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Analyzing junior high school students' mathematical reasoning in problem-solving: A focus on the Treffinger-oriented learning approach

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ABSTRACT

Mathematical reasoning is a fundamental ability that helps students solve various problems. This study aims to analyze students' mathematical reasoning in solving word problems using the Treffinger approach. This qualitative research with a phenomenological design involves researchers as the main instrument and uses mathematical reasoning tests and interview guides. The results showed that students tend to reread problems and annotate them before presenting mathematical ideas, indicating the positive influence of the Treffinger approach. Students use scratch sheets to justify answers and check the validity of their problem-solving steps, although some students still make mistakes in previous stages. The results show that the Treffinger approach encourages students' reasoning abilities in solving problems, from presenting the problem to checking the validity of the answer. However, students' understanding of mathematical concepts is also essential in problem-solving. This research implies an alternative solution for teachers to improve students' mathematical reasoning abilities through classroom learning.

Analisis penalaran matematis siswa SMP dalam pemecahan masalah: Fokus pada pendekatan pembelajaran berorientasi *Treffinger*

Kata Kunci:

Penalaran matematis
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Penyelesaian masalah
Treffinger

ABSTRAK

Penalaran matematis adalah kemampuan fundamental yang membantu siswa dalam memecahkan berbagai masalah. Tujuan penelitian ini adalah menganalisis penalaran matematis siswa dalam menyelesaikan soal cerita menggunakan pendekatan *Treffinger*. Penelitian kualitatif dengan desain fenomenologi ini melibatkan peneliti sebagai instrumen utama dan menggunakan tes penalaran matematis serta pedoman wawancara. Hasil penelitian menunjukkan bahwa siswa cenderung membaca ulang soal dan memberi notasi sebelum menyajikan gagasan matematika yang menunjukkan pengaruh positif dari pendekatan *Treffinger*. Siswa menggunakan lembar coretan untuk membenarkan jawaban dan memeriksa kesahihan langkah-langkah pemecahan masalah, meskipun beberapa siswa masih melakukan kesalahan pada tahap sebelumnya. Hasil menunjukkan bahwa pendekatan *Treffinger* mendorong kemampuan penalaran siswa dalam memecahkan masalah, dari penyajian masalah hingga pemeriksaan kesahihan jawaban. Namun demikian, pemahaman konsep matematika siswa juga penting dalam pemecahan masalah. Penelitian ini berimplikasi menjadi solusi alternatif untuk meningkatkan kemampuan penalaran matematis siswa melalui pembelajaran di kelas.

1. INTRODUCTION

Mathematics is a discipline characterized by a unique structure and systematization [1]. It is a collection of numbers and formulas and an activity involved in active, dynamic, and generative problem-solving processes. Moreover, mathematics plays a role in developing reasoning skills, critical thinking attitudes, objectivity, and openness [2]. Understanding and mastering mathematics are crucial for students, especially when facing the rapid pace of change in science and technology. Mathematical reasoning plays a crucial role in an individual's thinking process, particularly when learning mathematics [3], [4]. This is because students must be able to communicate their mathematical ideas or concepts clearly and effectively. Mathematical reasoning involves formulating and understanding mathematical statements, engaging in logical thinking, and reaching valid conclusions based on available mathematical information [5], [6].

Mathematical reasoning is essential in solving mathematical problems [7], [8]. People with good mathematical reasoning skills can better identify problems, plan problem-solving strategies, and reach appropriate solutions [9]. This implies that mathematical reasoning is the foundation for deep mathematical understanding and the ability to solve problems in creative ways. The better someone's mathematical reasoning skills are, the better their understanding and mastery of mathematics will be. This ultimately impacts the mathematical learning outcomes achieved by the individual [10]. Therefore, teachers need to ensure that students acquire knowledge and skills in mathematical reasoning, as this will equip them with strong intellectual abilities. Consequently, they will become competent and critical thinkers, ready to face mathematical challenges and problems in everyday life.

Students should be able to use mathematics as a means of communication, both in the classroom and everyday life [11], [12]. This means that mathematics is not just about following procedures or examples without understanding; instead, it becomes a language that students can use to express their ideas clearly [13], [14]. However, if students do not develop reasoning skills well, mathematics will only feel like a meaningless sequence of mechanical steps. Therefore, enhancing students' mathematical reasoning abilities is imperative, especially considering the current situation in the education world, where data shows that students' mathematical reasoning skills still need to improve. Many students perceive mathematics as difficult and uninteresting [15], whereas only a few genuinely find satisfaction in understanding mathematics as a discipline that trains their reasoning abilities. Further efforts are needed to change this perception and develop a deeper understanding of mathematics so that students can not only solve mathematical problems but also apply their reasoning skills in various aspects of everyday life.

There are various ways to enhance students' reasoning abilities in the context of mathematics learning. One practical approach involves teachers stimulating students to think logically [16]. This can be done by providing various problems relevant to students' daily situations and transforming them into mathematical forms. Thus, students can relate mathematical concepts to their experiences in daily life, which helps them understand the meaning of mathematics more deeply. Students can also develop their reasoning abilities by learning to analyze problems based on steps that align with mathematical theorems and concepts [17]. This involves a deep understanding of mathematical concepts and how to apply them in real-life situations. This process enables students to develop critical thinking skills and formulate valid solutions.

Furthermore, one method that can be used in mathematics learning is the Treffinger learning model. This model can be an effective tool for developing students' reasoning abilities. The Treffinger model emphasizes problem-based approaches and creative problem-solving in mathematics learning [18], [19]. By presenting exciting problems and challenges that require creative thinking, students can engage in a more profound mathematical reasoning process. To develop students' reasoning abilities in mathematics learning, holistic and diverse approaches such as those mentioned above can be crucial components in efforts to enhance students' understanding and skills in mathematics [20].

Based on the discourse above, this article aims to analyze students' mathematical reasoning abilities in solving mathematical word problems using the Treffinger approach. Using word problems allows a more accurate understanding of how students can apply mathematical reasoning in real-world situations, an essential aspect of mathematical problem-solving. This identification will provide valuable insights into aspects needing improvement in mathematics teaching. The significance of this research also lies in its ability to measure the effectiveness of the Treffinger learning method in enhancing students' mathematical reasoning abilities. This information will assist teachers and curriculum developers in designing more relevant and supportive learning strategies in the context of everyday mathematical problem-solving.

Various previous studies on mathematical reasoning abilities have been conducted. Rusmianingrum & Setyaningsih [21] and Febriandi [22] investigated the mathematical reasoning abilities of junior high school students in solving mathematical word problems. At the same time, Setialesmana [8] analyzed students' mathematical problem-solving abilities based on Gagne's problem-solving steps. These various studies assessed students' mathematical reasoning abilities using various mathematical problems in situations or teaching methods typically employed by teachers. Given the heterogeneity of learning objectives, the teaching methods used do not facilitate students' mathematical reasoning abilities. Consequently, some of these studies need a specific discussion involving the Treffinger-oriented learning approach in research on mathematical reasoning. Therefore, this research aims to analyze students' mathematical reasoning abilities in solving mathematical word problems using the Treffinger approach. Considering previous studies and the research objectives, the novelty of this study lies in integrating two essential aspects of mathematics education: mathematical reasoning and the Treffinger teaching method. Through this approach, this research provides a more comprehensive understanding of how students can develop mathematical reasoning abilities in the context of everyday mathematical problem-solving. Thus, this research can contribute to mathematics education and develop more effective learning methods.

Contribution to the literature

This research contributes to:

- Providing an overview of the effectiveness of the Treffinger teaching method in enhancing students' mathematical reasoning abilities.
- Identifying specific issues in mathematics education, particularly in problem-solving.
- Developing mathematical education theory by exploring and measuring students' mathematical reasoning abilities in the context of mathematical problem-solving using the Treffinger method approach.

2. METHOD

To analyze mathematical reasoning abilities in depth, qualitative research was chosen as the research approach. This approach helps researchers describe existing realities as accurately as possible [22]. In this approach, data is analyzed, focusing on understanding the context, processes, and meanings contained within it [23]. Therefore, this qualitative approach allows researchers to understand more deeply how students face and solve mathematical problem-solving challenges with this specific method. Subsequently, this research utilized phenomenology as the research design. Phenomenology is used to uncover an individual's experiences and then represent them [24], in this case, the mathematical reasoning abilities of students in solving word problems. The choice of a phenomenological research design is based on the research objective of conducting an in-depth analysis of students' mathematical reasoning abilities through their experiences, including their learning methods. Thus, this research design will enable researchers to answer research questions with the necessary depth to understand the phenomenon under investigation.

The research instruments utilized were both primary and auxiliary instruments. The researchers' role was the primary instrument having complete control over the research implementation. Then, the auxiliary instruments in this research include mathematical word problems, observation guidelines, and interview guidelines. These word problems are used to assist researchers in analyzing students' mathematical reasoning abilities. The problems comprise integer operation materials as the content and structure based on mathematical reasoning indicators adopting the theory from Hendriana [25].

Mathematical reasoning test instrument:
Shelma wants to send several items (A, B, C) to a certain destination using a cargo service. The items have weights of 1500 g, 1 kg, and 3500 g respectively. The shipping fees from this cargo service are based on the weight of an item, with the following rates:

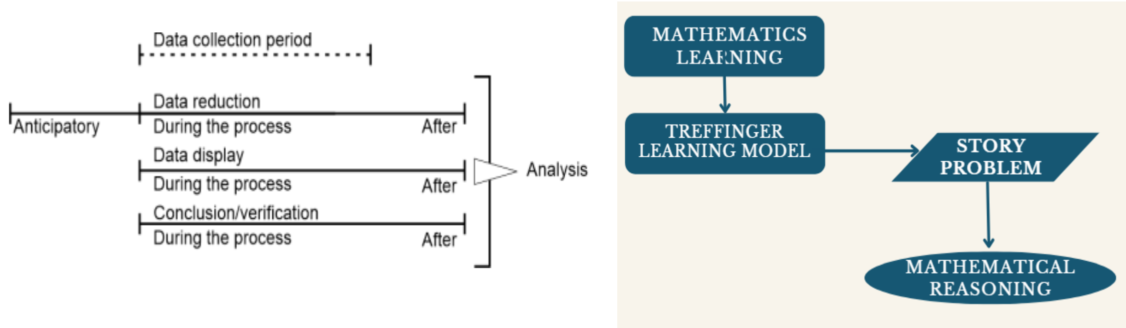
Weight (g)	Cost (Rp)
<500	4000
500 – 1000	7000
1001 – 3500	13500
3501 – 5000	24500
>5000	37000

Based on the given shipping cost table, the cargo service offers several shipping options as follows:
1. Shipping item A, B, and C separately.
2. Combining the shipment of item A and B, while item C is sent separately.
3. Combining the shipment of item A and C, while item B is sent separately.
4. Combining the shipment of item B and C, while item A is sent separately.
5. Combining the shipment of item A, B, and C.

To save on expenses, Shelma should choose the shipping option that minimizes the total cost. Then, we need to arrange the shipping models from the most expensive to the least expensive.

After the questions were created, the questions were then validated by several experts in the field, namely two mathematics education lecturers and one teacher at a junior high school in Surakarta. Meanwhile, the interview guidelines were used as a form of confirmation to analyze the mathematical reasoning abilities more profoundly and as a form of data triangulation besides using word problems.

Data collection in this research was conducted using tests and interviews carried out at one of the junior high schools in Surakarta. Seventh-grade students were selected for research subjects using purposive sampling techniques. Purposive sampling is a method of selecting research subjects with a specific purpose according to the research objectives [24]. From this technique, two students were selected as research subjects. The selection of subjects was based on the implementation of Treffinger learning conducted on the selected subjects and on the heterogeneity of answers. The data analysis of this research was conducted inductively, as outlined by Miles & Huberman [26], with steps including data collection, reduction, data display, and concluding the final stage. The data analysis process is depicted in Figure 1a. The research process involved implementing the Treffinger learning model with the study participants. Data were collected through word problems and in-depth interviews, which were reduced, displayed, and analyzed to assess students' mathematical reasoning abilities. This process is clearly illustrated in Figure 1b.



(a) (b)

Figure 1. (a) Research Flow Data Analysis Process [27] and (b) Research Flow Diagram

3. RESULTS AND DISCUSSION

The research findings illustrate how students go through the problem-solving process in mathematical word problems. Based on the indicators used, these findings outline steps ranging from presenting ideas, making hypotheses, performing mathematical manipulations, compiling evidence, drawing conclusions, and verifying validity. The exposition of subject responses is detailed for each indicator.

3.1 Presenting Ideas

This indicator demonstrates the subjects' ability to analyze mathematical ideas or statements through various forms, such as oral, written, visual, or diagrammatic. Based on Figure 2, Subject 1 demonstrates the ability to visualize the problem information comprehensively, from the information about the weight of each item to the categorization of costs based on the item's weight. This condition has also been confirmed through in-depth interviews conducted by the researcher.

Researcher : *"How do you start solving the problem?"*
Subject 1 : *"I usually start by writing down the given data and the data being asked for. This helps me work on the problem more easily and focus on only the important information in the problem."*
Researcher : *"Then, how do you analyze the problem?"*
Subject 1 : *"I mark some important information by making annotations on the problem sheet when understanding the information, then I write down this important information on the answer sheet."*

The excerpt from the interview above validates that Subject 1 can analyze the information contained in the problem and transform it into mathematical statements. Subject 1 demonstrates this response by marking specific points on the problem sheet while understanding the information before writing it down on the answer sheet.

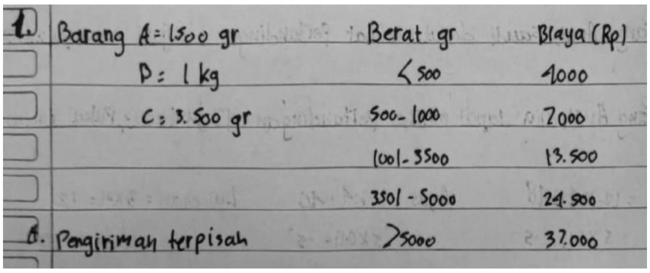


Figure 2. Subject 1's Answer Sheet on Problem Number 1 in Presenting Mathematical Statements

Translate:
Item A: 1500 gr
Item B: 1 kg
Item C: 3.500 gr

Weight (g)	Cost (Rp)
<500	4000
500-1000	7000
1001-3500	13.500
3501-5000	24.500
>5000	37.000

Another finding is the difference in response between Subject 2 and Subject 1. Subject 2 tends to only read the problem once before working on their answer sheet immediately. As a result, the subject does not express the information contained in the problem in mathematical statements on their answer sheet. This condition can be observed in Figure 3. Furthermore, the researcher has confirmed this condition through in-depth interviews conducted with Subject 2.

Researcher: *"How did you solve the problem?"*

Subject 2: *"When I am working on the problem, I directly pour out my answer on the answer sheet without hesitation, but beforehand, I make sure to read the problem carefully and thoroughly so that I can understand the context and requirements of the problem well."*

Researcher: *"Why didn't you write down the problem information on your answer sheet?"*

Subject 2: *"For me, what is very important is the process of solving the problem and the steps I take to obtain an accurate answer. Additionally, I also consider time efficiency because I realize that there are still many problems to solve, and I am worried that I will not have enough time to solve them all if I spend too much time on just one problem."*

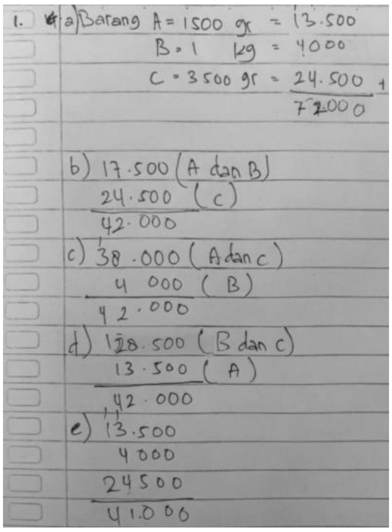


Figure 3. Subject 2's Answer Sheet on Problem Number 1 in Presenting Mathematical Statements

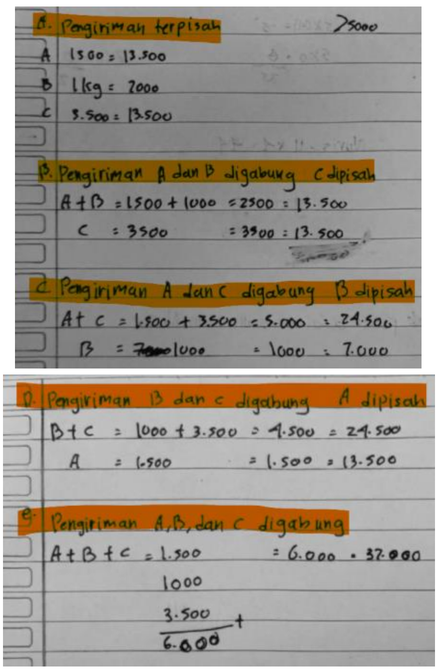
The excerpt from the interview above reflects that Subject 2 adopts an efficient approach to solving problems, where they read the problem information only once and then quickly write down the answer obtained on the answer sheet. This action is done without considering the writing of known information from the problem because it is deemed unnecessary. Subject 2 focuses on calculations and concluding to optimize their time efficiency, especially considering the number of problems that need to be solved. Therefore, it is confirmed that the subject does not record the information obtained from the problem on their answer sheet. However, through the interview, the Subject can recall and mention various information in the problem.

3.2 Making Assumptions

This indicator aims to reveal the ability to make assumptions, hypotheses, or problem-solving plans as the initial step in solving mathematical problems. Various responses were found for this indicator. Firstly, the response of Subject 1 demonstrates the ability to apply this indicator by writing down the known information in mathematical statements before devising a calculation plan. Specifically, the subject notes down various possible shipping costs per shipping model, as seen in Subject 1's answer sheet in Figure 4. Subject 1's approach is further reinforced through data from interviews conducted by the researcher, providing in-depth information about mathematical reasoning ability on this indicator.

Researcher: *"What process do you go through to solve the problem?"*
Subject 1: *"I start by identifying the information known from the problem, then comparing it with the mathematical knowledge I have learned previously. After that, I try to devise a calculation plan by considering various shipping models and the associated possible costs."*

Through the interview, it is deeply confirmed how students engage in mathematical reasoning when solving word problems. This includes identifying challenges faced and the strategies applied to overcome them. The interview also reveals the importance of students' ability to analyze complex mathematical problems, apply relevant knowledge, and use logic and critical thinking to find solutions. In conclusion, the interview provides essential insights into the practice of mathematical reasoning among students, demonstrating how they approach and solve mathematical challenges using various strategies.



Translate:
A. Separate shipping
1. 1.500=13.500
2. 1 kg=7.000
3. 3.500= 13.500

B. Shipments A and B are combined, and C is separated
A+B= 1.500+1.000=2.500=13.500
C= 3.500= 3.500= 3.500

C. Shipments A and C are combined, B is separated
A+C=1.500+3.500=5.000=24.500
B= 1000= 1000= 7000

Translate:
D. Shipments B and C are combined or separated
B+C= 1.000+3.500= 4.500= 24.500
A= 1500= 1500=13.500

E. Shipments A, B, and C are combined
A+B+C= 1.500+1000+3.500=6.000=37.000

Figure 4. Subject 1's Answer Sheet on Problem Number 1 in Proposing Allegations

Subject 2, similar to Subject 1, records various possible shipping costs for each available model. However, there are shortcomings in their writing, particularly in options a and e, which could potentially lead to errors in the final calculations. These errors stem from the unclear information recorded, which could lead to misinterpretation during analysis. This condition is visualized in Figure 5, demonstrating these shortcomings in Subject 2's answer sheet. Furthermore, the researcher confirms this issue through an interview, deepening the understanding of how deficiencies in writing and presenting information can affect the accuracy of calculation results. The interview serves as a crucial means to understand potential errors that may occur in the process of solving mathematical problems, particularly in assessing and comparing shipping options based on cost.

Researcher:	*"From our analysis of your answers, we noticed shortcomings in the writing of options a and e. Could you explain your thought process when recording this information?"*
Subject 2:	*"I tried to gather all the information as quickly as possible and consider all possibilities. In that process, I lost important details because I was too focused on speed rather than accuracy."*

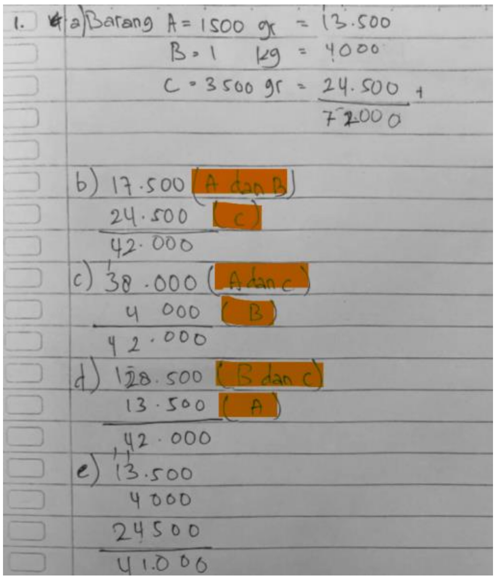


Figure 5. Subject 2's Answer Sheet on Problem Number 1 in Proposing Allegations

In the interview with the researcher, Subject 2 acknowledges that errors in recording details for specific shipping options jeopardize the accuracy of their calculations. Subject 2 attempts to address this by double-checking the data against the given problem, emphasizing the importance of clarity and caution in gathering information. Awareness of these errors prompts Subject 2 to adopt a more systematic and reflective strategy in the initial analysis, demonstrating a commitment to improving problem-solving skills. This interview highlights the value of accuracy, learning from mistakes, and adapting strategies as keys to developing a deeper mathematical understanding.

3.3 Mathematical Manipulation

In the context of mathematical reasoning abilities, particularly regarding the mathematical manipulation indicator, subjects are expected to perform mathematical operations such as calculations or transformations to find solutions. Based on the analysis of the collected data, only one response was obtained, provided by Subject 1. The answer given by Subject 1 demonstrates their ability to manipulate calculations effectively, as seen in Figure 6. The figure illustrates the calculation process and the solution obtained by Subject 1, emphasizing the significant role of the addition concept in mathematical manipulation. From the figure, it is clear that Subject 1 understands the concept of addition and can apply it accurately in the mathematical manipulation required for the problem. This proficiency is further reinforced through an interview conducted by the researcher.

Researcher: *"How do you determine the steps needed to solve the problem?"*
Subject 1: *"I looked at the problem and immediately knew I had to add those numbers. I tried to add them one by one to get the answer."*
Researcher: *"Do you use any specific strategies in performing addition?"*
Subject 1: *"No, I just follow the basic addition process that I've learned. I'm confident with the basics, so I can get the right answer."*

This brief interview shows that Subject 1 has a strong understanding of the addition concept as part of mathematical manipulation and can efficiently apply it to solve problems. Subject 1's confidence in the fundamentals of addition and the ability to apply them without requiring specific strategies demonstrate strength in conceptual understanding and efficiency in mathematical reasoning. Subject 1 emphasizes the importance of mastering the basics of mathematics, such as addition, which serves as a foundation for further mathematical manipulation. This highlights the value of solid conceptual understanding and the ability to apply it practically, which are crucial in developing mathematical reasoning abilities.

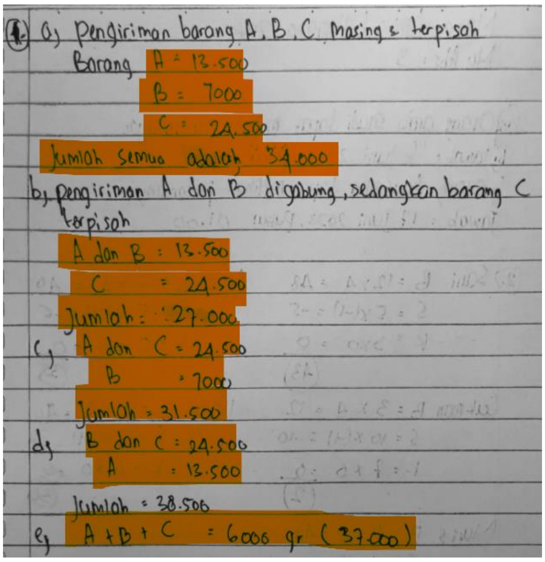


Figure 6. Subject 1's Answer Sheet for Problem Number 1 in Mathematical Manipulation

Translate:

A. Delivery of goods A, B, C each separately. Goods:
A= 13.500
B= 7.000
C= 24.500
The total number is 34.000
B. Delivery of goods A and B combined, while goods are separate
A+B= 13.500
C= 24.000
The total = 34.000
C. A and C= 24.000
B= 7.000
The total = 31.500
D. B and C= 24.500
A= 13.500
The total = 38.500
A+B+C= 6000gr (37.500)

3.4 Constructing Evidence

The indicator used to measure the ability to compile evidence is the subject's ability to provide solid reasoning or evidence to support the validity of the proposed solution. In this context, evidence of the reasoning supporting the validity of the solution is gathered through interviews, where subjects are expected to explain their thought process in solving the problem, including how they perform calculations and arrive at answers. After data reduction, only one type of response was obtained.

The subjects in this study used worksheets to record their calculation processes as evidence of their problem-solving skills. However, some subjects chose to convey their thought processes only through interviews. Subject 1 stated in the interview that they did not use scratch paper when solving problem number one. The subject explained the reason behind this approach during the interview.

Researcher: *"You chose not to use scratch paper when working on question number one. Can you explain why you chose this approach?"*
Subject 1: *"Certainly. I feel more comfortable and freer to explore and process information and calculations in my mind. I believe it helps me focus more on the problem-solving process rather than writing it down."*
Researcher: *"Interesting. How do you ensure you do not lose important details or make mistakes in your calculations?"*
Subject 1: *"I usually try to visualize the problem and its calculations in my mind and repeat the process several times. If I feel unsure, I will redo my thought process from the beginning. It is like having a mental whiteboard where I can erase and rewrite until I find the right solution."*

The interview with Subject 1 revealed a unique problem-solving approach, where the subject relies on their mental abilities to solve mathematical problems without visual scratch paper. This indicates that mathematical problem-solving does not always have to be followed by visual representations and that understanding concepts and mental visualization abilities also play a crucial role in mathematics. Subject 1 emphasized that this method helps them stay focused and efficient while sharpening their memory and visualization abilities.

3.5 Drawing Conclusions

The indicator of mathematical reasoning in concluding is the ability to draw conclusions based on the mathematical statements provided. Concluding relies on the calculations that have been performed. If the calculations are correct, then the conclusions drawn will be accurate. This was evident in Subject 1, who could conclude accurately because the calculations performed in the previous indicators or stages were also correct. The answer sheet of Subject 1 is presented in Figure 6.

One of the factors influencing errors in concluding is a need for more focus when understanding information. This can hurt the calculation process. In Figure 7, we can see the visualization of the answer provided by Subject 2. From this, it is evident that a lack of focus plays a significant role in the errors that arise while understanding and processing information. Errors in concluding due to a lack of understanding of information become the main factor in these mistakes. The researcher confirmed this through in-depth interviews with Subject 2.

Researcher: *"I noticed errors in your conclusions. Did a lack of focus affect you?"*
Subject 2: *"I struggled to focus, which affected my understanding of comprehending information related to shipping costs based on weight."*

This interview highlights how a lack of focus affects Subject 2's mathematical reasoning ability in understanding and processing information. This is evident through errors in concluding, as seen in Figure 8. Subject 2 acknowledges that internal factors such as interest and fatigue can affect their level of focus. From this discussion, the researcher gained valuable insight into how the psychological conditions of the subjects affect their cognitive performance, opening the way for intervention strategies to improve focus and concentration in learning or information-processing situations.

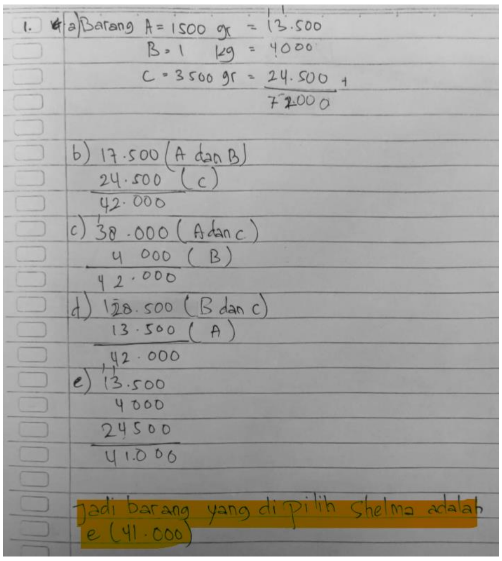


Figure 7. Subject 2's Answer Sheet on Problem Number 1 in Drawing Conclusions

Translate:
So the item chosen by Shelma is e (41.000)

3.6 Checking Validity

The ability measured through this indicator highlights how far students can assess and test the validity of mathematical arguments or reasoning presented. This indicator encompasses students' evaluation of various aspects of solving mathematical problems, including the accuracy of the information provided, the application of relevant mathematical concepts, the accuracy of calculations, and the logic behind the conclusions drawn. There is one response provided by a subject, whom we refer to as Subject 1. This subject methodically examines each stage of the problem-solving process. The process begins with verifying the information provided, followed by an examination of the mathematical concepts applied, an evaluation of the calculations performed, and ends with an assessment of the conclusions reached. During the interview, the subject explains how this mathematical reasoning process is carried out.

Researcher:	*"Can you explain how you verify the accuracy of what you obtained from problem number 1?"*
Subject 1:	*"I check if my conclusions are logical based on the steps I have taken. I also consider if there are any unconscious assumptions that I may have made that could affect the results. If I find any errors, I return to the previous step to correct them."*

This interview concludes Subject 1's approach to solving mathematical problems and the importance of mathematical reasoning ability. The mathematical reasoning process described by Subject 1 highlights a layered approach to handling mathematical problems. It begins with a deep understanding of the given information, followed by identifying and applying relevant mathematical concepts. Accuracy in calculations is vital, but the next important step is critically evaluating the conclusions reached, considering the logic and underlying assumptions. This process is often an iterative cycle, requiring revision and correction to arrive at correct and logical conclusions.In presenting mathematical ideas, the subjects' responses show variations in visualizing problem information. Subject 1 utilizes visualizations, such as tables, to present data in a structured manner, facilitating analysis and reducing potential errors. Subject 2, on the other hand, prefers a direct approach, answering the problem without separately writing down initial information.

Nevertheless, both responses indicate rereading problem information and annotating the problem sheet before presenting mathematical ideas. This finding highlights the importance of balanced strategies in mathematical learning. While time efficiency is crucial, especially in exam contexts, a deep understanding of the problem and rewriting important information can reduce the risk of errors [27]. Mathematical problem-solving should emphasize the importance of deep conceptual understanding and practical problem-solving strategies [28]. These differing response variations maintain the significance of this indicator's critical point: understanding problem information and being able to present it in mathematical language. Saxton [29] state that mathematical reasoning ability in understanding information is needed to weigh and select the necessary information to answer the problem. The study by Hasanah et al. also shows that good mathematical reasoning ability can understand and present information in appropriate mathematical language. This indirectly relates to Treffinger's learning model, which trains students' creativity in problem-solving [19], as demonstrated by the variation in subject responses in solving problems.

The analysis of findings regarding students' ability to make hypotheses highlights a crucial aspect of mathematical reasoning and its relevance to the Treffinger learning model. The Treffinger model, which emphasizes the development of creativity and critical thinking through independent decision-making and problem-solving, can serve as a practical framework for interpreting these results. The ability to make hypotheses is an essential initial step in mathematical reasoning, involving the formulation of hypotheses or problem-solving plans. Findings from student responses indicate variations in the application of this ability. Subject responses in this regard show responses by noting various possible shipping prices in making hypotheses for solving the problem, then writing down and analyzing the known information and formulating various calculation plans, demonstrating the application of deep mathematical reasoning. This aligns with key aspects of the Treffinger model, namely emphasizing independent problem identification and strategy development to find solutions [30]. However, the shortcomings in writing by Subject 2, especially in presenting shipping price options, underscore the importance of clarity and precision in mathematical reasoning processes. It indicates opportunities to apply the Treffinger model in providing constructive feedback, encouraging reflection, and improving strategy. In this context, identifying and correcting mistakes become integral parts of the learning process, in line with Treffinger's learning principle that encourages students to take calculated risks and learn from mistakes [31].

The ability to manipulate mathematics efficiently reflects a solid understanding of basic and advanced mathematical concepts [32]. Subject responses indicate how they manipulate mathematics by performing calculations according to their experience and understanding. The level of practice in solving problems also indirectly influences data manipulation skills. In the context of Treffinger learning, which emphasizes creativity in problem-solving, the more room students have for exploring their thoughts, the stronger and more accessible it is to remember the steps they take in solving problems [33]. This is consistent with the responses obtained.

Constructing evidence requires more than just performing correct calculations; it involves developing logical reasoning to convince others of a solution's truth [34]. This includes a deep understanding of the involved mathematical concepts, the ability to connect these concepts with the given problem, and communication skills to convey thought processes clearly and systematically. Subject 1, who chooses not to use scratch paper and relies on mental processes, demonstrates strong visualization skills and confidence in their internal reasoning abilities. This approach shows that mathematical problem-solving can be highly personal and varied, depending on how individuals are most comfortable processing information and navigating solutions. However, communicating thought processes remains at the core of practical evidence construction. Subject 2, on the other hand, uses scratch paper to aid their calculation process, highlighting the value of visual representation in clarifying and organizing problem-solving steps. This approach is suitable for situations where detail and accuracy in calculations are crucial, such as in exam contexts or when dealing with complex problems. The different approaches subjects take in constructing evidence suggest that mathematics teaching and learning must be flexible and adapt to diverse learning styles [35]. Teachers can leverage Treffinger's principles to encourage students to develop evidence-construction skills that strengthen their understanding of concepts and use that knowledge practically [36]. In line with the research conducted, mathematical reasoning skills are crucial for students in solving mathematical problems, enabling them to complete more steps in the problem-solving process [8].

In concluding, the success of Subject 1 is directly related to their ability to perform calculations correctly and their deep conceptual understanding. This indicates that a solid conceptual understanding and accurate application of mathematical operations is the main foundation for drawing valid mathematical conclusions. On the other hand, Subject 2 needs help drawing accurate conclusions, primarily due to the lack of focus that affects their understanding of problem information. Errors that arise early on continuously lead to errors in subsequent problem-solving processes, resulting in inaccurate conclusions [37].

In examining the validity of an argument, the response obtained is that subjects check all the skipped problem-solving steps to verify the calculations. This step highlights the importance of mathematical reasoning skills in mathematics education. Examining the validity of an argument is not just about finding the "correct" answer; it is more about understanding the process that leads to that answer, verifying the accuracy of the steps taken, and ensuring that the conclusions drawn are logical and supported by solid evidence [38]. Mathematics teaching and learning should encourage students to develop both skills, preparing them for exams and using mathematical reasoning in everyday life.

4. CONCLUSION

This research shows that the Treffinger learning approach significantly enhances students' mathematical reasoning abilities in solving mathematical word problems. Through phenomenological analysis, it was found that students using this approach tend to reread and annotate the problems before expressing their mathematical ideas, use scratch paper to justify their answers, and check the validity of their solution steps. These results affirm that the Treffinger approach helps students understand the problems and develop and verify their solutions effectively. The implications of these findings suggest that the Treffinger method can be a valuable teaching strategy for teachers to enhance students' mathematical reasoning skills, providing them with a creative and analytical framework to tackle complex mathematical challenges in the classroom context. This study is limited by the selection of material used in the word problems to measure students' mathematical reasoning abilities. Future research could use different materials or types of mathematical problems. Additionally, expanding the study of students' mathematical reasoning abilities by using different teaching models besides Treffinger is recommended to observe these abilities from various perspectives.

AUTHOR CONTRIBUTION STATEMENT

DFH contributed to this article comprehensively, from conceptualization to drafting and templating. SH contributed valuable input and feedback on the writing process, which is also greatly appreciated.

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