are methods that can be used to accelerate adiabatic quantum dynamics (Nakamura et al., 2020).

The fast-forward method is developed to accelerate adiabatic quantum dynamics guaranteed by regularization provisions (Masuda & Nakamura, 2022). The fast-forward method is formulated on the Schrödionger equation. The fast-forward method's main step is adding Hamiltonian to the wave function (Sugihakim et al., 2021). The fast-forward method is one way to achieve adiabaticity and prepare the desired final state (Masuda et al., 2022). The fast-forward method was initially proposed to accelerate quantum dynamics in general (Setiawan, et al., 2023). However, the fast-forward method was developed again to accelerate the dynamics of quantum adiabatic (Setiawan et al., 2023). In addition to the fast-forward method, other methods can accelerate adiabatic dynamics, namely shortcuts to adiabaticity.

Shortcuts to adiabaticity are a fast way to reach the final stage of a slow adiabatic change with the controlling parameters of a system (Guéry-Odelin et al., 2019). Shortcuts to adiabaticity are used to accelerate the dynamics of cauliflower by providing a temporary Hamiltonian as a control parameter to obtain an adiabatic state in a short time (Ainayah et al., 2023). In addition to prioritizing speed, the shortcuts to the adiabaticity method also prepare many routes for control parameters to minimize energy consumption (Elisa et al., 2023). These two methods are also an alternative to control the dynamics of quantum systems without requiring a long time (Dari et al., 2024). Fast forward and shortcuts to adiabaticity can be used to accelerate the dynamics of quantum, such as electron spin motion (Hegade et al., 2021).

Quantum spin is one of the fundamental properties of nanoparticles that describes the particle's intrinsic angular momentum. Electron spin or spin is the most widely reviewed spin. In electron spin, there are only two eigenstates: spin up (↑) and spin down (↓). These two eigenstates can be expressed as a two-row matrix. Spin-up is expressed as $\begin{pmatrix}1\\0\end{pmatrix}$, and spin-down is expressed as $\begin{pmatrix}0\\1\end{pmatrix}$ (Griffiths, 2005). Electron spin is one example of the most basic system in quantum physics. This system is referred to as a two-level system (Wang et al., 2023). In a two-level system, there are only two energy levels. Two-level systems are widely reviewed and developed for technological developments such as qubit development (Ivakhnenko et al., 2022). One model that reviews two-level systems is the Landau-Majorana-Zener model.

Landau-Majorana-Zener is one of the models used to review spin motion with a magnetic field (Ivakhnenko et al., 2022). In the Landau-Majorana-Zener model, there is a magnetic field that will be used as a spin-driving field (Berry, 2009). Therefore, the Landau-Majorana-Zener system can be applied using a fast-forward method to review the electron spin adiabatically. The magnetic field in the Landau-Majorana-Zener system will be used as the initial Hamiltonian or driving energy to drive the electron spin adiabatically.

In the adiabatic quantum theorem, the system must remain in the same eigenstate during adiabatic quantum dynamics (Setiawan, 2019). This study will review how the fast-forward method on the Landau-Majorana-Zener system accelerates the spin motion adiabatically. This research aims to determine the wave function solution, regularization term, and additional Hamiltonian to accelerate the spin dynamics adiabatically.

METHODS

The method used in this research is qualitative, involving conducting a literature

study or literature review on adiabatic quantum theory. This study will also conduct a theoretical review of the Landau-Majorana-Zener system and a fast-forward method to accelerate adiabatic spin dynamics. The steps in this research are as in Figure 1.

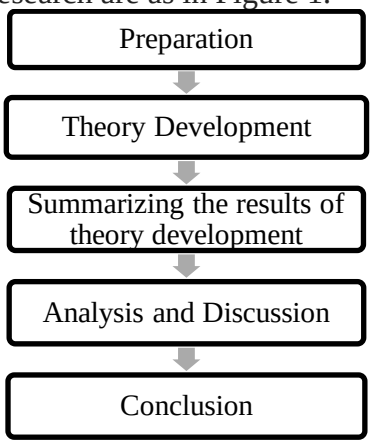

Figure 1. Research Steps

1. Preparation
   At this stage, preparations are made, such as collecting literature in books, journals, and other references regarding quantum theory, Schrödinger equation, quantum adiabatic, fast forward method, and Landau-Majorana-Zener system. The book used in this research is Introduction to Quantum Mechanics, Second Edition.
2. Theory Development
   At this stage, theory development will be carried out, such as derived from the Schrodinger equation to find the wave function solution and the initial Hamiltonian of the Landau-Majorana Zener system.
3. Summarizing The Results of Theory Development
   At this stage, the results of theory development are applied to find the energy and wave function. The energy and wave function depend on the eigenvalue and eigenvector. Thus, at this stage, the eigenvalue will be obtained, which is the energy, and the eigenvector will be the wave function in the Landau-Majorana-Zener system.
4. Analysis and Discussion
   At this stage, the eigenvalues and eigenvectors obtained in the previous stage will be used to find regularization terms and additional Hamiltonians to move the spin adiabatically. Then, the results of the calculation of the regularization term and additional Hamiltonian will be validated and simulated using Wolfram Mathematica.
5. Conclusion
   At this stage, the results of the analysis of the calculations and discussion will be summarized to get answers to the problem formulations in this study.

RESULTS AND DISCUSSION

In the Landau-Majorana-Zener system, a magnetic field drives the spin from the up and down states adiabatically. This driving magnetic field can be written as follows (Berry, 2009).

$$\mathbf{B_0}(t) = B_{min}\hat{\mathbf{e}}_x + vt\hat{\mathbf{e}}_z \quad (1)$$

B_{min} is the driving energy or minimum magnetic field in the x-axis. V is the system's speed, and t is the time required to change direction from the south pole to the north pole in the Landau-Majorana-Zener model (Berry, 2009). With B_{min} dependent on time (t) and v = constant, the Hamiltonian of the Landau-Majorana-Zener system is as follows.

$$H_0(t) = \frac{1}{2}\boldsymbol{\sigma} \cdot \mathbf{B_0}(t) \quad (2)$$

with $\boldsymbol{\sigma}$ being the Pauli matrix

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (3)$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (4)$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (5)$$

By substituting equation (1) and the Pauli matrix into equation (2), we get the initial Hamiltonian in equation (6).

$$H_0(t) = \frac{1}{2}\begin{pmatrix} vt & B_{min} \\ B_{min} & -vt \end{pmatrix} \quad (6)$$

Using adiabatic dynamics, $R(t) = R_0 + \epsilon t$. With ϵ being the adiabatic constant ($\epsilon \ll 1$) and R_0 being the time multiplier (Setiawan, et al., 2023). The Hamiltonian with time parameter R(t) can be rewritten in equation (7).

$$H_0(R(t))=\frac{1}{2}\begin{pmatrix} v\,R(t) & B_{min}(R(t)) \\ B_{min}(R(t)) & -v\,R(t) \end{pmatrix} \quad (7)$$

The eigenvalue shows the energy level of the Landau-Majorana-Zener system. The eigenvalues in this system are as follows.

$$\lambda_{\pm} = \pm\frac{1}{2}\sqrt{B_{min}(R(t))^2 + v^2R(t)^2} \quad (8)$$

Two eigenvalues indicate two energy levels, namely positive and negative eigenvalues, that indicate the ground and the first excited states (Pingak et al., 2019). Take the negative eigenvalue to get the negative eigenvector or wave function at the ground state.

$$\Psi_0\,(R(t)) = \begin{pmatrix} C_1(R(t)) \\ C_2\,(R(t)) \end{pmatrix} = \begin{pmatrix} \frac{P-S}{Q} \\ \frac{J}{Q} \end{pmatrix} \quad (9)$$

with

$$J = B_{min}(R(t)) \quad (10)$$

$$P = v\,R(t) \quad (11)$$

$$S = \sqrt{B_{min}(R(t))^2 + v^2R(t)^2} \quad (12)$$

and

$$Q = \sqrt{2J^2 + 2P^2 - 2PS} \quad (13)$$

In spin systems, accelerated dynamics can be applied through regularization. The regularization term guarantees that a system moves adiabatically. The formula for the regularization term is written in the following equation (Setiawan, 2019).

$$\widetilde{\mathcal{H}}_n\begin{pmatrix} C_1(R) \\ \vdots \\ C_N(R) \end{pmatrix} = i\hbar\begin{pmatrix} \frac{\partial C_1}{\partial R} \\ \vdots \\ \frac{\partial C_N}{\partial R} \end{pmatrix}\left(\sum_{j=1}^{N} C_j^*\frac{\partial C_j}{\partial R}\right)\begin{pmatrix} C_1(R) \\ \vdots \\ C_N(R) \end{pmatrix} \quad (14)$$

Equation (14) is a wave function solution derived from the Schrodinger equation (Natasha et al., 2024) (Ningrum et al., 2023) (Supriadi et al., 2023). The Schrodinger equation is used to obtain the wave function of the Landau-Majorana-Zener system (Afidah et al., 2024) (Kharismawati, 2020). The Schrodinger Equation solution describes the shape of the electron trajectory and the wave function that describes the particle behaviour (Supriadi et al., 2023).

Because the electron spin reviewed in this study is one spin, the wave function solution for the Landau-Majorana-Zener system in equation (14) can be rewritten in equation (15).

$$\widetilde{\mathcal{H}}\begin{pmatrix} C_1(R) \\ C_2(R) \end{pmatrix} = i\hbar\begin{pmatrix} \frac{\partial C_1}{\partial R} \\ \frac{\partial C_2}{\partial R} \end{pmatrix}\left(C_1^*\frac{\partial C_1}{\partial R} + C_2^*\frac{\partial C_2}{\partial R}\right)\begin{pmatrix} C_1(R) \\ C_2(R) \end{pmatrix} \quad (15)$$

To obtain the regularization term, we can use equation (15), which is the regularization term, which can be written as a two-by-two matrix, as in equation (16).

$$\widetilde{\mathcal{H}} = \begin{pmatrix} \widetilde{\mathcal{H}}_{11} & \widetilde{\mathcal{H}}_{12} \\ \widetilde{\mathcal{H}}_{21} & \widetilde{\mathcal{H}}_{22} \end{pmatrix} \quad (16)$$

For the energy to be in a real system, the system must be a Hermit. In a Hermit system, $\widetilde{\mathcal{H}}_{11} = -\widetilde{\mathcal{H}}_{22}$ and $H_{21}^* = \widetilde{\mathcal{H}}_{12}$ (Setiawan, 2019). By supposing

$$a = \frac{\partial C_1}{\partial R} \quad (17)$$

$$b = \frac{\partial C_2}{\partial R} \quad (18)$$

will be obtained $\widetilde{\mathcal{H}}_{11}$ and $\widetilde{\mathcal{H}}_{12}$

$$\widetilde{\mathcal{H}}_{11} = i\hbar\,a\frac{P-S}{Q} - i\hbar\,b\frac{J}{Q} \quad (19)$$

$$\widetilde{\mathcal{H}}_{12} = i\hbar\,a\frac{J}{Q} + i\hbar\,b\frac{P-S}{Q} \quad (20)$$

and

$$\xi\,(t) = 0 \quad (21)$$

The regularization term can be obtained using equation (15), and this system can be written in equation (22).

$$\widetilde{\mathcal{H}} = \begin{pmatrix} i\hbar\,a\frac{P-S}{Q} - i\hbar\,b\frac{J}{Q} & i\hbar\,a\frac{J}{Q} + i\hbar\,b\frac{P-S}{Q} \\ -i\hbar\,a\frac{J}{Q} - i\hbar\,b\frac{P-S}{Q} & -i\hbar\,a\frac{P-S}{Q} + i\hbar\,b\frac{J}{Q} \end{pmatrix} \quad (22)$$

By using the time multiplier $\bar{v}(t) = \epsilon\bar{\alpha}$, where ϵ is an adiabatic factor close to zero, and $\bar{\alpha}$ is a time scale factor to infinity so. The accelerated Hamiltonian can be written using the following equation (Benggadinda & Setiawan, 2021).

$$\mathcal{H} = v(t)\,\widetilde{\mathcal{H}} \quad (23)$$

So we get $\mathcal{H}$ as follows.

$$\mathcal{H}=\begin{pmatrix} v(t)\left(i\hbar a\frac{P-S}{Q}-i\hbar b\frac{J}{Q}\right) & v(t)\left(i\hbar a\frac{J}{Q}+i\hbar b\frac{P-S}{Q}\right) \\ v(t)\left(-i\hbar a\frac{J}{Q}-i\hbar b\frac{P-S}{Q}\right) & v(t)\left(-i\hbar a\frac{P-S}{Q}+i\hbar b\frac{J}{Q}\right) \end{pmatrix} \quad (24)$$

By using the time scale factor $R(\Lambda)(t))=R_0+\bar{v}\left[t-\frac{T\sin\left(\frac{2\pi t}{T}\right)}{4\pi}\right]$, H_{FF} can be written using the following equation (Setiawan et al., 2023).

$$H_{FF} = H_0\,(R(\Lambda)(t)) + \mathcal{H}(R(\Lambda)(t)) \quad (25)$$

Furthermore, by using equation (25), we will get H_{FF} as in equation (26).

$$H_{FF}=\begin{pmatrix} \frac{P}{2}+v(t)\left(i\hbar a\frac{P-S}{Q}-i\hbar b\frac{J}{Q}\right) & \frac{J}{2}+v(t)\left(i\hbar a\frac{J}{Q}+i\hbar b\frac{P-S}{Q}\right) \\ \frac{J}{2}+v(t)\left(-i\hbar a\frac{J}{Q}-i\hbar b\frac{P-S}{Q}\right) & -\frac{P}{2}+v(t)\left(-i\hbar a\frac{P-S}{Q}+i\hbar b\frac{J}{Q}\right) \end{pmatrix} \quad (26)$$

with

$$J = B_{min}(R(\Lambda)(t)) \quad (27)$$

$$P = v\,R(\Lambda)(t)) \quad (28)$$

and

$$S = \sqrt{B_{min}(R(\Lambda)(t))^{\,2} + v\,R(\Lambda)(t))^{\,2}} \quad (29)$$

After that, with parameters $B = B_0-R$, $B_0=10$, $v=50$, $T= 1$, $R = \bar{v}\left[t-\frac{T\sin\left(\frac{2\pi t}{T}\right)}{2\pi}\right]$ and $\sigma = -25$, the graphs of C_1 and C_2 will be obtained before adding the additional Hamiltonian as shown in Figure 2.

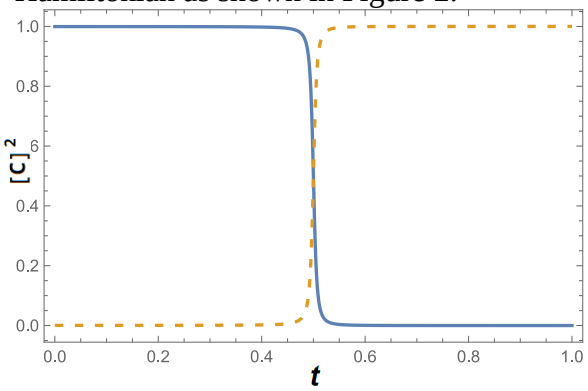

Figure 2. Graphs $|C_1|^2$ and $|C_2|^2$ before the additional Hamiltonian is added.

The initial state is in Figure 2. The blue line, $|C_1|^2$, moves from the top or spin-up state (↑), and the yellow line, $|C_2|^2$, moves from the bottom or spin-down state (↓). The sequence ends with $|C_1|^2$ in the spin-down state (↓) and $|C_2|^2$ in the spin-up state (↑).

Furthermore, by increasing the parameter $B_0=100$, with $B = B_0-R$, $v = 50$, $T = 1$, $R = \bar{v}\left[t-\frac{T\sin\left(\frac{2\pi t}{T}\right)}{2\pi}\right]$ and $\sigma = -25$, the graphs of C_1 dan C_2 will be obtained after adding the additional Hamiltonian as shown in Figure 3.

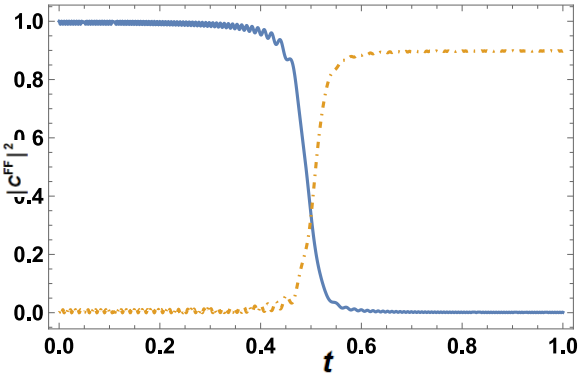

Figure 3. Graphs of $|C_1|^2$ and $|C_2|^2$ after the additional Hamiltonian is added.

After adding Hamiltonian, as in Figure 3, the blue line, which is $|C_1|^2$, moves from the top or spin-up state (↑), and the yellow line, which is $|C_2|^2$, moves from the bottom or spin-down state (↓), then ends with $|C_1|^2$ in the spin-down state (↓) and $|C_2|^2$ in the spin-up state (↑). This state is the same as before adding the additional Hamiltonian in Figure 2.

From the comparison of Figure 2 and Figure 3, there is a difference in the gap and the final position of the graph after changing from the spin-up and spin-down states. In Figure 3, there is a larger gap, and the final position is slightly different from the graph before adding the Hamiltonian. This happens because the spin dynamics when the additional Hamiltonians are added are accelerated, so it takes a shorter time than before the additional Hamiltonians are added.

After comparing the wave function graph before and after adding the additional Hamiltonian, the wave function graph after

adding the additional Hamiltonian is close to the same wave function graph before adding the additional Hamiltonian. This means that the adiabatic dynamics can be maintained after adding the additional Hamiltonian, but it requires more energy than before adding the additional Hamiltonian. The magnetic field needed to create the same graph as the wave function graph before adding the additional Hamiltonian, namely B_0 = 100 Tesla. The energy required to accelerate the adiabatic spin dynamics is quite large, so the costs are also quite high. To overcome this, it is necessary to minimize the energy required so that the costs are also smaller.

CONCLUSION AND SUGGESTIONS

This research obtained the wave function solution, regularization term, and additional Hamiltonian for the Landau-Majorana-Zener system. This regularization term and additional Hamiltonian guarantee that the electron spin in the Landau-Majorana-Zener system moves adiabatically and is accelerated. However, accelerating the adiabatic dynamics of the electron spin requires a large energy, which is B0 = 100 Tesla.

Suggestions for further research are made to minimize the energy needed to accelerate adiabatic spin dynamics, thereby reducing the costs required. Further research can be done by increasing the number of electrons related to adiabatic spin dynamics research.

ACKNOWLEDGMENTS

Thank you to the Physics Education Study Program, which has allowed the author to participate in the Merdeka Belajar Kampus Merdeka (MBKM) research 2024 program.

AUTHOR CONTRIBUTIONS

AK designed the research, analyzed the data, and wrote the article. IS and DH provided critical and detailed assistance in planning and conducting the research.

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