

## ANALYSIS OF STUDENT REVERSIBLE THINKING SKILLS ON GRAPH CONCEPT

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### Article Info

#### Article history:

Received: June 29, 2020

Accepted: July 30, 2020

Published: July 31, 2020

#### Keywords:

Graph concept  
Ramsey number  
Reversible thinking

### ABSTRACT

The ability of reversible thinking in mathematics has less attention, but most of the mathematical subject is reversible. This type of research was qualitative descriptive which aimed to analyze student's reversible thinking skills on the graph concept. The subject of research were students of mathematics education, Faculty of Teacher Training and Education, Lampung University taking the graph theory in the academic year 2019/2020. Research subjects were 40 people selected by the total sampling technique. The research data was collected through tests, namely the question of Ramsey number. Data were analyzed descriptively with 3 stages, namely data reduction, data display, and conclusion. The results showed that 42.5% of students had reversible thinking skills and 57.5% of students did not have them. Based on the results of this study, it was concluded that most students did not have reversible thinking skills on graph concepts.

## ANALISIS KEMAMPUAN BERPIKIR *REVERSIBLE* MAHASISWA PADA KONSEP GRAF

### ABSTRAK

#### Kata Kunci:

Konsep graf  
Bilangan Ramsey  
Berpikir *reversible*

Kemampuan berpikir *reversible* dalam matematika kurang mendapat perhatian, padahal sebagian besar materi matematika bersifat *reversible*. Jenis penelitian ini deskriptif kualitatif bertujuan untuk menganalisis kemampuan berpikir *reversible* mahasiswa pada konsep graf. Subjek penelitian adalah mahasiswa pendidikan matematika, FKIP Universitas Lampung yang menempuh kuliah teori graf pada tahun akademik 2019/2020. Subjek penelitian sebanyak 40 orang yang dipilih dengan teknik sampling total. Data penelitian dikumpulkan melalui tes, yaitu soal bilangan Ramsey. Data dianalisis secara deskriptif dengan 3 tahapan, yaitu reduksi data, penyajian data, dan penarikan kesimpulan. Hasil penelitian menunjukkan bahwa 42,5% mahasiswa memiliki kemampuan berpikir *reversible* dan 57,5% mahasiswa tidak memilikinya Berdasarkan hasil penelitian ini, disimpulkan bahwa sebagian besar mahasiswa tidak memiliki kemampuan berpikir *reversible* pada konsep graf.

### 1. INTRODUCTION

Mathematics is one of the subjects given at all levels of education (from elementary school to college). To successfully learn mathematics, students need to understand the characteristics of mathematics itself, namely having abstract objects and deductive patterns

[1], containing logically ordered structures [2], and global rules [3]. Although mathematics has characteristics that seem difficult when compared to other subjects, learning mathematics can provide many benefits for students, both in everyday life and in thinking skills for students themselves. Benefits for students in everyday life, among others, can improve their ability to think logically [4], systematic [5], critical [6], and creative [7]. Meanwhile, the benefits of thinking skills for students themselves are increasing the ability of problem-solving, reasoning and proof, communication, connections, and representation [8], and an attitude of appreciating the benefits of mathematics [9].

The benefits of mathematics continue to be developed by experts, which are not only based on the characteristics of mathematics in general (daily life and thinking skills for students themselves) but are also developed based on the characteristic aspects of the scope of their mathematics material. Experts and researchers have developed skills based on the characteristic aspects of the sphere of mathematics, such as reflective thinking on the derivative of functions [10] and systems of two-variable linear equations [11]; thinking abstraction in mathematical statistics material [12]; intuitive thinking on material opportunities [13]. The three thinking skills are closely related to the material being studied. For example, the reflective ability is closely related to derived material, namely the nature of set reflection. The nature of set reflection is one of the fundamental materials of derived functions. Abstraction thinking ability is also closely related to the scope of mathematical statistics material, namely the abstraction of random, discrete, and continuous variables. Furthermore, intuitive ability is closely related to the material scope of opportunities, namely the estimation of the probability of occurrence. This description shows that mathematical thinking skills related to aspects of the scope of the material have been researched by experts/researchers, but are still limited (not even available) in certain areas of material, especially in the scope of universities. For example, the ability to think that until now has not been studied by experts is the ability to think is reversible. Even though this reversible thinking ability is often found in both low and advanced mathematics material. Low mathematics material can be found in mathematical operations between addition and subtraction, and in advanced mathematical material, it can be found in the inverse nature of matrices, set complement, and implications for logic.

The scope of mathematics material taught from elementary school to college is very large, and each scope of material has different characteristics. This study is focused on students' reversible thinking skills on the concept of graphs. Reversible thinking is the ability to think in reverse. Thinking in reverse is usually associated with mathematical operations, for example, is  $2 - 1 = 1 - 2$ ?. In this question, if the students answer the question correctly, the students are said to have the ability to think reversibly, and conversely, they do not have the reversible ability if they answer the question incorrectly. This reversible thinking ability needs to be possessed by students because it can stimulate the internalization of mental actions in students' schemata. This mental internalization strengthens students mentally when facing problems [14]. To date, several studies have been conducted to examine reversible thinking skills, such as assessing MTs students' reversible thinking abilities on linear equations [15], and examining students' reversible reasoning in solving mathematics problems in junior high school students [16]. Research on the ability to think reversible is still limited to the school level, and not yet at the college level. The scope of mathematics topics related to reversible thinking is more in college than in school.

Graph theory is a branch of mathematics whose studies include objects called points and sides, and graph theory is also called graph theory. The concepts in graph theory have many benefits in developing science and solving problems of everyday life. Some of the

benefits of the concept of graphs include graphs for social network analysis [17], determination of flight schedules and routes [18], construction of public transportation routes [19], software engineering [20], distribution of rice for the poor [21], creation of test schedules [22], and setting traffic lights [23]. Considering that the concept of the graph has many benefits, students' ability to think about this graph concept is very necessary. One of the thinking skills in the graph concept is the ability to think reversibly. This reversible thinking ability is found in the graph concept, namely Ramsey numbers. The absence of research that explains students' reversible thinking skills on the concept of graphs is a problem that needs attention. Therefore, this study aimed to analyze students' reversible thinking skills on the concept of graphs. The benefit of this research is expected to be one of the references for teachers in conducting lecture evaluations, as well as the development of teaching materials for graph theory courses in universities.

**2. METHOD**

This type of research was qualitative descriptive [24] which aimed to analyze student's reversible thinking skills on the graph concept. The subject of research were students of mathematics education, Faculty of Teacher Training and Education, Lampung University taking the graph theory in the academic year 2019/2020. Research subjects were 40 people selected by the total sampling technique. The research data was collected through tests, namely the question of Ramsey number. The selection of students in higher education as research subjects was based on the consideration that students already have a better knowledge of mathematical operations than primary and secondary school students. Data were analyzed descriptively with 3 stages, namely data reduction, data display, and conclusion. This research used 3 stages of activity, namely preparation, implementation, and analysis. The flow of activity stages can be seen in Figure 1.

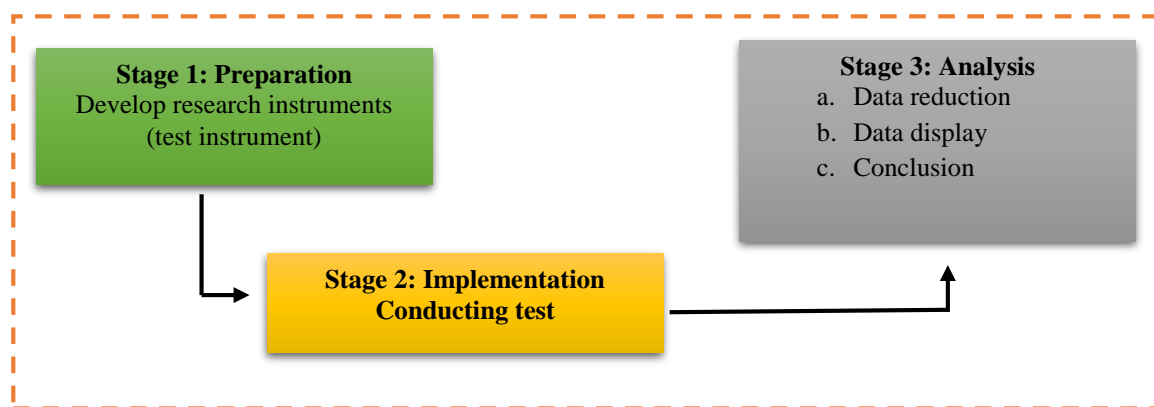


Figure 1. The Flow of Research Stages

The preparation stage is to compile a research instrument, in the form of a test description. Before the test instrument was used in the study, the content validity test was carried out by asking for an assessment from 2 experts (colleague lecturers). The results of the assessment of the two experts who assessed the accuracy of the concept and language stated that the test instrument was feasible. Given, this research is qualitative and the results of the research are not to be generalized, the reliability test or otherwise was not carried out [25]. The following are the test instruments used in the study presented in Table 1.

**Table 1.** The Ramsey Number Test Instrument and Its Relationship with Reversible Thinking Ability

No	Problems	The Relation of Problems with Reversible Thinking Ability
1	What is the definition of the Ramsey number?	The definition of the Ramsey number contains the ability to think reversibly, namely the term $G$ graph and $G'$ graph. Students who can distinguish graph $G$ and graph $G'$ are said to have the ability to think reversibly; and vice versa if you cannot differentiate means not having the ability to think reversibly.
2	Is $r(2,3) = r(3,2)$ ?	To be able to solve the problem, it can be done using formulas and graphs (points and lines). Students who can complete $r(2,3)$ and $r(3,2)$ in the same way are said to have reversible thinking; and vice versa if not the same way means not having the ability to think reversible.

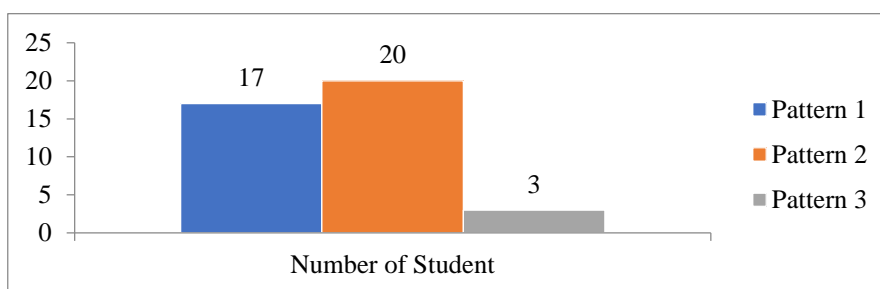
The implementation stage is to conduct tests. The analysis stage is processing the test result data. Data were analyzed descriptively with 3 stages, namely data reduction, data presentation, and conclusion drawing [26]. The data reduction stage is to summarize, encode, and classify data based on the similarity of answer patterns. Furthermore, presenting data from the results of this data reduction in several forms, namely narrative, tables, and figures. The final stage is concluding; to describe students' reversible thinking skills on the concept of graphs. The description of students' reversible thinking skills is based on an analysis of students' answers. The student's answers were assessed by following the assessment rubric in Table 2 below.

**Table 2.** Rubric for Assessment of Student Answers

No	Student Answers	Score
1	Answers 1 and 2 are correct, and the answers to both are consistent (consistent)	3
2	Answers 1 and 2 are correct, but the answers are inconsistent (inconsistent)	2
3	One of the right/wrong answers.	1
4	Both answers are wrong	0

### 3. RESULTS AND DISCUSSION

The result of this research is a description of students' reversible thinking ability on the concept of graph theory. The results of data analysis from the answers of 40 students were carried out by data reduction, which was based on the similar pattern of the answer to question no. 2 (consistent and inconsistent) and associated with answer no.1. Consistent means that students can think reversibly, and vice versa inconsistent means that students do not. have the ability to think reversible. There are 3 patterns of student answers, namely (1) correct and consistent answers, (2) correct and inconsistent answers, (3) wrong answers. The number of student answers for each pattern is presented in Figure 2 below.

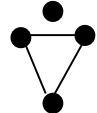
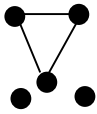



**Figure 2.** Number of Student Answers Based on Answer Patterns

3.1 Answer Pattern 1: Correct and consistent answers

The answer pattern 1 is the student who answers question number 2 correctly and the answers are consistent; meaning that students can think reversible. 17 students answered pattern 1 (42.5%). All students used graphs (point and line) to determine  $r(2,3)$ , but in 2 ways, namely graphs and formulas; with details of 14 students using graphs and 3 students using formulas. The results of the data analysis showed that the correct and consistent students had the correct answer in question 1. The following is an example of correct and consistent answers in determining  $r(3,2)$  using a graph as shown in Table 3.

Table 3. Examples of Correct and Consistent Student Answers (Graphical Method)

Problems	Student Answers
What is the definition of the Ramsey number?	Given any positive integer $m$ and $n$ , Ramsey's number $r(m, n)$ is the smallest positive integer $r$ applicable to any graph $G$ where point $r$ contains $K_m$ as a subgraph $G$ or $G'$ contains $K_n$ with $K_m$ and $K_n$ is a complete graph.
Is $r(2,3) = r(3,2)$ ?	<p>a. <math>r(2,3) = \dots</math>                      Looking for a graph <math>G</math> with a minimal point <math>r</math> whose properties <math>G</math> contains <math>K_2</math> or <math>G'</math> contains <math>K_3</math>.  <math>r = 1</math> (<math>G</math> does not contain <math>K_2</math> and <math>G'</math> does not contain <math>K_3</math>, so <math>r \neq 1</math>)</p> <p><math>G</math>: ●  <math>G'</math>: ●  <math>r = 2</math> (<math>G</math> does not contain <math>K_2</math> and <math>G'</math> does not contain <math>K_3</math>, so <math>r \neq 2</math>)</p> <p><math>G</math>: ● — ●  <math>G'</math>: ● — ●  <math>r = 3</math> (<math>G</math> does not contain <math>K_2</math> and <math>G'</math> contains <math>K_3</math>, so <math>r = 3</math>)</p> <p><math>G</math>: ● ●  <math>G'</math>: ●                        So <math>r(2,3) = 3</math></p> <p>b. <math>r(3,2) = \dots</math>                      Looking for a graph <math>G</math> with <math>r</math> a minimal point whose properties <math>G</math> contains <math>K_3</math> or <math>G'</math> contains <math>K_2</math>.  <math>r = 1</math> (<math>G</math> does not contain <math>K_3</math> and <math>G'</math> does not contain <math>K_2</math>, so <math>r \neq 1</math>)</p> <p><math>G</math>: ●  <math>G'</math>: ●  <math>r = 2</math> (<math>G</math> does not contain <math>K_3</math> and <math>G'</math> does not contain <math>K_2</math>, so <math>r \neq 2</math>)</p> <p><math>G</math>: ● — ●  <math>G'</math>: ● ●  <math>r = 3</math> (<math>G</math> does not contain <math>K_2</math> and <math>G'</math> contains <math>K_3</math>, so <math>r = 3</math>)</p> <p><math>G</math>: ●    <math>G'</math>: ● ●                        So <math>r(3,2) = 3</math></p>

Based on Table 3, it can be seen that there is a similar way between the answers to question no 2 determining  $r(2,3)$  and  $r(3,2)$  using the graphical method. The similarity of these methods shows consistent answers, and this means that students can think reversibly. Likewise, some students answered using the formula to determine  $r(2,3)$  and  $r(3,2)$ , as in Table 4 below.

**Table 4.** Examples of Correct and Consistent Student Answers (Formula Method)

Problems	Student Answers
What is the definition of the Ramsey number?	Given any positive integers $m$ and $n$ , Ramsey's number $r(a, b)$ is the smallest positive integer $r$ applicable to any graph $G$ where point $r$ contains the complete group $(K_a)$ or $G$ contains the complete group $(K_b)$ . The value of $r$ is determined by the formula:  $r(a,b) \leq \binom{a+b-2}{a-1}; \text{ with } \binom{a+b-2}{a-1} = \frac{(a+b-2)!}{(a-1)!((a+b-2)-(a-1))!}$
Is $r(2,3) = r(3,2)$ ?	<p>a. <math>r(2,3) = \dots</math>                      Given that <math>a = 2</math> and <math>b = 3</math>. Then:</p> $\binom{a+b-2}{a-1} = \frac{(2+3-2)!}{(2-1)!((2+3-2)-(2-1))!}$ $= \frac{3!}{1! \cdot 2!} = \frac{6}{2} = 3$ <p>So <math>r(2,3) = 3</math></p> <p>b. <math>r(3,2) = \dots</math>                      Given that <math>a = 2</math> and <math>b = 3</math>. Then:</p> $\binom{a+b-2}{a-1} = \frac{(3+2-2)!}{(3-1)!((3+2-2)-(3-1))!}$ $= \frac{3!}{2! \cdot 1!} = \frac{6}{2} = 3$ <p>So <math>r(3,2) = 3</math></p>

When compared between the answers to the graph method and the formula, it can be seen that the graph method is a more difficult way than the formula method. Because, to answer the problem graphically requires more mathematical reasoning, such as doing trial error (trial and error) which is not certain; while the formula method does not require much reasoning but only accuracy in calculation. Also, if it is related to the answer to question no. 1, it turns out that the students who used the graphical method were the students who correctly answered a question no 1 (the definition of Ramsey's number); On the other hand, students who use the formula method are students who incorrectly answer question no.1. These findings indicate that students who have a good understanding of a definition will have the ability to solve problems well too. This is in line with previous research which stated that understanding mathematical concepts begins with the ability to understand the definition of the concept [27]. These results indicated that students who use mathematical reasoning have more reversible thinking skills than students who use mathematical calculations.

The findings of this research can be used as a recommendation for mathematics learning or lectures that teachers/lecturers need to improve students' mathematical reasoning abilities. Several research results explain that mathematical reasoning ability can

be improved through methods, such as learning with mind-mapping strategies [28], Van Hiele learning model [29], realistic mathematics realistic approach [30], open-ended approach assisted by visual basic applications [31], a problem-based learning model [32], a discovery learning model [33]. The results of this study indicate that students' mathematical reasoning abilities can be improved by implementing various strategies, approaches, or learning models by teachers /lecturers.

**3.2 Answer Pattern 2: Correct and inconsistent answers**

Answer pattern 2 is the student who correctly answers question 2, but the answer is not consistent; meaning that students cannot think reversibly. This inconsistent answer is shown by the different ways students determine  $r(2,3)$  and  $r(3,2)$ , that is, students used the graphical method when determining  $r(2,3)$ , but when determining  $r(3,2)$  they used other means (theorems, tables, formulas). 20 students answered pattern 2 (50%); with details of 12 person ways theorems and 8 person ways tables. The following is an example of student answers using the theorem method as in Table 5.

**Table 5.** Examples of Correct and Inconsistent Student Answers (Theorem Method)

Problem	Student Answers
Is $r(2,3) = r(3,2)$ ?	b. $r(3,2) = \dots$ There is a theorem in Ramsey's number for $r(2, n)$ and $r(n, 2)$ , namely: $r(2, n) = r(n, 2)$ . So for $r(3,2) = r(2,3) = 3$ . So $r(3,2) = 3$

Based on Table 5, it can be seen that students used the theorem which stated that  $r(2,3) = r(3,2)$ . It is not permissible to directly use a theorem to solve problems. Because a theorem may be used to show the truth of another theorem or verify a new theorem. The reason students use the theorem is that they do not understand the rules of using the theorem or are unable to determine  $r(3,2)$  using a graph. These results indicate that determining  $r(2,3)$  is easier for students to do than determining  $r(3,2)$ . Furthermore, examples of student answers using the table method in Table 6.

**Table 6.** Examples of Correct and Inconsistent Student Answers (Table Method)

Problem	Student Answers																				
Is $r(2,3) = r(3,2)$ ?	b. $r(3,2) = \dots$ In addition to the method in point a looking for $r(2,3)$ , the table method can be used: Table of known $r(m, n)$ values: <table border="1" style="margin-left: 20px;"> <tr> <td>m/n</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>2</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>3</td> <td>3</td> <td>6</td> <td>9</td> <td>14</td> </tr> <tr> <td>4</td> <td>4</td> <td>9</td> <td>18</td> <td></td> </tr> </table> So $r(3,2) = 3$	m/n	2	3	4	5	2	2	3	4	5	3	3	6	9	14	4	4	9	18	
m/n	2	3	4	5																	
2	2	3	4	5																	
3	3	6	9	14																	
4	4	9	18																		

Based on Table 6, it can be seen that students used a table which stated that  $r(2,3) = r(3,2)$ . The table only presents the conclusions of the calculations so that the use of the table cannot be justified. The reason why students used this table is because they did not understand the use of tables to solve questions.

3.3 Answer Pattern 3: Wrong answers

Answer pattern 3 is the students’ answer wrong; it means that students cannot solve the  $r(2,3)$  and  $r(3,2)$  questions. There were only 3 students who answered pattern 3 (7.5%). The following is an example of a student’s wrong answer in Table 7.

Table 7. Examples of Wrong Student Answers

Problem	Student Answers
Is $r(2,3) = r(3,2)$ ?	$r(3,2) = \dots\dots$ $r = 1$ (G does not contain K3 and G ‘does not contain K2, so $r \neq 1$ ) G : ● G’ : ●  $r = 2$ (G does not contain K3 but G ‘contains K2, so $r = 2$ ) G : ● ● G’ : ● — ●  So $r(3,2) = 2$

In Table 7, it can be seen that there are student errors in determining  $r(3,2)$ . The location of the error is that students do not try to find  $r(3,2)$  for  $r = 3$ . The student’s answer is correct when determining  $r(3,2)$  for  $r = 1$  and  $r = 2$ , but wrong for  $r = 3$ . It can be seen that students immediately conclude that the valid  $r$  is  $r = 2$ , without trying to calculate for  $r = 3$ . The result of this answer also shows that the student did not try first for  $r = 3$ , and so on. The weakness of students in trying when solving questions cannot be separated from how brave they were when they were at school. At school, students do not want to accept the challenges given by the teacher. The teacher’s challenge is in the form of completing assignments, presentations, performance, or examinations [34]. According Permendikbud No. 81A about Curriculum Implementation states that trying activities is one part of the 5 stages of the scientific approach in the 2013 curriculum [35]. Thus, this trial activity already exists in the 2013 curriculum learning activities and has been commonly done by students while in previous schools. Then, what the teacher needs to do is improving students attitudes to dare to try in various activities during learning since school.

The attitude to dare to try needs to be instilled since students are in schools, such as being brave to try, ask questions, or complete questions/assignments. Several studies have recommended these methods. Ways to increase students’ courageous attitudes are teachers implementing strategies, models, or cooperative script learning approaches [36], cooperative type STAD [37], media-assisted think pair share [38], problem-posing and problem-solving approaches [39]. This study explains that teachers can use various ways of increasing courage in learning, namely teachers using various strategies, models, or learning approaches that are tailored to the courageous attitude they want to develop.

The important thing about the ability to think reversible based on student answers is the suitability of the correct answers between questions no.1 and no.2. That is, if the answer is correct in question no.1 then the answer will be correct and consistent in no.2, but if the answer is wrong in question no.1 then answer no.2 is correct and inconsistent. These results indicate that the cause of inconsistent answers is that students are unable to write a definition of a concept. These results also indicate that the ability to write a definition (define) a concept can have a positive effect in solving problems related to the application of the concept. Therefore, the ability to write the definition of a concept is very basic, and the first step in writing the definition of a concept correctly is to understand the definition of a concept [40]. A concept in mathematics is manifested in the form of a definition, and



this definition becomes a barrier/scope that distinguishes a concept from other concepts. The formation of this definition is the result of conventions (agreements) from mathematicians and applies globally which are compiled using formal and standard language. However, often the use of formal and standard language makes it difficult for students to understand the definition because it requires reasoning in understanding it (mathematical reasoning). This mathematical reasoning can be trained by cultivating an attitude to dare to try or not to be afraid to do wrong math activities during learning, such as the attitude to have the courage to argue, ask questions, solve problems. This description is in line with the results of previous research that the cause of low mathematical reasoning is that students do not dare to ask opinions, do not dare to answer questions, and do not dare to present answers to the questions they are working on. Furthermore, to improve students' mathematical reasoning is to provide opportunities for students to explore their knowledge and investigate various problems [41]. So, mathematical reasoning and the attitude to dare to do mathematical activities are important elements in improving reversible thinking skills.

#### 4. CONCLUSION

Reversible thinking is reverse thinking that needs to be developed in mathematics learning in elementary school students to college students. Reversible thinking skills in students need to be analyzed because this can provide a real picture of students' mathematical abilities at the previous level. The results showed that 42.5% of students had reversible thinking skills, and 57.5% of students did not. To improve students' reversible thinking skills, it is recommended that lecturers develop mathematical reasoning and attitudes to dare to carry out activities in a class by implementing various strategies, approaches, or learning models. So it can be concluded that most students cannot think reversibly on the concept of graphs, especially Ramsey numbers.

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