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Algebraic reasoning of high school students in solving inverse function problems: Viewed from mathematical resilience

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ABSTRACT

Algebra is one of the essential branches of mathematics that often presents challenges for students, especially when faced with inverse function problems. This study aims to describe algebraic reasoning in solving inverse function problems from the perspective of students' mathematical resilience. This qualitative research uses a case study design involving 33 eleventh-grade students from a public high school in West Bandung Regency. The instruments used were a mathematical resilience scale and an algebraic reasoning test. Data analysis was conducted through data reduction, presentation, and conclusion drawing. The results showed that 15.15% of students had low resilience, 72.72% had moderate resilience, and 12.12% had high resilience. There were different characteristics in algebraic reasoning among students in these resilience categories. The implications indicate that targeted learning strategies can enhance algebraic reasoning, especially for moderate and low-resilience students.

Penalaran aljabar siswa SMA dalam menyelesaikan soal fungsi invers: Ditinjau dari resiliensi matematis

Kata Kunci:

Penalaran aljabar
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ABSTRAK

Aljabar merupakan salah satu cabang penting dalam matematika yang sering kali menantang bagi siswa, terutama ketika siswa dihadapkan pada masalah fungsi invers. Penelitian ini bertujuan untuk mendeskripsikan penalaran aljabar dalam menyelesaikan masalah fungsi invers dari sudut pandang ketahanan matematis siswa. Penelitian kualitatif ini menggunakan desain studi kasus yang melibatkan 33 siswa kelas XI salah satu SMA Negeri di Kabupaten Bandung Barat. Instrumen yang digunakan adalah skala ketahanan matematis dan tes penalaran aljabar. Analisis data dilakukan melalui reduksi data, penyajian, dan penarikan kesimpulan. Hasil penelitian menunjukkan bahwa 15,15% siswa mempunyai resiliensi rendah, 72,72% resiliensi sedang, dan 12,12% resiliensi tinggi. Terdapat karakteristik berbeda dalam penalaran aljabar di kalangan siswa dalam kategori ketahanan ini. Implikasinya menunjukkan bahwa strategi pembelajaran yang ditargetkan dapat meningkatkan penalaran aljabar, khususnya bagi siswa dengan ketahanan sedang dan rendah.

1. INTRODUCTION

Inverse functions are one of the topics studied in Senior High School in the Merdeka Curriculum. An inverse function is a function that "reverses" a function. If the function f applied to input x and resulted $in\ y$, then applying its inverse g to y resulted in x, and vice versa [1]. Inverse functions have an important role in mathematics. Inverse functions are needed in analysis, geometry, and statistics. The inverse function can also be used to solve various problems, namely, finding the value of x from $y = f(x)$, changing the form of the function equation, determining the point of intersection of two lines, and determining the turning point of a curve.

The conceptual understanding that students have about inverse functions can be characterized by students' ability to explain and describe the concept of inverse functions, knowing the steps to determine the inverse of a function, especially paying attention to the relationship between bijective functions and inverse functions, and providing arguments about the reasons why a function has an inverse if the function is bijective [2]. Students with difficulties in achieving a meaningful understanding of the inverse function will have challenges conceptually and cognitively using a variety of representations [3], [4]. Students' prior knowledge of inverse operations or providing analogies from real-life situations may be useful for building a foundation, although not sufficient to enhance their conceptual understanding of inverse functions. Some ways to enhance students' meaningful learning are by employing a variety of appropriate representation systems [5] [6], testing concepts through conceptually focused and cognitively challenging tasks, connecting inverse functions to the concept of 'one-to-one and to' functions and the concept of the function itself, and ensuring active student involvement in the knowledge construction process [7].

Students can develop a deeper understanding of the inverse function through covariant and bidirectional reasoning processes by thinking quantitatively and qualitatively, analyzing the relationship between two quantities, thinking causally, thinking functionally, and using abductive reasoning [8]. Research found that students understand the inverse function as a function that reverses the original function. However, students develop a more complex understanding through covariant and bidirectional reasoning. They understand that the inverse function and the original function represent the same relationship, and students understand that the graphical representation of the inverse function is a convention, not a mathematical rule [8]. This more complex understanding is based on the algebraic reasoning performed by students. They consider the relationship between inputs and outputs using quantitative and qualitative thinking. They also use causal thinking to consider how inputs and outputs affect each other. Furthermore, they use functional thinking to consider how the inverse function is used to invert the original function. Finally, they use abductive thinking to conclude the nature of the relationship between inputs and outputs based on observations of some examples [8].

Algebraic reasoning is the ability to think, which involves the development of mathematical thinking by building a definition or understanding of symbols and algebraic operations [9]. Kaput, Blanton & Moreno state that algebraic reasoning relates to a child's ability to think logically about quantities (known or unknown) and the relationship between them [10]. Algebraic reasoning refers to the psychological processes involved in solving problems that mathematicians can easily express using algebraic notation. This emphasizes the implicit cognitive processes that may be carried out among younger students when involved in problem-solving (such as noticing structural relationships and making generalizations) and shows that some may involve variables and arithmetic rules [11]. Algebraic reasoning in this research is students' ability to make decisions or draw conclusions in solving algebraic problems by considering generalization, representation, and justification [12].

Algebraic reasoning on inverse functions can be a complicated task for students, as inverse functions cannot be understood in just one simple way [13]. The research results show that someone who understands the concept of function and inverse function can learn the subject matter, communicate ideas and solutions, and connect mathematical ideas [14], [15]. While exploring these related concepts, students will be encouraged to think independently and devise original strategies in their work with functions and inverses. Thus, students must have an attitude that can face challenges in learning [14], [15]. Students with a resilient attitude in overcoming mathematics challenges are called mathematical resilience [16], [17]. Mathematical resilience is a positive construct that will enable students to overcome obstacles when learning mathematics and develop a positive attitude towards mathematics [18], [19]. Mathematical resilience in this research is the attitude of students who view mathematics as useful in life, so they never give up and have self-confidence in learning mathematics and solving mathematical problems. Mathematical resilience skills can help students overcome difficulties in learning inverse functions [20], [21]. This ability can help students to show that learning mathematics is useful for the future (value of mathematics), demonstrate an attitude of perseverance, confidence, hard work, and not easily giving up when facing mathematics problems (struggle), and demonstrate a desire to socialize, assist, and discuss with peers (collaboration) when facing challenges [18].

Studies on algebraic reasoning and mathematical resilience have been conducted previously: (1) The algebraic reasoning process based on student characteristics, such as mathematical anxiety [22], type of student intelligence [23], and SOLO taxonomy [24]; (2) the relationship between mathematical resilience and communication skills [25], problem-solving abilities [26], and learning outcomes [27]; (3) student errors in solving word problems based on resilience [28]; and 4) the contribution of mathematical resilience to developing attitude assessment rubrics in mathematics learning [29].

Based on previous studies on algebraic reasoning and mathematical resilience, which have been described previously, there has been no research regarding the relationship between algebraic reasoning and mathematical resilience. There needs to be more in-depth research related to students' algebraic reasoning in solving inverse function problems by examining it from mathematical resilience. The formulation of this research is how is students' algebraic reasoning in solving inverse function problems seen from students' mathematical resilience? Thus, this research aims to describe the algebraic reasoning process of students with different levels of resilience in solving inverse function problems.

Contribution to the literature

This research contributes to:

- Developing and validating instruments to measure algebraic reasoning and mathematical resilience.
- Identifying differences in the characteristics of algebraic reasoning among students with varying levels of mathematical resilience.
- Enriching the literature on the interaction between cognitive abilities and affective attitudes in mathematics learning.

2. METHOD

This research was a qualitative descriptive study with a case study design. This research aims to describe algebraic reasoning in inverse function topics in terms of students' mathematical resilience. The subjects in this study were 33 eleventh-grade students in one of the public high schools in West Bandung Regency. The instrument was a test instrument prepared based on representation, generalization, and justification [12]. The analysis of the validity and reliability of the algebraic reasoning test instrument was performed and declared valid and reliable. The test instrument's aspects and indicators are explained in Table 1.

Table 1. Algebraic Reasoning Test Instrument

<table>
<tr><th>Indicator</th><th>Question & Aspect of Algebraic Reasoning</th></tr>
<tr><td>

Students can identify relevant information, relate the information, and model relationships through algebraic representations.
Students can present the given problem in the form of an algebraic representation, which includes coordinate graphs and equations that relate the context to the information given.
The student can provide initial representations or add to existing representations by carrying out calculation procedures in algebraic or arithmetic forms and changing the form of expressions or equations into equivalent forms.
Students can conclude the existing context.
Students can create general shapes from the specific shapes given.
Students can provide explanations or justifications.

</td><td>
1. Given $f(x) = \frac{5-2x}{3}$.
a. Find the range of f if it is known that the domain f is {0, 1, 2, 3, 4}! (Aspect Representation)
b. Draw the relationship between the domain and range in cartesian coordinates, with the $x-$axis for the domain and the y-axis for the range! (Aspect Representation)
c. Based on the image in answer (b), the function f has an inverse; explain! (Aspect Justification)
d. Determine $f^{-1}(x)$ and provide explanations for each step! (Aspect Justification)

2. A fabric merchant has a calculation to determine the price for x meters of cloth, namely $f(x)$ rupiahs. This price is obtained from each meter of cloth sold multiplied by the production costs of IDR 15.000,00 plus a profit of IDR 50.000,00. The following is data on the selling prices of these cloth traders if given in a table.
<table>
<tr><th>Fabric length (m)</th><th>Selling price ($f(x)$)</th></tr>
<tr><td>1</td><td>15.000 (1) +50.000= 65.000</td></tr>
<tr><td>2</td><td>15.000 (2) + 50.000= 80.000</td></tr>
<tr><td>3</td><td>15.000 (3) + 50.000= 95.000</td></tr>
<tr><td>4</td><td>15.000 (4) + 50.000= 110.000</td></tr>
<tr><td>5</td><td>15.000 (5) + 50.000125.000</td></tr>
<tr><td>…</td><td>…</td></tr>
<tr><td>x</td><td>$f(x) = \cdots$</td></tr>
</table>
a. Based on the table, find the formula $f(x)$! (Generalization & Representation Aspects)
b. Based on the table, $f(x)$ has an inverse. Explain! (Justification Aspects)
c. Determine $f^{-1}(x)$ by providing explanations for each step! (Representation & Justification Aspects)
</td></tr>
</table>

The mathematical resilience response questionnaire was prepared based on mathematical resilience indicators. The mathematical resilience indicators used are: (1) students show that learning mathematics is useful for the future; (2) show an attitude of perseverance and confidence, work hard, and don't give up easily when facing mathematical problems [18]; (3) show a desire to socialize, easily provide help, and discuss with peers; (4) use the experience of failure in completing mathematics tasks to build self-motivation; (5) have curiosity and reflect on their work related to mathematical tasks; (6) generate new ideas/methods and look for creative solutions when completing mathematics assignments. The questionnaire instrument consisted of 29 statements with answer options:

Strongly Agree (SA), Agree (A), Disagree (D), and Strongly Disagree (SD). On positive statements, SA score = 4; A score= 3; D score: 2; SD score = 1, and vice versa for negative statements. Analysis of the validity and reliability of the mathematical resilience scale has been carried out and declared valid and reliable. The level of students' mathematical resilience is in Table 2.

Table 2. Mathematical Resilience Categories [30]

Interval Limits	Category
$x < (\bar{x} - SD)$	Low
$(\bar{x} - SD) \leq x < (\bar{x} + SD)$	Medium
$(\bar{x} + SD) \leq x$	High

Data analysis is carried out by reducing the data, then presenting the data, and ending with concluding the data. Answers to the algebraic reasoning aspect, the percentage is found by using the formula:

$$P = \frac{f}{N} \text{x} 100\% \qquad (1)$$

Information:
P: Percentage of students who answered
f: The number of students who answered
N: The overall number of students

3. RESULTS AND DISCUSSION

Thirty-three students were the subjects involved in this research. The research used a mathematical resilience questionnaire and an algebraic reasoning test. Based on the questionnaire results, a recapitulation was obtained with a maximum score of 90, minimum score of 74, average of 82, and standard deviation of 2.67. From these results, it can be determined that $\bar{x} - SD = 82 - 2.67 = 79.33$ and $\bar{x} + SD = 82 + 2.67 = 84.67$.

Table 3. Recapitulation of Students' Mathematical Resilience Levels

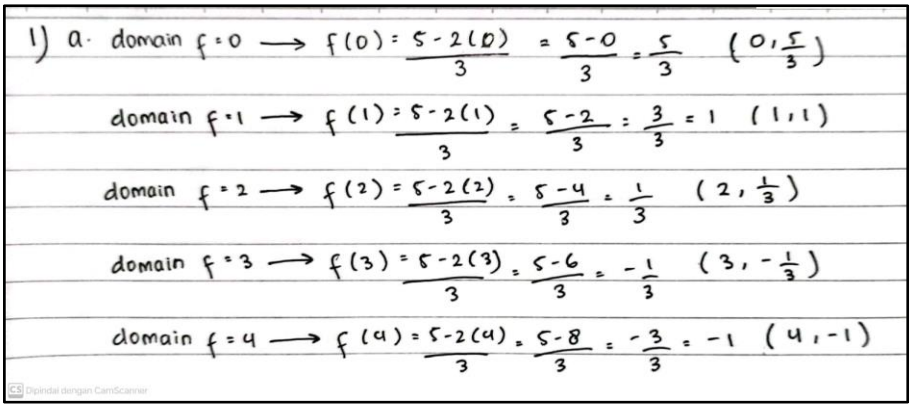
Interval Limits	Category	Students	Percentage
$x < 79.33$	Low	5	15.15%
$79.33 \leq x < 84.67$	Medium	24	72.72%
$84.67 \leq x$	High	4	12.12%

Based on Table 3, 15.15% of students are in the low resilience category, 72.72% are in the medium resilience category, and 12.12% are in the high resilience category. Each student is given an algebraic reasoning test with three aspect indicators measured: representation, generalization, and justification. Some student work from high, medium, and low mathematical resilience student groups was analyzed to see the characteristics of algebraic reasoning. Students with high mathematical resilience are coded as HMR, students with medium mathematical resilience are coded as MMR, and students with low mathematical resilience are coded as LMR.

3.1 Representation Aspect

The first aspect of representation with indicators is the student's ability to identify relevant information, relate this information, and model relationships through algebraic representation. This indicator is shown by students who can present the range of a function. Students in the high resilience category showed several different answers. All HMR students can determine the range by substituting each domain member in its function.

However, only 40% of HMR students can correctly write the range in a set. The remaining students only look for function values.



1) a. domain f : 0 → $f(0) = \frac{5-2(0)}{3} = \frac{5-0}{3} = \frac{5}{3}$ $(0, \frac{5}{3})$

domain f : 1 → $f(1) = \frac{5-2(1)}{3} = \frac{5-2}{3} = \frac{3}{3} = 1$ $(1, 1)$

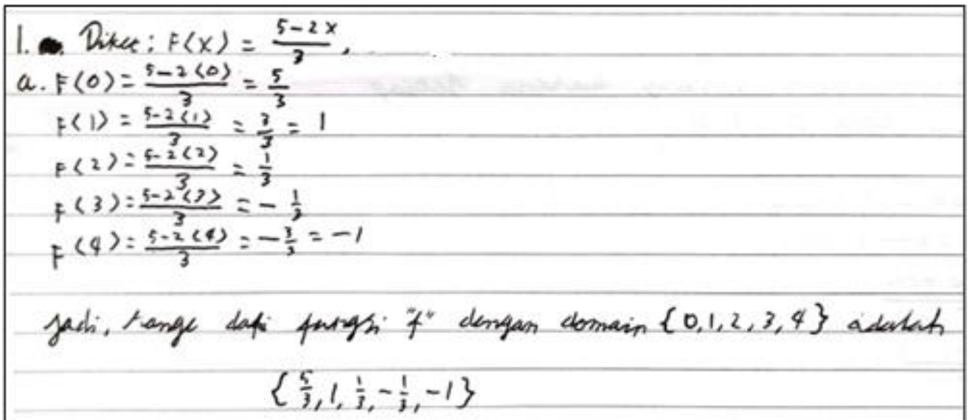
domain f : 2 → $f(2) = \frac{5-2(2)}{3} = \frac{5-4}{3} = \frac{1}{3}$ $(2, \frac{1}{3})$

domain f : 3 → $f(3) = \frac{5-2(3)}{3} = \frac{5-6}{3} = -\frac{1}{3}$ $(3, -\frac{1}{3})$

domain f : 4 → $f(4) = \frac{5-2(4)}{3} = \frac{5-8}{3} = \frac{-3}{3} = -1$ $(4, -1)$

Figure 1. HMR-1's Answer about the Range

In Figure 1, HMR-1 only looks for function values without writing that the range is a set of function values. If you look at the domain, $f: 0$ is not correct. It should be $x = 0$ for $f(0)$. HMR-1 also do not write the range but writes pairs of domain members and range members.



1. Dik : $F(x) = \frac{5-2x}{3}$

a. $F(0) = \frac{5-2(0)}{3} = \frac{5}{3}$

$F(1) = \frac{5-2(1)}{3} = \frac{3}{3} = 1$

$F(2) = \frac{5-2(2)}{3} = \frac{1}{3}$

$F(3) = \frac{5-2(3)}{3} = -\frac{1}{3}$

$F(4) = \frac{5-2(4)}{3} = -\frac{3}{3} = -1$

Jadi, range dari fungsi "f" dengan domain {0,1,2,3,4} adalah $\{\frac{5}{3}, 1, \frac{1}{3}, -\frac{1}{3}, -1\}$

Translate Known: $F(x) = \frac{5-2x}{3}$ $F(0) = \frac{5-2(0)}{3} = \frac{5}{3}$ $F(1) = \frac{5-2(1)}{3} = \frac{3}{3} = 1$ $F(2) = \frac{5-2(2)}{3} = \frac{1}{3}$ $F(3) = \frac{5-2(3)}{3} = -\frac{1}{3}$ $F(4) = \frac{5-2(4)}{3} = \frac{-3}{3} = -1$	So, the range of the function "f" with domain {0, 1, 2, 3, 4} is $\{\frac{5}{3}, 1, \frac{1}{3}, -\frac{1}{3}, -1\}$

Figure 2. HMR-2's Answer about the Range

In Figure 2, HMR-2 can determine the range by finding the function value and then writing that the range is a set of function values. Based on these two answers, HMR students in representation with indicators can carry out calculation steps in algebraic or arithmetic forms and change the form of expressions or equations into other equivalent forms. This is confirmed by the HMR-3 answer in determining the inverse of a function.

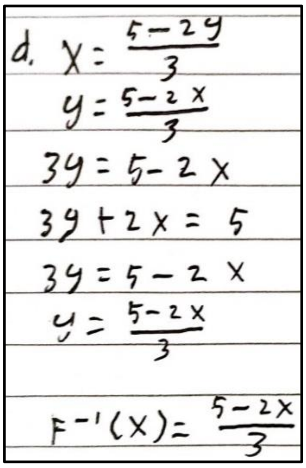


Figure 3. HMR-3's Answer about Determining the Inverse

In Figure 3, HMR-3 is asked to determine the inverse of the function $f(x) = \frac{5-2x}{3}$ through the correct calculation steps. However, in the first step, there is no explanation for why $x = \frac{5-2y}{3}$ became $y = \frac{5-2x}{3}$.

The next aspect of representation, HMR students can present the problem through coordinate graphs and equations that connect the context with the information provided. This is shown by the indicator that the HMR student can present the problem as a coordinate graph related to information about the domain and range of the function $f(x) = \frac{5-2x}{3}$. As many as 60% of HMR students can present the domain and range of a function in the form of cartesian coordinates. As many as 40% of HMR students presented the domain and range of a function in the form of tables and ordered pairs $(x, f(x))$, which did not match the instructions in the problem.

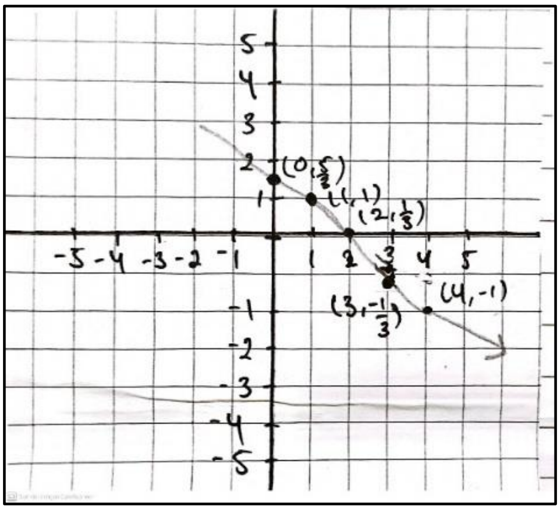
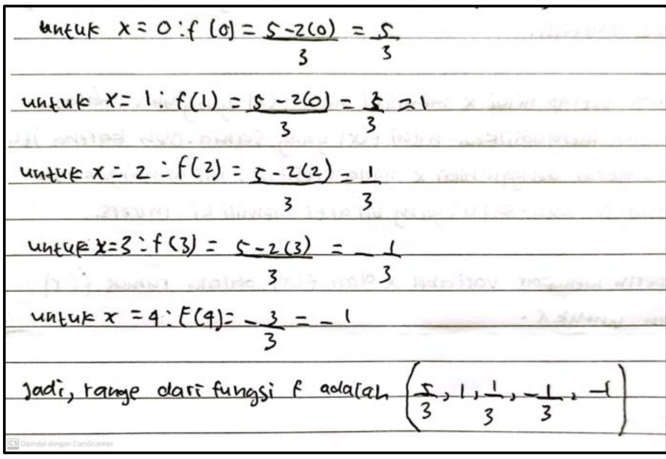


Figure 4. HMR-2's Answer about Coordinate Graph

In Figure 4, HMR-1 can present a problem given an algebraic representation in the form of a coordinate graph of the domain and range of a function that has been determined. However, if you look closely, some points are written less accurately.

Students in the medium resilience category (MMR) with indicators can identify relevant information, relate the information, and model relationships through algebraic representations, which students show can present the range of a function. As many as

37.5% of MMR students wrote the range and showed how to find the function value. As many as 54.2% of MMR students only looked for the range by determining the function value.



Translate:
For $x = 0$: $f(0) = \frac{5-2(0)}{3} = \frac{5}{3}$
For $x = 1$: $f(1) = \frac{5-2(0)}{3} = \frac{3}{3} = 1$
For $x = 2$: $f(2) = \frac{5-2(2)}{3} = \frac{1}{3}$
For $x = 3$: $f(3) = \frac{5-2(3)}{3} = -\frac{1}{3}$
For $x = 4$: $f(4) = \frac{5-2(4)}{3} = \frac{-3}{3} = -1$

So, the range of the function f is $\{\frac{5}{3}, 1, \frac{1}{3}, -\frac{1}{3}, -1\}$

Figure 5. MMR-1's Answer about the Range

In Figure 5, MMR-1 can write the range by finding the function value. However, if you look at the work procedure, there are still errors in the process and result of determining the function value for the domain $x = 1$. Based on the answer, the representation aspect with indicator MMR-1 can carry out calculation procedures in algebraic or arithmetic forms and change the form of expressions or equations into other equivalent forms, which has not been demonstrated properly. Another student's answer (MMR-2) supports the indicator shown in the steps in finding the inverse of the function $f(x) = \frac{5-2x}{3}$.

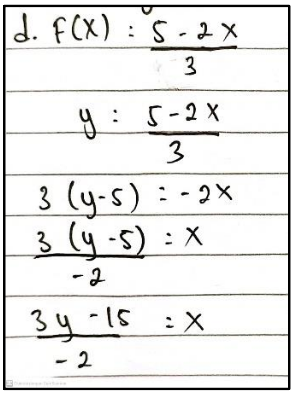


Figure 6. MMR-2's Answer Determines the Inverse Function

In Figure 6, MMR-2 carries out the procedure for determining the inverse of the function $f(x) = \frac{5-2x}{3}$. It can be seen in the third step that students are not yet able to carry out algebraic fraction operations procedures.

The next aspect of representation is that as many as 29.17% of MMR students presented the problem in the cartesian coordinate form, and as many as 45.83% of MMR students presented the problem in another form (not following the question instructions). In Figure 7, students already understand the problem and describe cartesian coordinates according to the points formed from the domain and range.

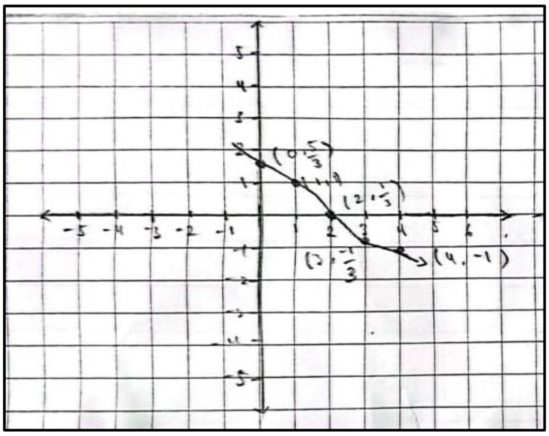
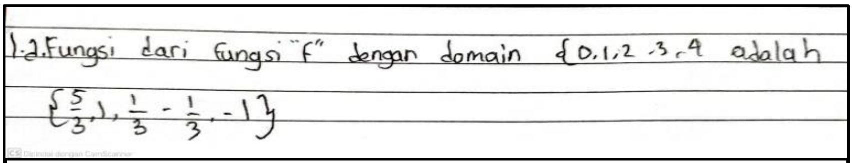


Figure 7. MMR-1's Answer about Coordinate Graph

Students in the low mathematical resilience (LMR) category showed that 25% could write the range but did not show the process of finding function values, 25% could find the range by determining function values only, and the rest did not answer.



Translate:
1. a. The function "f" with domain {0, 1, 2, 3, 4} is
$\{\frac{5}{3}, 1, \frac{1}{3}, -\frac{1}{3}, -1\}$

Figure 8. LMR-1's Answer about the Range

In Figure 8, LMR-1 can write the range correctly. However, his answer is incorrect regarding language and does not go through the process of finding function values.

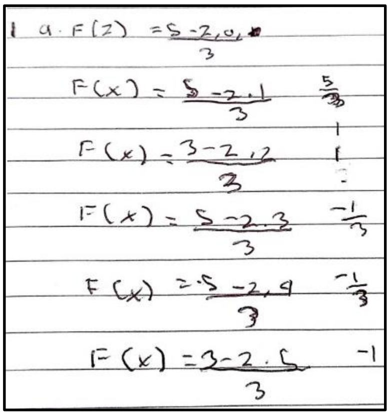


Figure 9. LMR-2's Answer about the Range

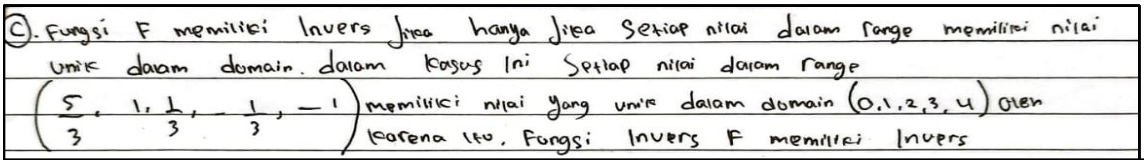
In Figure 9, LMR-2 can determine the function value for each domain member. However, some of the writing is not correct. He writes $f(x)$ even though the value of x has been substituted with domain members. If you look at the work procedures, there are procedures for calculating arithmetic forms that are not quite correct, namely the lack of writing the equal sign and errors in writing numbers. This is also shown in the LMR-2 answer in the aspect of representation with indicator. Students can carry out calculation procedures in algebraic or arithmetic forms and change the form of an expression or equation into another equivalent form which is not correct. The LMR students also show this can’t determine the function's inverse. In the next aspect of representation, 25% of LMR students could present functions in cartesian coordinates, but it was incorrect; 25% of LMR students presented it as a collection of points, and the rest did not answer.

These findings align with the fact that a growth mindset will characterize students with mathematical resilience. This emphasizes that this mindset believes in the desire and effort to learn mathematics. Students with high resilience will change the mindset of "I cannot" to "I can". Students who lack mathematical resilience will experience anxiety and helplessness in learning. Mathematical resilience is considered to be in the opposite direction to student helplessness. Mathematical resilience is part of the resilience that allows students to overcome their anxiety and allows them to learn [31].

In the representation aspect in number 2, the student can identify relevant information, relate the information, and model relationships through algebraic representation. The student can identify relevant information, link the information, and model relationships through algebraic representation by writing equations from data. Students with a high mathematical resilience (HMR) can all present function formulas from the information provided in the table. 79.17% of MMR students can present function formulas from the information provided through tables. Meanwhile, 25% of LMR students could present function formulas from the information provided. This finding aligns with the research [28] that students with low resilience mostly make comprehension errors, students with medium levels of resilience predominantly make transformation errors, and students with high resilience solve more questions correctly. However, some students were seen making process skill errors.

3.2 Generalization Aspect

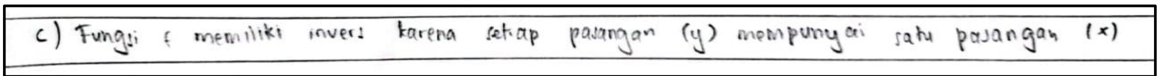
The first aspect of generalization is shown by the indicator that the student can conclude from the graph regarding the condition for a function to have an inverse. As many as 60% of HMR gave reasons for the requirement that a function have an inverse.


c. Fungsi F memiliki Invers jika hanya jika Setiap nilai dalam range memiliki nilai unik dalam domain. dalam kasus ini Setiap nilai dalam range ($\frac{5}{3}$, 1, $\frac{1}{3}$, $-\frac{1}{3}$, -1) memiliki nilai yang unik dalam domain (0,1,2,3,4) oleh karena itu, Fungsi Invers F memiliki Invers

Function F has an inverse if only if every value in the range has a unique value in the domain. In this case, every value in the range $\{\frac{5}{3}, 1, \frac{1}{3}, -\frac{1}{3}, -1\}$ has a unique value in the domain {0, 1, 2, 3, 4}. Therefore, the inverse function F has an inverse.

Figure 10. Conclusions from HMR-1

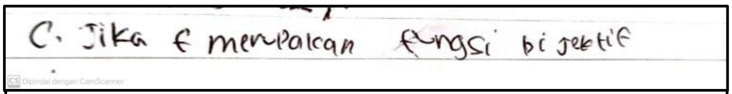
In Figure 10, HMR-1 can conclude that the function f will have an inverse if every value in the range has a value in the domain. 12.5% of MMR students gave a reason for the requirement that a function have an inverse, and the rest did not answer.


c) Fungsi f memiliki invers karena setiap pasangan (y) mempunyai satu pasangan (x)

Translate:
Function f has an inverse because every pair (y) has one pair (x).

Figure 11. Conclusions from MMR-1

In Figure 11, MMR-1 can conclude that the function f will have an inverse because every pair of y has one pair in x. This shows that students wrote the conclusion based on the context of the graph at previously determined cartesian coordinates. Fifty per cent of LMR students gave a reason for the condition that a function has an inverse; the rest did not answer.

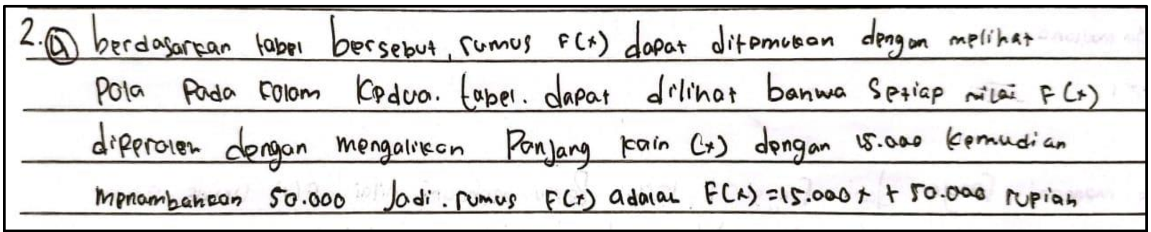

C. Jika f merupakan fungsi bijektif

Translate:
If f is a bijective function

Figure 12. Conclusions from LMR-1

In Figure 12, LMR-1 knows that the condition for a function to have an inverse if the function f is a bijective function. However, the LMR-1 answer does not relate to graphs in cartesian coordinates.

The second aspect of generalization is shown by the indicator determining the function formula from the available data. All students with high mathematical resilience can write function formulas based on the data provided.


2. a) berdasarkan tabel tersebut, rumus f(x) dapat ditemukan dengan melihat pola pada kolom kedua. tabel. dapat dilihat bahwa setiap nilai f(x) diperoleh dengan mengalikan panjang kain (x) dengan 15.000 kemudian menambahkan 50.000 Jadi. rumus f(x) adalah f(x) = 15.000x + 50.000 rupiah

Translate:
Based on the table, the formula $f(x)$can be found by looking at the pattern in the second column. The table shows that each value of $f(x)$ is obtained by multiplying the length of the fabric (x) by 15000 and then adding 50000. So the formula $f(x))$ is $f(x) = 15000x + 50000$

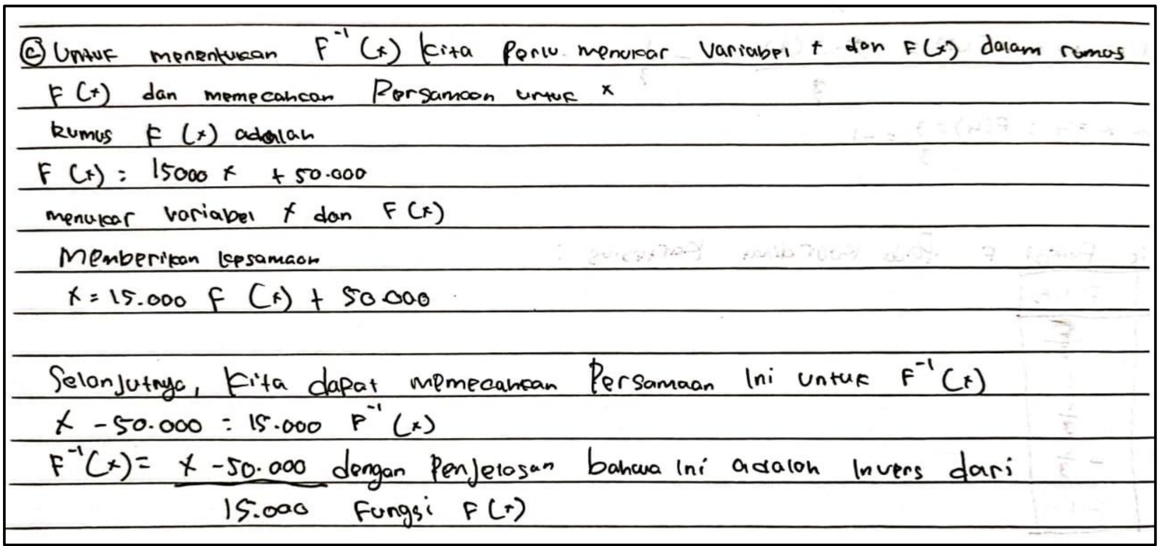
Figure 13. HMR-1's Answer about the General Form

In Figure 13, HMR-1 can be generalized by explaining that to get the general form $f(x) = 15000x + 50000,$ each value of $f(x)$ is obtained by multiplying the length of the fabric (x) by 15000 and adding 50000. 79.17% of MMR students and 50% of LMR students could determine the function formula without explaining, and the rest did not answer.

These findings also show students' abilities to solve problems. Students with a high level of mathematical resilience can face and overcome obstacles and negative situations related to problem-solving because they can successfully train themselves when they experience difficulties solving mathematical problems [26].

3.3 Justification Aspect

Indicators show the justification aspect. The student can explain how to determine the inverse function. As many as 60% of HMR students can explain how to determine the inverse function.



Translate:
To determine $f^{-1}(x)$, we need to swap the variables f and $F(x)$ in the formula $F(x)$ and solve the equation for x.
The formula $F(x)$ is
$F(x) = 15000x + 50000$
swap the variables x and $F(x)$
Gives similarity
$x = 15000\, f(x) + 50000$

Next, we can solve this equation for $f^{-1}(x)$
$x - 50000 = 15000\, f^{-1}(x)$
$f^{-1}(x) = \frac{x-50000}{15000}$ with the explanation that this is the inverse of the function $f(x)$

Figure 14. The Explanation Determines the Inverse Function of HMR-1

In Figure 14, HMR-1 shows and explains the procedure for determining the inverses of a function to obtain an inverse function. However, all students with medium mathematical resilience (MMR) and low mathematical resilience (LMR) did not explain how to determine the inverse function.

The justification aspect related to students' communication abilities in providing explanations can be seen in the research results, which show that students with medium and low resilience have not been able to explain the inverse. This follows research [25], which found that students' mathematical resilience in mathematics learning had a significant effect on students' mathematical communication abilities.

Students' algebraic reasoning in solving inverse function problems in each category of mathematical resilience has different characteristics. (1) In students with high resilience, the representation aspect is shown by students being able to determine the range by determining the functional value for each member of the domain and can write the range in set form; students can carry out algebraic or arithmetic calculation procedures, change the form of an expression or equation into another equivalent form properly, and students can present problems given an algebraic representation in the form of a coordinate graph of the domain and range of a function that has been determined. The generalization aspect is demonstrated by students being able to make conclusions from the graphs that have been made regarding the conditions for a function to have an inverse. Students can generalize to get the function equation's general form from the data given. In the justification aspect, students can explain how to determine the inverse function. (2) For Students with medium resilience, the representation aspect is shown by students being able to determine the range and write it correctly. However, it has not been demonstrated well in terms of carrying out algebraic or arithmetic calculation procedures and changing the form of an expression or equation into another equivalent form. Students already understand questions and describe cartesian coordinates according to the points formed from the domain and range. In the generalization aspect, students can write conclusions based on the graphic context at previously determined cartesian coordinates. Students have not shown the justification aspect. (3) Students with low resilience in the representation aspect, are shown by writing the range without showing the process of determining function values; students are not yet correct in carrying out algebraic or arithmetic calculation procedures and changing the form of an expression or equation into another equivalent form. Students with low resilience have not demonstrated aspects of generalization and justification. Considering these findings, further research can be conducted by designing appropriate learning to improve algebraic reasoning in medium and low-resilience students.

4. CONCLUSION

This study found differences in the characteristics of algebraic reasoning among students with varying levels of mathematical resilience in solving inverse function problems. Students with high resilience could determine the range of the function, correctly perform algebraic or arithmetic procedures, and present problems in appropriate algebraic representations. In contrast, students with medium and low resilience exhibited weaknesses in representation, generalization, and justification. These findings indicate that mathematical resilience is crucial to students' algebraic reasoning abilities. The implications of this study suggest the need for targeted instructional designs to enhance algebraic reasoning in students with medium and low resilience. Developing learning strategies that focus on increasing mathematical resilience can help students face challenges in learning mathematics, improve their algebraic reasoning abilities, and ultimately enhance overall mathematics learning outcomes.

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AUTHOR CONTRIBUTION STATEMENT

AYF contributed to writing the article, analyzing research methods, data collection and processing, implementing data analysis, and preparing the results and discussion sections. RR contributed to providing direction and guidance in topic development, assisting in writing and editing, and providing input on relevant references and literature. SP contributed to providing direction and guidance in topic development, assisting in writing and editing.

REFERENCES

[1] M. Chirimbana, “The effect of a problem based learning approach on the teaching and learning of composition and inverses of functions in a foundation programme,” Ph.D. dissertation, Dept. Education, Stellenbosch University, 2014. [Online]. Available: https://scholar.sun.ac.za/items/90cf7d5f-de3d-4430-b6a7-7ad8fe2c3a16
[2] L. Delastri, Purwanto, Subanji, and M. Muksar, “Students ’ conceptual

understanding on inverse function concept," in *International Conference on Mathematics and Science Education*, 2018. pp. 1-7, doi: 10.1088/1742-6596/1157/4/042075.

[3] K. Chan and Y. Zhou, "Effects of cooperative learning with dynamic mathematics software (DMS) on learning inversely proportional functions," *Int. J. Emerg. Technol. Learn.*, vol. 15, no. 20, pp. 210–225, 2020.

[4] M. Ikram, I. N. Parta, and H. Susanto, "Exploring the Potential Role of Reversible Reasoning: Cognitive Research on Inverse Function Problems in Mathematics," *J. Educ. Gift. Young Sci.*, vol. 8, no. 1, pp. 591–611, 2020.

[5] D. Castillo, J. Carrión, C. Chamba, Y. Jiménez, M. J. Rodríguez-Álvarez, and V. Lakshminarayanan, "Teaching math: A review of effective teaching and learning strategies in higher education,", vol. 1, no. 1, pp. 1-32, 2024, doi : 10.21203/rs.3.rs-4708199/v1.

[6] S. A. Licorish, H. E. Owen, B. Daniel, and J. L. George, "Students' perception of Kahoot!'s influence on teaching and learning," *Res. Pract. Technol. Enhanc. Learn.*, vol. 13, no. 1, pp. 1–23, 2018.

[7] I. Bayazit and E. Gray, "Understanding Inverse functions: The relationship between teaching practice and student learning," in *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, 2004, vol. 2, no. 1986, pp. 103–110.

[8] T. Paoletti, "Reasoning about relationships between quantities to reorganize inverse function meanings: The case of arya," *J. Math. Behav.*, vol. 57, no. 1, pp. 1-24, 2020, doi: 10.1016/j.jmathb.2019.100741.

[9] C. Kieran, "Algebraic thinking in the early grades: What is it," *Math. Educ.*, vol. 8, no. 1, pp. 139–151, 2004.

[10] A. Twohill, "Algebraic reasoning in primary school: Developing a framework of growth points," in *Proceedings of the British Society for Research into Learning Mathematics*, 2013, vol. 33, no. June, pp. 55–60.

[11] D. W. Carraher, A. D. Schliemann, B. M. Brizuela, and D. Earnest, "Arithmetic and algebra in early mathematics education," *J. Res. Math. Educ.*, vol. 37, no. 2, pp. 87–115, 2006.

[12] J. R. Lepak, J. L. W. Wernet, and R. A. Ayieko, "Capturing and characterizing students' strategic algebraic reasoning through cognitively demanding tasks with focus on representations," *J. Math. Behav.*, vol. 50, no. 1, pp. 57–73, 2018, doi: 10.1016/j.jmathb.2018.01.003.

[13] R. Even, "The inverse function: Prospective teachers' use of 'undoing,'" *Int. J. Math. Educ. Sci. Technol.*, vol. 23, no. 4, pp. 557–562, 1992, doi: 10.1080/0020739X.1992.10715689.

[14] J. Nolasco, "The struggle with inverse functions doing and undoing process," California State University, 2018.

[15] D.-D. Andrea, "Investigating student leaning and building the concept of inverse function," The State University of New Jersey, 2011.

[16] M. N. Hanifah and U. Fatmahanik, "Students' learning difficulties in learning mathematics in view of mathematical resilience," in *Syekh Nurjati International Conference on Elementary Education*, 2023, pp. 277–291, doi : 10.24235/sicee.v1i0.14631.

[17] A. O. A. Awofala, "A validation of the mathematical resilience scale for twelfth graders through confirmatory factor analysis and its relationship with achievement in mathematics in Nigeria," *SN Soc. Sci.*, vol. 1, no. 8, pp. 1-17, 2021.

[18] Johnston-Wilder and C. Lee, "Developing mathematical resilience," in *BERA Annual Conference 2010*, University of Warwick, Sept. 1-4, 2010, pp. 1–16.
[19] C. Lee and S. Johnston-Wilder, *The Construct of Mathematical Resilience*. Elsevier Inc : Belanda, 2017.
[20] A. N. A. Hindi and I. Muthahharah, "Teacher's perception of student's mathematics learning difficulties," *Daya Mat. J. Inov. Pendidik. Mat.*, vol. 9, no. 1, pp. 170–177, 2021.
[21] S. Rohmah, T. A. Kusmayadi, and L. Fitriana, "Problem solving ability of junior high school students viewed by mathematical resilience," *Univers. J. Educ. Res.*, vol. 8, no. 7, pp. 3026–3033, 2020.
[22] M. A. Rudin and M. T. Budiarto, "MATHE dunesa," *J. Ilm. Pendidik. Mat.*, vol. 8, no. 2, pp. 232–237, 2019.
[23] P. E. Kobandaha, Y. Fuad, and Masriya, "Algebraic reasoning of students with logical-mathematical intelligence and visual-spatial intelligence in solving algebraic problems," *Int. J. Trends Math. Educ. Res.*, vol. 2, no. 4, pp. 207–211, 2019, doi: 10.33122/ijtmer.v2i4.138.
[24] N. Authary and Nazariah, "Pelevalan penalaran aljabar siswa dalam memecahkan masalah matematika berbasis taksonomi structure of the observed learning outcome (SOLO)," *J. Numer.*, vol. 6, no. 2, pp. 274–282, 2019.
[25] Rifdah and N. Priatna, "The relationship between mathematics resilience and mathematics communication skills," in *International Conference on Mathematics and Science Education*, 2019, pp. 1-6, doi: 10.1088/1742-6596/1521/3/032037.
[26] D. Attami, B. Budiyono, and D. Indriati, "The mathematical problem-solving ability of junior high school students based on their mathematical resilience," in *International Conference on Mathematics and Science Education*, 2020, pp. 1-8, doi: 10.1088/1742-6596/1469/1/012152.
[27] Laelasari, Darhim, and S. Prabawanto, "Analysis of students ' mathematics resilience abilities on linear program material," in *MSCEIS*, 2018, pp. 1-5, doi: 10.1088/1742-6596/1280/4/042005.
[28] A. Haerani, K. Novianingsih, and Turmudi, "Analysis of students ' errors in solving word problems viewed from mathematical resilience," *J. Teor. dan Apl. Mat.*, vol. 5, no. 1, pp. 246–253, 2021.
[29] A. Suri and T. Herman, "How are the contributions of mathematics resilience for developing attitude rubric to assess mathematics learning ?," in *International Converence on Mathematics and Science Education*, 2019, pp. 1-6, doi: 10.1088/1742-6596/1521/3/032055.
[30] S. Arikunto, *Prosedur Penelitian*. Jakarta: Rineka Cipta, 2012.
[31] N. H. Fitrah, N. Fatihah, and A. Madihie, "Resilience in mathematics , academic resilience , or mathematical resilience ?: An overview," *Univers. J. Educ. Res.*, vol. 8, pp. 34–39, 2020, doi: 10.13189/ujer.2020.081905.