INDONESIAN JOURNAL OF SCIENCE AND MATHEMATICS EDUCATION
https://ejournal.radenintan.ac.id/index.php/IJSME/index
08 (1) (2025) 54-65
E-ISSN: 2615-8639
DOI: 10.24042/ijsme.v8i1.23753
March 2025

Exploring students' concept images of fractions: A hermeneutic phenomenology study

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Article history:

Submitted: August 8, 2024
Accepted: January 29, 2025
Published: March 30, 2025

Keywords:

concept image, constant comparative method, fraction concept, hermeneutic phenomenology

ABSTRACT

Understanding the concept of fractions is a crucial aspect of mathematics learning; however, students often develop diverse mental representations of this concept. This study aims to describe students' concept image in understanding fractions, particularly their interpretation. A qualitative approach with the hermeneutic phenomenology method was employed, where data were collected through written tests and interviews with eight selected students. Data analysis was conducted using the Constant Comparative Method (CCM), with validation through triangulation and peer debriefing. The findings reveal that students possess various concept images of fractions, including fractions as separate numbers, as numbers consisting of a numerator and denominator, as a single entity, as a part of a divided whole, and as numbers related to other mathematical concepts. However, students' understanding of their concept images remains limited, potentially leading to errors in solving fraction problems. The diversity of concept images reflects differences in students' learning experiences, highlighting the need for more varied and contextual teaching approaches in presenting fraction concepts. These findings imply the need for more effective instructional strategies to optimize students' understanding and minimize future learning barriers.

Eksplorasi citra konsep siswa tentang pecahan: Sebuah studi fenomenologi hermeneutik

Kata Kunci:

citra konsep, metode perbandingan konstan, konsep pecahan, fenomenologi hermeneutik

ABSTRAK

Pemahaman konsep pecahan merupakan salah satu aspek penting dalam pembelajaran matematika, namun siswa sering kali memiliki representasi mental yang beragam terhadap konsep ini. Penelitian ini bertujuan untuk mendeskripsikan concept image siswa dalam memahami konsep pecahan, khususnya dalam aspek interpretasinya. Penelitian ini menggunakan pendekatan kualitatif dengan metode fenomenologi hermeneutik, di mana data dikumpulkan melalui tes tertulis dan wawancara terhadap delapan siswa terpilih. Analisis dilakukan menggunakan Constant Comparative Method (CCM), dengan validasi melalui triangulasi dan peer debriefing. Hasil penelitian menunjukkan bahwa siswa memiliki beragam concept image tentang pecahan, termasuk pecahan sebagai bilangan yang terpisah, bilangan dengan pembilang dan penyebut, angka tunggal, bagian dari suatu keseluruhan yang terbagi, serta bilangan yang terkait dengan konsep lainnya. Namun, pemahaman siswa terhadap concept image yang mereka miliki masih terbatas, sehingga berpotensi *menyebabkan kesalahan dalam menyelesaikan soal pecahan. Keanekaragaman concept image ini mencerminkan perbedaan pengalaman belajar siswa, yang menegaskan perlunya pendekatan pembelajaran yang lebih variatif dan kontekstual dalam menyajikan konsep pecahan. Temuan ini memberikan implikasi bagi pengembangan strategi pembelajaran yang lebih efektif guna mengoptimalkan pemahaman siswa dan mengurangi hambatan belajar di masa depan.*

Contribution to the literature

This research contributes to:

- Investigating the difference of concept image on students' fraction concept.
- Enriching the literature on students' concept image on the concept of fractions.
- Developing a didactical design based on students' concept images for learning fraction concepts.
- Offering empirical evidence of students' concept images to teachers shows that there is still a gap between concept definition and students' concept images.

1. INTRODUCTION

In mathematics, fractions have an important role as one of the concepts that students must master [1]–[4]. This important role is in line with using fractions in life, not only related to mathematical concepts but also impacts other disciplines [5]–[7]. Given the importance of the concept of fractions in mathematics learning in Indonesia, the concept of fractions is one of the materials that must be learned by Indonesian students at the elementary and junior high school levels. At the junior high school level in Indonesia, the concept of fractions is studied together with the concept of integers with basic competencies, including the ability to explain and calculate the operation of the fraction so that it can solve the problems related to fractions in a context.

The fraction concept is one of the concepts studied in mathematics, and it is unique. In mathematics, the fraction concept is denoted as $\frac{a}{b}$, which is interpreted in several ways. A fraction can be interpreted in five interpretations: (1) part-whole, namely as a whole (either discrete or continuous) which is divided into several equivalent parts; (2) measure, referring to the use of the unit fraction in determining the distance from the starting point; (3) division (quotient) which is related to the problem of dividing operations between two integers; (4) ratio, indicating the index of comparison between two quantities; and (5) operator, namely fraction as something that works and modifies something [8], [9]. Thus, the variety of interpretations of fractions shows that the concept of fractions can be viewed not only from one aspect but also can be used in various problem contexts.

However, learning the concept of fractions in the early stages provides a challenge; fractions involve a new type of number (rational numbers), and fractions represent the relationship between two numbers [10]. At the junior high school level, the concept of rational numbers is a new thing because students are still limited to whole numbers, so in the process of learning the concept of fractions, teachers are required to be able to link the concept of whole numbers that have been learned with the concept of fractions. With this challenge, Simon et al. identified four limitations related to the interpretation of the meaning of fractions from students who do not understand the concept of fractions so that it can have an impact on the emergence of difficulties in students [10]. The limitations referred to by Simon *et al.* [10] are: (1) lack of fraction as quantity, where students only understand fractions as an arrangement but do not understand fractions as a quantity, size, or amount, which can have an impact on the meaninglessness of fractions; (2) fraction as two numbers, namely students' limitations in seeing fractions as two numbers, where students only understand fractions as pairs of numbers without a certain relationship between them; (3) limited part-whole; and (4) difficulty understanding referent units, namely the development of the meaning of fractions as part of the totality at hand.

The limitations, as stated by Simon *et al.* [10] above, are that understanding the meaning of fractions can provide stumbling blocks for students in learning fractions. To gain understanding, students must be involved in learning, including subjective mental actions that provide opportunities for students to gain learning experiences [11], [12]. McNeil and Alibali stated that the learning experience obtained by students will form a new understanding that provides an overview of concepts that can be repeated and consistent for a relatively long period [13]. On the other hand, some research results on the concept of fractions show that there are still errors experienced by students in understanding the concept of fractions, including using them to solve relevant problems, including junior high school students [4], [14]–[18].

The errors experienced by students indicate that the concept of fractions, including the meaning of fractions, still needs to be improved. Bruner states that knowledge formed based on initial experiences that do not follow the information found can negatively impact learning difficulties at the beginning [13]. In fact, the learning experiences obtained by students can be an important element in constructing the knowledge obtained because these experiences are a form of a concept image [19].

Concept image describes the total cognitive structure, properties, processes, and processes and cognitive structures associated with a concept [20]. In contrast to the concept definition, a concept image is obtained through a series of cognitive processes, so according to Vinner, a concept image can be a visual representation of a concept or in the form of a collection of impressions or experiences gained [21]. Thus, the concept image of students is unique for each individual and varies between individuals [22]. In addition, the concept image has a cumulative nature allows it to change according to time, individual maturity, and new stimuli and experiences obtained by students [23], [24].

Several studies on fraction concepts have been conducted, especially in Indonesia [3], [4]. In Roni *et al.* [3], the research focuses on developing a Learning Trajectory (LT) to help students understand fractions' division in grade 5. Through the LT, Roni *et al.* [3] specifically explored the concept of division of fractions. Meanwhile, Unaenah *et al.* [4] explored learning obstacles that occur in students, especially related to epistemological obstacles. These researchers are limited to exploring the learning obstacles that occur to students in the concept of fractions. However, students' concept image of the concept of fractions is another important thing that must be investigated to provide future information on how students' understanding of the concept of fractions impacts how the learning process should be carried out. In learning, students use concept images to understand a concept or solve a problem. Thus, knowing students' concept images, including fractions, will be very important. Concept images that deviate from mathematical ideas or concepts can hurt student understanding, resulting in low student learning outcomes. Therefore, the main purpose of this research is to explore and deeply understand students' concept image of fraction material. The study conducted is expected to provide an overview of the concept image that can be used as a foundation in preparing the learning design to be carried out, especially on the concept of fractions. Essentially, this research aims to build upon previous work by focusing on the cognitive framework students employ when dealing with fractions, rather than solely on learning trajectories or obstacles.

2. METHOD

2.1 Research Design

This study aimed to investigate students' concept image of fractions, namely the definition of fractions. This research was conducted using a hermeneutic phenomenological approach. This approach was chosen because it allowed researchers to understand the subjective experiences of individuals in-depth and capture the meaning from students' perspectives regarding the definition of fractions. After receiving ethics approval, the research was conducted. In this context, hermeneutic phenomenology helps explore how students understand the concept of fractions based on their own experiences and how they shape and interpret the concept in the learning process. Koh and Owen stated that qualitative research emphasizes how things happen and focuses on how a person understands and interprets experiences [25].

Meanwhile, phenomenology refers to disclosing a person's perception of experience and meaning [26]. In the context of qualitative research, Grbich states that phenomenology is an approach used to understand the meaning and essence of experience [27]. However, phenomenology cannot be understood without the meaning of phenomenology, so hermeneutics is needed [28]. Hermeneutics refers to the interpretation of meaning [29], so hermeneutic phenomenology is a paradigm in describing and interpreting experience and the study of meaning and meaning (interpretation of meaning) related to the experience gained [30], [31]. The following is a flowchart of the research process carried out in Figure 1.

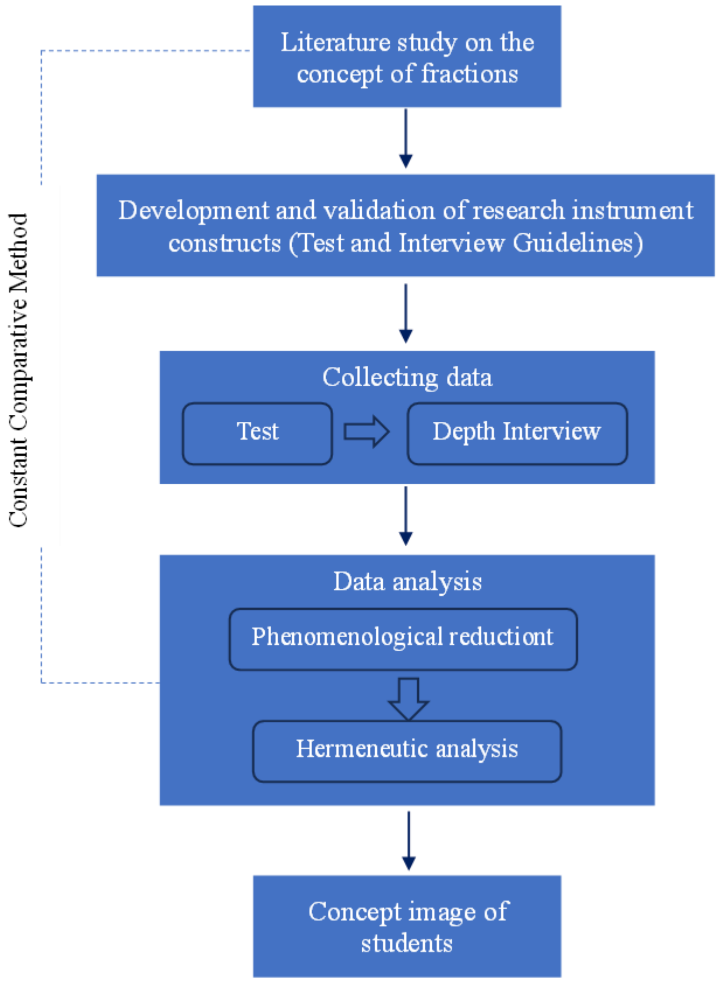

Figure 1. Flowchart of Research Process

2.2 Participants and Data Collection

The participants in this study were 29 seventh-grade junior high school students in Surakarta. After receiving consent from parents and students, the research was conducted. The 29 students were asked questions related to the meaning of fractions. Furthermore, based on the results of the written answers given, seven students were selected to be interviewed further by the researcher so that the concept image of the understanding of fractions was obtained. The selection of the seven students was based on the consideration of answers and communication skills based on recommendations from the mathematics teachers of these students. In the aspect of student answers, the quality of understanding is considered, which is selected based on the ability of students to answer questions related to the concept of fractions. The main considerations are answers that show depth of understanding, interesting conceptual errors, or diversity of thinking. In addition, the diversity of answers was also considered.

Furthermore, in terms of communication skills, students' ability to express their thoughts, both orally and in writing, was considered. In this study, the data collected were qualitative data from task-based interviews. The main instrument in this research is the researcher himself, and the auxiliary instrument is an unstructured interview guideline based on student answers and mathematics tests. In this study, the object of research is the description of students' concept image in understanding the meaning of fractions. In this study, the object of research is the students' conceptual description of understanding the meaning of fractions. The math test used is a description test with indicators in the form of Explain the basic concept of fraction definition. The test has gone through internal validation through expert judgment.

2.3 Data Analysis

The main purpose of phenomenological analysis is to present an analytical and in-depth description of the phenomenon under study, namely, students' deep experience in understanding the meaning of fractions. This study used data analysis techniques using the Constant Comparative Method (CCM). The constant Comparative Method is a qualitative data analysis based on comparing data from various previous sources [32]. Hendriyanto *et al.* [33] state that the data analysis process using CCM consists of four stages: (1) Coding – assigning codes to different elements based on content; (2) Comparison – analyzing codes across sections to identify similarities and differences; (3) Conceptualization – grouping codes into categories and developing formulations; and (4) Theoretical Saturation – further refining and analyzing categories until no new concepts emerge.

Checking the validity of data in qualitative research was crucial to ensuring accuracy and reliability. In this study, triangulation and peer debriefing methods were used. Triangulation was carried out using source triangulation, comparing student answers and interview results, including comparing answers between students. Meanwhile, peer debriefing was carried out by involving other peer researchers to review the research process and results.

3. RESULTS AND DISCUSSION

This section presents the results of students' concept images for the concept of fractions. The concept of fractions is in the form of understanding of fractions (meaning of fractions), including how students interpret the meaning of fractions. Table 1 presents the meaning of fractions from 29 participants, which are then grouped based on the similarity of the meaning conveyed. Essentially, the analysis focuses on how students conceptualize and articulate their understanding of fractional values.

Table 1. Participants' Answers to the Meaning of Fractions

No	Concept Image	Code	Frequency
1	Fraction refers to the number being split apart	α	3
2	A fraction is a number consisting of a numerator and denominator	β	3
3	Fraction as a single number	γ	5
4	One whole piece that is divided into equal parts	δ_1	7
5	One whole piece that is divided into several parts	δ_2	8
6	Numbers used for counting questions and promoting numbers	θ	3

Based on Table 1, there are six meanings of fractions understood by students that show the concept image of students, where two of them, δ_1 and δ_2, have similar meanings. Still, the meaning of δ_2 is not specifically stated whether the intended parts are equal. In addition, Table 1 shows that most students interpret fractions as δ_2, then δ_1, and γ. Based on the similarity of meaning in Table 2, it is found that the students' concept image of the meaning of fractions from 29 students is diverse, where most students interpret fractions as a part of the set of real numbers as presented in the meaning of α, β, γ, and θ. This indicates a spectrum of student understanding, ranging from basic part-whole interpretations to more abstract numerical representations.

Lamon stated that the meaning or interpretation of fractions consists of five meanings, namely parts of whole, quotients, ratios, operators, and measures [8]. The meaning of fraction in α and β refers to a fraction as a number expressed as $\frac{a}{b}$, with an as the numerator and b as the denominator. The existence of the fraction meaning refers to the fraction notation often used, namely $\frac{a}{b}$. The meaning of fraction as a number broken down or consisting of a numerator and enumerator is in line with the research results presented by Isnawan *et al.* [16]. One of the meanings of fractions understood by students, in the categories of students with high, medium, and low abilities, is that fractions are interpreted as numbers consisting of numerators and denominators [16]. Furthermore, the meaning of fractions is grouped as the meaning of fractions in the ratio interpretation [16].

Interview with 1st Student

Researcher	:	*What do you think a fraction is?*
Student	:	*Fractions are two numbers organized by denominator and numerator* (β)
Researcher	:	*Why can you explain the meaning or definition of a fraction as two numbers consisting of a denominator and a numerator?*
Student	:	*Because of that memory.*

Interview with 2nd Student

Researcher	:	*What do you think fractions are? What are they like?*
Student	:	*Fractions are numbers that are broken down (being split apart) to be smaller than the main value* (α).
Researcher	:	*A number is broken down to be smaller than the main value. What does it mean to be broken down to less than the main value? You can illustrate the statement on paper. You explained what it means for a number to be broken down to less than its primary value. What is the main value, and can you explain what is broken down into smaller values?*
Student	:	*For example, 20% of 100 can be split into 20 per 100.*

Van de Walle et al. state that all fractions are ratios. However, the reverse meaning does not apply [34]. Students' concept of fraction as a ratio is entirely based on the form or notation, $\frac{a}{b}$, which is understood as a comparison between a and b. According to Clark *et al.* [35], this condition is illustrated in the relationship that fraction is a subset of ratio, which is stated in the following Venn diagram.

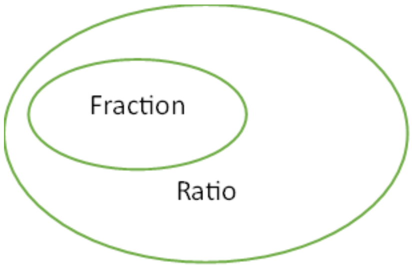

Figure 2. Fraction as a Subset of Ratio

Students' interpretation of the concept of fractions, as presented in Figure 2 above, shows a generalization of the meaning of fractions. The meaning of fractions as a ratio, as in Figure 2 above, is often found in mathematics learning [35]. Tall and Vinner stated that the concept image formed is a mental picture connected to the concept, including the processes associated with the concept [20]. The existence of concept images in α and β for a fraction as a ratio occurs due to previous learning experiences or previous concepts. The meaning of fraction as a ratio cannot just happen, but this is due to learning experiences, including the teacher's influence. The meaning of fraction as a ratio is one of the popular choices teachers use in teaching fractions [35]. However, Clark et al. underlined that fraction as a ratio should only refer to the notation used to express the ratio itself [35]. This is because the ratio is a relative comparison in the sense of multiplication.

Furthermore, the meanings of fractions that students have as concept images are γ and θ. Students' concept image on γ and θ shows that a fraction is a number; precisely, a fraction is a number or symbol of a number. In this case, the notation refers to rational number notation. In this context, the $\frac{a}{b}$ notation is not interpreted as a separate part but as a whole unit that shows a certain value. Grossnickle and Reckzeh stated that the development of fractions is based on the form of fractional numbers, which is a synonym of rational numbers $\frac{a}{b}$ [36]. Even some experts, such as Brumfiel [37] and Willerding [38], straightforwardly stated that a fraction is a number (written notation) and a rational number is a concept. Thus, the concept images γ and θ found in students refer to the notation of a number value.

Interview with 3rd and 4th Student

Researcher	:	*What do you think a fraction is?*
Student 3	:	*Fractions are numbers usually used to calculate a question and promote a number* (θ).
Student 4	:	*Fraction is a fractional number of a single number* (γ).

Based on the interview results above, it is revealed that students interpret fractions as a symbol of a number of a number (value). However, students could not further explain the meaning of the statement. This shows that students only understand fractions as a representation of a number in the form of numbers. Concept images γ and θ refer to the interpretation of fractions as part of a measure. In this case, if the fraction is interpreted as a measure, then a fraction can be understood as a single number, including if the form $\frac{a}{b}$ with $a > b$ becomes a possible and reasonable thing [10]. In addition, fraction as the measure will bring students to a further interpretation of fractions, namely as a size relationship not only limited to a part-whole relation [39]. However, students still have not fully mastered the meaning of fractions as a measure [10]. This indicates that fractions have difficulties, including that students experience a lack of fractions as quantity, as stated by Simon *et al.* [10].

The next students' concept images of fractions are δ_1 and δ_2, where both are distinguished only by the result of the division. Concept image δ_1 states that a fraction is a whole part divided into equal parts, while δ_2 does not state whether the parts are equal. This difference highlights a crucial understanding of the fundamental definition of fractions.

Interview with 5th Student

Researcher : *What do you think fractions are?*
Student : *One whole part is divided into several equal parts* (δ_1).
Researcher : *How many are there? How many parts can it be divided into?*
Student : *Whatever, the important thing is to be the same.*

Interview with 6th Student

Researcher : *What do you think fractions are?*
Student : *A whole part that is divided into equal parts* (δ_1).
Researcher : *How many parts are there? Should it be divided into two or three or whatever?*
Student : *Divided by three.*
Researcher : *Can you divide it with others as well?*
Student : *Divided by four.*
Researcher : *Is there anything you can't do, such as dividing by more than 10?*
Student : *Can do two, can do four.*
Researcher : *Can it be up to you? For example, I divide this one unit into 1000.*
Student : *Not allowed.*
Researcher : *Why?*
Student : *Because one can only be a few parts.*
Researcher : *Yes, how many parts are there?*
Student : *Five.*
Researcher : *How much is it not allowed?*
Student : *More than 10, probably.*

Interview with 7th Student

Researcher : *What do you think fractions are?*
Student : *A fraction is one part of a whole divided into several parts* (δ_2).
Researcher : *A whole part that is divided into several parts. How many parts is it divided into? It's up to you to divide it into two or how many parts.*
Student : *Whatever.*
Researcher : *Does each part have to be the same or different?*
Student : *Different.*
Researcher : *Can it be a shame?*
Student : *Probably.*

The interview results on 5th and 6th students stated that a fraction is a whole part divided into several equal parts (δ_1). However, the 6th student found that the statement "several parts" is still limited to a certain number, namely a maximum of 10. This understanding refers to an illustration. For example, if given a circle-shaped object, the circle can only be broken down or divided into 10 equal parts. Unlike the 5th student, the term "several parts" is not limited to a certain number of parts, but the 5th student underlines that they must be equal. In this case, the 6th student's concept image still limits that a part can only be divided into a maximum of 10 equal parts, so that if illustrated, the fraction is denoted by $\frac{a}{b}$, with $1 \leq b \leq 10$, while the 6th student does not limit the value of b in the form of the fraction. The 7th student expressed the fraction as a whole part divided into several parts (δ_2), with the term "several parts" being the same or different, but did not further explain the conditions for each.

The interpretation of fractions as a whole divided into parts refers to the interpretation of fractions as parts of wholes or parts of sets. The part-whole refers to a whole divided into several equivalent parts [9]. Behr et al.'s opinion refers to the concept image as understood by the 5th and 6th students. Other opinions from Hecht and Yagi [40] and Kieren [41] state that part-whole or partition refers to how many objects or collections are represented by a fraction symbol. Thus, this concept image refers to the relationship between the number of parts and the total number of whole parts that have been divided.

Referring to Table 2, concept images δ_1 and δ_2 are the most concept images owned by students, namely 15 students out of 29 students. This shows that students' learning experience in interpreting fractions from the learning process is carried out by introducing fractions as parts of a whole, both by the teacher and in the textbook. This follows what Lamon [8] revealed: that the part-whole meaning is the most common concept introduced in the learning process. However, Lamon underlined that in the learning process, understanding the part-whole concept can lead to weak conceptual understanding [8]. In line with that, the part-whole concept can be difficult for students due to students' limitations in understanding the part-whole concept [10]. In this concept, students see that a fraction is a full collection of parts that make up the whole. Still, according to Stafylidou and Vosniadou [42], it will lead to an inability to conceptualize the wrong fraction. This is the case with the 6th student and the 7th student above, where the 6th student still limits the part in question, while the 7th student does not state it as the same part.

The diversity of students' concept images on the meaning of fractions shows the diversity of experiences gained by students during the learning process. However, the concept images that students have are not fully utilized by students in solving the given problems. Vinner stated that concept image represents what is seen as related to a particular concept, so it can be raised as a reaction to a question [21]. Thus, the formation of concept images in students will be better if the learning experiences provided are also correct so that they can direct students to the correct informal definition. However, the research that has been done is still limited to the meaning of fractions owned by students as a concept image.

In contrast, other fraction concepts, such as their properties and operations, have not been studied. Therefore, the next research related to the concept image on the properties or calculation operations of fractions can be done. In addition, the results obtained are expected to be used as references in developing a didactic design on the fraction concept.

According to the research results, there is a diversity of students' concept images of fractions in the form of fraction definitions. However, the results also reveal that the students themselves do not fully understand the students' concept images. This raises the potential for learning obstacles in the future when students are faced with problems that have different contexts. Thus, the research results obtained imply the need for more diversity in the concept of fractions, especially in the definition of fractions, including various contexts in the process of presenting the concept of fractions. Nevertheless, the researcher realizes the limitations of the research we conducted. The research is still limited to students from one school, so the results obtained provide the possibility of other concept images when the respondents are expanded. For this reason, we suggest that future studies expand the range of respondents involved in the research to make the concept image obtained more comprehensive. In addition, we suggest expanding the concept of fractions to be studied, not only limited to the definition but also to a wider scope of fraction concepts following curriculum development.

4. CONCLUSION

The fraction concept, especially the interpretation of the meaning of fractions, is very diverse. Based on the results obtained, students' concept image of the fraction concept for the meaning of fraction is categorized into five: fraction as a number being split apart, a number consisting of numerator and denominator, and fraction as a single number. This whole part is divided into several equal and/or unequal parts, and a fraction is related to a number. However, referring to the interpretation of the meaning of fractions according to Lamon in 2020, students' concept images are categorized into three, namely fractions as ratio, measure, and part-whole. The diversity of students' concept images on the concept of fractions is inseparable from the learning experience obtained by students, so the learning process and the correct mastery of the concept of fractions by the teacher can have a positive impact on students' concept images. This implies an opportunity for teachers to know the needs of students in understanding the concept of fractions so that they can design constructive learning in the concept of fractions. In addition, it is hoped that it can be an opportunity for the teachers to find out the needs of students in understanding the concept of fractions so that they can design constructive learning in the concept of fractions. In addition, future research is expected to expand the study of students' concept image on the concept of fractions, which not only focuses on the definition of fractions but can also be studied on other fraction concepts, such as fraction operations.

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