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# Students' procedural knowledge on simplex method in linear programming: An explanatory sequential design

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**ABSTRACT**
This study employed a sequential explanatory mixed-methods design to assess students' procedural knowledge in linear programming using the simplex method through quantitative tests, qualitative interviews, and Focus Group Discussions (FGD). The objective was to uncover students' understanding of the step-by-step procedure in the simplex method, with results indicating a low level of knowledge (38.31%) in solving linear programming problems. Thematic analysis from the qualitative phase identified students' perceptions of the problem-solving process. Based on these findings, it is recommended that educators adopt more in-depth and multifaceted teaching strategies to enhance students' procedural understanding. This research has implications for developing teaching strategies to improve students' procedural knowledge in linear programming.

## Pengetahuan prosedural siswa pada metode simpleks dalam pemrograman linear: Sebuah desain explanatori sekuensil

***Kata Kunci:***

Desain sekuensial penjelasan
Pemrograman linier
Pendidikan matematika
Aspek prosedural
Metode simpleks

**ABSTRAK**
Penelitian ini menggunakan desain metode campuran sekuensial eksplanatori untuk menilai pengetahuan prosedural siswa dalam pemrograman linier menggunakan metode simpleks, melalui tes kuantitatif, wawancara kualitatif serta Focus Group Discussions (FGD). Tujuannya adalah mengungkap pemahaman siswa mengenai prosedur langkah demi langkah dalam metode simpleks, dengan hasil menunjukkan tingkat pengetahuan yang rendah (38,31%) dalam pemecahan masalah pemrograman linier. Analisis tematik dari fase kualitatif mengidentifikasi persepsi siswa terhadap proses pemecahan masalah. Berdasarkan temuan ini, disarankan agar pengajar mengadopsi strategi pengajaran yang lebih mendalam dan multi-faset untuk meningkatkan pemahaman prosedural siswa. Penelitian ini berimplikasi pada pengembangan strategi pengajaran yang dapat meningkatkan pengetahuan prosedural siswa dalam pemrograman linier.

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## 1. INTRODUCTION

The simplex method is one of the ways of solving linear programming problems. This method is an iterative procedure for finding the optimal solution of a linear model [1]. This is a topic where procedural knowledge is necessary to complete the step-by-step process of the solution. In a study conducted by Pale [1] in Kenya, he administered a test to 240 high school students from 10 different schools to determine the subject areas where the level of procedural knowledge is low. He conducted a test containing five questions for several Math fields and found that students only got an average of 1.3455 in matrices and 1.3194 in linear programming. The student's scores are relatively low compared to the number of items given per topic. Students are having difficulty learning the procedural concepts of this topic. In Jakarta, Indonesia, Khairunnisa and Darhim [2] explored the procedural and conceptual challenges students face in linear programming, and they found that students have problems with both procedural and conceptual understanding. Their study shows that students have problems finding connections between and among the variables that make up the function and the constraints. Out of the 55 students in their study, only 43 percent could see the relationships and correctly write the equations or constraints of the linear programming problem.

Research on the causes of difficulty of students in matrices was also conducted by Maharaj and Ntuli [3] in South Africa. Some of their research required respondents to solve word problems involving matrices, utilizing the elementary row operation. In an interview, they found that students could hardly arrange the given quantities in a matrix. The students have difficulty rewriting equations to a matrix and correctly arranging them to their designated rows or columns. In this study, the simplex method is utilized to solve linear programming, and one of the steps is to rewrite the equations to matrices.

Further, Bautista et al. [4] conducted an error analysis on linear algebra involving vectors, matrices, linear equations, and determinants in the Philippines. The study findings identified specific errors in solving the abovementioned topics with the following themes: wrong operations, computational errors, defective algorithms, and other random issues or answers. This research identified specific errors on a different topic, including matrices used in the simplex method. The identified errors are errors in the solution, hence part of the procedural process.

Linear programming is part of the mathematics curriculum, and teachers should pay attention to the students' low level of procedural knowledge when learning this topic. Neglecting this would mean failure to attain the learning outcome that will later affect the students in higher math concepts. Knowledge in a simplex method is a pre-requisite and will have a domino effect in attaining the program outcome since these outcomes are aligned. The result of this study will help determine the student's procedural knowledge of the topic, making it easier for the teachers to identify where or what to focus on in the teaching-learning process to enrich the student's procedural understanding. Teachers can design teaching methods and techniques tailored to the student's needs, making it easier for teachers and students to handle this lesson.

Moreover, the findings contribute significantly to a deep understanding of the student's level of procedural knowledge in learning simplex methods in linear programming through this explanatory sequential design. The outcome of this study provides possibilities for curriculum review and enhancement to ensure the attainment of mathematical competencies among students, especially in courses that require basic knowledge and competencies in linear programming. This study can be noted as valuing for all college learners. The researcher aimed to determine the students' procedural knowledge level in learning linear programming through the simplex method and to determine their standpoints on the salient points of the step-by-step process of the solution.

Numerous studies related to students' procedural knowledge have been conducted, including analysis of students' conceptual and procedural understanding [2], analysis of errors in linear algebra [4], and learning linear programming with mathematical modeling [5]. However, research has not yet examined the understanding of students' procedural knowledge using the simplex method in linear programming that focuses on sequential design.

The researcher aimed to determine the students' procedural knowledge level in learning linear programming through the simplex method and to determine their standpoints on the salient points of the step-by-step process of the solution. Previous studies only focus on the graphical solution of the topic and two types of broad understanding in linear programming [6], [7], focusing on procedural and conceptual [2], [8]. Hence, there is a shortage in the conduct of an explanatory type of research investigating the level of procedural knowledge of the step-by-step process of the solution in a simplex method in linear programming.

**Contribution to the literature**

This study contributes to filling the research gaps related to, among others:

- The findings of this study can serve as a guide for teachers in presenting procedural content for the Simplex method, including the importance of reviewing key concepts before starting the discussion.
- Highlighting learning challenges and offering a methodological framework that can be adapted for similar studies in mathematics education and beyond.

## 2. METHOD

This study utilizes a mixed-methods research design. This type of research design focuses on collecting, analyzing, and mixing quantitative and qualitative data in a series of studies. Its rationale is that combining quantitative and qualitative approaches provides a better understanding of the problem rather than utilizing them separately [9], [10].

The mixed-method design employed in this study is explanatory sequential, wherein quantitative and qualitative data are integrated from philosophical assumptions, data collection, and data analysis [11]. Mixing these approaches allows us to explain and understand the significant and non-significant results of the quantitative phase further through the qualitative data collected. Explanatory sequential mixed-method design is a two-phase process in which the quantitative result is enriched using the extracted qualitative data [12]. Furthermore, the quantitative result of the study is elaborated, explained, and made sense through the qualitative data. In this study, the test was conducted first (quantitative phase) to determine the student's difficulty in a particular part of the solution. After these difficulties and their existence were identified in the quantitative phase, these were further understood by collecting qualitative data through in-depth interviews (IDI) and Focus Group Discussion (FGD) with the chosen respondents. This way, the understanding of the problematic issue is explored more comprehensively.

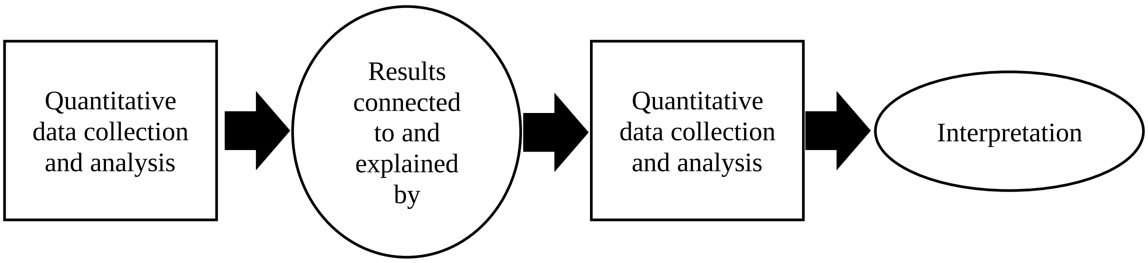


**Figure 1.** Explanatory sequential design

Figure 1 below shows the explanatory sequential design process done in this study. This is the Participant Selection Model formulated by Creswell and Plano Clark [10], wherein a two-phased study starts with the quantitative phase to gain a general understanding of the problem, followed by the qualitative phase to have an in-depth understanding of the respondent's viewpoint. The explanatory-sequential type of mixed-method design starts with collecting quantitative data [9]. This research made use of descriptive statistics, which was used to summarize data in an organized manner by describing the relationship between variables in a sample or population [7].

In this study, the quantitative data was gathered through test administration to the respondents. One linear programming problem was given to the students, and they were asked to solve it using the Simplex method. The questionnaire used in this phase helps determine the participants' level of procedural knowledge.

In the qualitative part of the study, a hermeneutical phenomenological approach was used to understand better the participants' standpoints on the salient points of the process. The exploration of the phenomenological approach is focused on a group of individuals who have all experienced the phenomenon [13]. The students who took the Mathematics in the Modern World Program were focused on this study. A phenomenological approach to qualitative research has two types: hermeneutical phenomenology and transcendental or psychological phenomenology.

**Table 2.** Flow of procedures

| Phase | Process | |
|---|---|---|
| | **Activities** | **Result** |
| Quantitative data collection | Test administration | Numeric data:<br>• Student's scores<br>• Points are garnered by the students in each |
| Quantitative data analysis | Descriptive statistics:<br>• Mean<br>• Standard deviation | Mean percentage (38.31) and standard deviation percentage (24.35) |
| Results connected to and explained by | Developing interview questions | Interview guide questions |
| Qualitative data collection | IDI and FGD | Interview responses |
| Qualitative data analysis | Reflexive thematic analysis | Core ideas and common themes:<br>• Manageable steps<br>• Computational error<br>• Poor retention and lack of mastery<br>• Procedural nature of the solution |
| Integration of the quantitative and qualitative results | Interpretation and explanation of the quantitative results through the qualitative results | Discussion and implication |

Table 2 explains the procedure for conducting this explanatory sequential mixed-method research. The sequence started with the quantitative part and its analysis, followed by the qualitative part and interpretation. In the quantitative part, data collection was done through test administration and then analyzed through descriptive statistics, specifically computing the quantitative data's mean percentage and standard deviation. The results were used as the basis for the IDI and FGD interviews. The questions were contextualized based on the student's answers to the test. The qualitative data of this study are interview responses from the respondents on their standpoints on the salient points of their answers in the steps of the solution. The qualitative data was then analyzed through thematic analysis, wherein core ideas were extracted from the interview responses and used to determine essential themes. After the qualitative data analysis, quantitative and qualitative results were integrated, where qualitative results were used to explain and confirm the quantitative results.

## 3. RESULTS AND DISCUSSION

The overall mean percentage of all the steps is 38.31 under the category level, low, with an overall standard deviation of 24.36, which tells us that the data is dispersed. The interpretation of the low mean level is that the students' procedural knowledge is poor. This result parallels the study of Khairunnisa and Darhim [2], where only 60 percent of the respondents could implement the step-by-step rules of linear programming. Since the procedural knowledge or the capability to follow procedures of the students in this lesson is poor, this tells us that it is indeed challenging for the students to perform the multi-step solution of linear programming utilizing the simplex method. Table 2 shows the level of students' procedural knowledge.

**Table 2.** Level of procedural knowledge of students

| Indicators | Mean Percentage | SD | Descriptive Level |
|---|---|---|---|
| Step 1: Rewriting the objective function in general form | 60.61 | 40.67 | Moderate |
| Step 2: Using the slack variables s and t and rewriting the inequalities as equation | 54.55 | 44.30 | Moderate |
| Step 3: Setting up the matrix | 53.03 | 36.60 | Moderate |
| Step 4: Determining the pivot column, row, and element | 43.94 | 34.71 | Moderate |
| Step 5.a: Computation – reducing the pivot element 1 (if it is already 1, as is) | 33.33 | 27.22 | Low |
| Step 5.b: Computation – performing the elementary row operations to make the elements on the same column as the pivot element zero. | 15.15 | 16.98 | Very Low |
| Step 6: Rewriting the new matrix | 7.58 | 14.30 | Very Low |
| Overall | 38.31 | 24.35 | Low |

In step 1, the students' procedural knowledge level is 60.61, categorized as moderate. This is the highest mean average among the six steps. The quantitative data result in this step confirms that the students know the operations and symbols used in this step and can use them in performing the process [14]. In the study of Khairunnisa and Darhim [2], this indicator also got 60 percent, the step with the highest mean percentage, similar to this study's result.

The second step of the solution uses the slack variables s and t and rewrites the inequalities into equations. The level of procedural knowledge in this step is 54.55, which is still moderate. This means that the student's procedural knowledge in this step is satisfactory. In the study of Khairunnisa and Darhim [2], a mean average of 51 percent was garnered. This is similar to the result of this paper, wherein the level is a little more than half. Since the data showed that the procedural knowledge was satisfactory, the inequality symbols were correctly interpreted, and the slack variables were correctly used to rewrite the inequalities to equalities.

Step 3 has a mean percentage of 53.03. In this step, students set up the initial matrix from the answers in steps 1 and 2. The mean 53.03 is under moderate level, just like steps 1 and 2. The procedural knowledge of the students in this step is satisfactory. In the context of this step, the moderate level tells us that students can set up the initial matrix. This result parallels the study of Khairunnisa and Darhim [2], garnering 60 percent in this indicator. This result contradicts the claim of the study of Maharaj and Ntuli [3] that this is part of linear algebra, where students' procedural knowledge is very low since most of the errors in the computation process are concentrated here.

The level of procedural knowledge in step four is moderate, with an average of 43.94. Their procedural level in this step is still satisfactory; therefore, the participants can correctly determine the matrix's pivot column, row, and element. This indicator, generally referred to as the ability to specify the key column and key row, was used in the study of Khairunnisa and Darhim [2] and also got a mean average of 60 percent, which was categorized as good. These parallel results tell us that students can follow through on determining pivot rows, columns, and pivot elements.

Steps 5a and 5b belong to the low-level category with a mean of 33.33 and 15.15, respectively. The interpretations of these mean percentages are poor and very poor because the participant's procedural knowledge level is poor. In this step, students struggle and commit mistakes in reducing the pivot element to 1 and making the other elements in the pivot column zero. This finding is supported by the research of Pale [1], whose findings stated that the performance in linear programming is very poor, garnering an average of only 1.3194 over 5. In the study of Khairunnisa and Darhim [2], this indicator is referred to as the transformations to obtain optimization with correct calculations, and the level of procedural knowledge is at 16 percent, which is very low. The study's result is parallel to this study's result.

Lastly, step 6 garnered an average of 7.58, the lowest among all the means. The level of this process is very low, implying that participants are grappling with rewriting the new matrix after performing the elementary row operation in step 5b. Since procedural knowledge is very poor, it can be interpreted that the participants lack the technical know-how to draw systematic steps to answer a problem correctly [15].

This study was anchored on Jean Piaget's theory of cognitivism. In this theory, learners are believed to stock information in the memory through schema [4]. This explains the decreasing mean percentages of the students in the multi-step solution of a linear programming problem using the simplex method. Since the procedure has six steps, students get higher scores or have higher chances of remembering the first few steps since the connections of the schema of the information are still strong; however, as the process continues, the connections of the information become weaker, which led to incorrect answers and low mean percentages in the latter steps.

Aside from this, we can also notice that steps 1 to 3 only consist of a few processes compared to the remaining steps. In step 1, the student only needs to transpose terms; in step 2, the student must rewrite the inequalities to equalities; and in step 3, it only requires arranging the numerical coefficients that were already identified. Since these steps only include a few processes, it is also more accessible for the learners to remember these things, and it can be managed by the internal factor or thought process, which was used during the test without the intervention of the external factor. This explains why these first three steps got the highest mean percentages, which are all more than 50 percent, which are all described as moderate. Steps 4, 5, and 6, which are the last three steps, also got the three lowest mean percentages because these steps are composed of lots of processes. In step 4, when we identified the pivot column, row, and element, there was a basis that needed to be understood and remembered, which is why, although this step is still categorized as moderate, its mean percentage still did not make it to 50 percent. In steps 5a and 5b, the elementary operation is performed, which is lengthy because of the many operations involved. Because of this, students have difficulty connecting information gathered and stored in the internal thought process, which is why these steps were categorized as low and very low.

In a Piagetian cognitivist classroom, it is firmly believed that each learner is unique and has unique characteristics [16]. This made sense of the varied interview responses wherein some students said the process was manageable while others disagreed.

In this study, the means in the quantitative data were explained by the qualitative results. The causes of the mistakes and struggles experienced by the students are computational errors, the procedural nature of the solution, poor retention, and lack of mastery. According to Kilag et al. [16], emphasis on the thinking process and focus on practices are primely important in a Piagetian classroom. That is, the thinking process of the learners on the concept is a priority, and that repetitive practice is always observed. The essential themes are explained by a classroom's characteristics that highlight the application of Cognitive theory. The computational errors of the students are attributed to the lack of requisite knowledge of the lesson. The thinking process of the learners should be focused on in such a way that the acquisition of the concepts is ensured. With this, the requisite concepts are learned, and problems in learning caused by a lack of requisite knowledge are avoided or at least lessened. Lastly, drills and opportunities to practice the concepts are highlighted here, which is the solution to poor retention and lack of mastery. The solution to forgetting and even the key to mastering procedures is to involve the learners in drills wherein a particular knowledge or skill is practiced [17].

### *3.1 Standpoints of the participants on the salient points on the steps*

The themes identified in the thematic analysis will be discussed individually in this part of the research. This is to further talk through the standpoints of the participants and how these affect their level of procedural knowledge in solving linear programming through the simplex method.

**Table 3**. Standpoints of the participants on the salient points of the steps

| Focal Point | Core Ideas | Essential Themes |
|---|---|---|
| Standpoints of the participants on the salient points of the steps | Step 1 is easy.<br>There is no confusion in this step.<br>The step is manageable.<br>Step 2 is easy.<br>The participant can perform the step with ease.<br>Step 3 is easy.<br>The students are okay with the process; they can perform the step correctly.<br>The participants were not confused in this step because columns and rows were labeled to ensure that coefficients were correctly placed in the matrix. | Manageable Steps |
| | Transposing the term with the variable z instead of x and y.<br>Confusion in changing of signs in the transposition method.<br>Confusion on where and how to insert the slack variable.<br>There are too many variables (which causes the said confusion).<br>The student got confused with the numerical coefficients of s and t.<br>Setting up the matrix is hard and confusing because one does not know where the inputs came from.<br>Input incorrect elements in the matrix due to incorrect identification of the numerical coefficient.<br>The respondent chose the row with the highest instead of the lowest positive quotient (of constant and its corresponding element in the pivot column)<br>Struggling in performing operations involving fractions.<br>They are having difficulty locating the pivot element.<br>Difficulty in performing operations with opposite signs and fractions. | Computational Error |

| | | |
|---|---|---|
| | The step is less complicated with the aid of a calculator to perform arithmetic operations. | |
| | The participant forgot what to do in step 2.<br>The participant thinks that they will be able to answer the step; it is just that they cannot recall how the step is done.<br>The participant finds it hard to solve this step on their own.<br>Interchanged the operations (multiplied instead of divided) because he was confused about whether to multiply or divide.<br>Forgot when to change the row label.<br>They get lost in multiplying the reciprocal to the elements (supposedly to the elements in the pivot row) because of too many inputs.<br>Forgetting the correct operation and getting confused due to Simplex's nature of having a long solution.<br>Gets confused and lost in performing the elementary row operation due to too many elements.<br>Getting lost in pairing the corresponding elements in the matrix when performing the elementary row operation.<br>They are familiar with the process but make mistakes because they get lost when performing the elementary row operation. | Poor retention and lack of mastery |
| | The participant got lost in this process since answers in steps 1 and 2 were incorrect.<br>The participant claims that if he had identified the correct pivot column, row, and element in step 3, he would have been able to answer this step correctly.<br>This step will be answered correctly if the correct matrix is set up.<br>This step was left unanswered because they could not answer step 5b.<br>Got the wrong answer in step 6 due to an incorrect answer in step 5b.<br>Did not proceed to step 6 because the solution in the previous steps was incomplete. | Procedural Nature of the Solution |

Table 3 presents the standpoints of the participants in the salient points of the steps. This consists of a thematic analysis of this research's qualitative data, collected through in-depth interviews and focus group discussions. The table shows that the seven steps were merged into one focal point in the first column. The second column contains the interview responses' core ideas, combined according to the common essential themes in the third column. In addition, the sample responses of the respondents in the IDI and FGD are presented here.

### *3.1.1 Manageable Steps*

This essential theme was extracted from steps 1, 2, and 3 of the solution. In these steps, the repetitive core ideas center is all about how easy, manageable, and clear the steps are. This theme explained the high means of the first three steps, all categorized under the moderate level in the quantitative phase. This means that their procedural knowledge is satisfactory. This theme was also negated in the paper of Ferryansyah et al. [18], wherein the respondents admittedly said that they had difficulty using and comprehending the use of notations and variables.

### *3.1.2 Computation Errors*

This theme emerged in steps 1, 2, 3, 4, and 5a. These computation errors were attributed to a lack of pre-requisite knowledge of the lesson. The participant's inability to acquire requisite knowledge for this lesson affects the procedural knowledge level. The lack of pre-requisite understanding is the primary cause of the learning problem experienced in this lesson [3], [19]. The study of Kilpatrick & Izsak [20] shows that students struggle to perform the necessary steps in solving problems in Linear Algebra because they have not acquired or mastered the knowledge or foundation they need to acquire.

This is supported by the study of Bautista et al. [4], wherein computational errors were evident, centered on wrong operations, defective algorithms, and other random issues. These errors are usually committed due to carelessness in the computational process, and one commits mistakes in carrying out the steps in simplifying [4]. In addition, Hidayanti [21] also found in his study that the computational errors on matrices are due to the student's carelessness in operating with the given matrices, lack of focus in operation, and confusion when too many rows and columns are involved.

### *3.1.3 Poor Retention and Lack of Mastery*

This third theme was also repetitively extracted from the core ideas of the interview responses. Poor Retention and Lack of Mastery are essential themes in steps 2, 3, 4, 5a, and 5b. In step 2, core ideas stated that the step would be answered had the student not forgotten the process. In step 3, the participant admitted that it was hard to answer the step alone and that she still needed a guide to solve the given problem correctly. In step 4, the student interchanged the operations since he forgot what came first. Lastly, in step 5b, wherein elementary row operation is part of the process, the respondents admitted that they got lost and tended to perform the operations to incorrect elements. This is because they have yet to become used to the process. The mastery of the respondents has not yet been developed.

This theme is supported by the study of Veloo et al. [22], where Mathematics difficulty is mainly traced to the student's forgetfulness. When students were asked about the computational, conceptual, and other errors they committed in their answers, they said that it was because they forgot how to perform the process or solution.

Forgetting is a natural tendency of the mind [17]. Even when we study, we still determine if we can retrieve everything we have read and reviewed when needed. To solve this problem, more practice is needed to transfer rote learning from short-term memory to long-term memory [17]. Drills are necessary to lessen the possibility of forgetting and increase the chance of transporting the information to long-term memory. The opportunity to practice concepts is a liability for both teachers and students. In the classroom, teachers provide problems for the students to practice; outside the classroom, it should be the student's responsibility and initiative to provide another opportunity, especially when they know he or she needs it.

### *3.1.4 Procedural Nature of the Solution*

Each step must be sufficiently understood and mastered to successfully acquire procedural knowledge. This essential theme emerged in several steps, like the other themes, particularly in steps 3, 4, and 6. The procedural nature of the solution means that the role of previous steps plays an important role in completing the following steps. The inability to answer correctly in the preceding means that the following steps must also be corrected. Naturally, this essential theme emerges in the later steps wherein precursory steps exist. In step 3, setting up the correct matrix requires correct answers in steps 1 and 2 since the inputs are based on those steps.

This theme parallels Bautista et al.'s study [4], who said that solving matrix operations consists of a series of steps and procedures and making a mistake in one step will have a domino effect on the next steps. Some responses in his study were irrelevant to the lesson since the students forgot to recall the correct procedure.

### *3.2 Integration of the Quantitative and Qualitative Data Results*

In an explanatory sequential design, the integration approach connects [23]. In this approach, the quantitative data results will be explained by the qualitative data. Table 4 contains the research area, quantitative and qualitative data results of the study and the nature of integration used.

**Table 4.** Joint display of quantitative and qualitative results

| Research Area | Quantitative Results | Qualitative Results | Nature of Integration |
|---|---|---|---|
| Level of Procedural Knowledge of the Participants in Each Step | Step 1: The mean is 60.61 (Moderate) with a standard deviation of 40.67.<br><br>Step 2: The mean is 54.55 (Moderate) with a standard deviation of 44.30.<br><br>Step 3: The mean is 53.03 (Moderate) with a standard deviation of 36.40.<br><br>Step 4: The mean is 43.94 (Moderate) with a standard deviation of 34.71.<br><br>Step 5a: The mean is 33.33 (Low) with a standard deviation of 27.22.<br><br>Step 5b: The mean is 15.15 (Very Low) with a standard deviation of 16.98.<br><br>Step 6: The mean is 7.58 (Very Low) with a standard deviation of 14.30. | Manageable Steps. (Essential Theme emerged in steps 1, 2, and 3)<br><br>Participants find the step easy and manageable. There was no confusion experienced on their end, and the step can be answered easily.<br>Computation Error. (Essential Theme emerged in steps 1,2, 3, 4, and 5a)<br><br>The computational errors committed in the steps were mainly attributed to the lack of requisite knowledge of the lesson.<br>Poor retention and lack of mastery. (Essential Theme emerged in 2, 3, 4, 5a, 5b)<br><br>This theme emerged in the latter parts of the step, where learners forgot how to perform it. This theme did not come out as a theme in Step 1. Poor retention and lack of mastery became more evident in the later steps, especially in step 5a, which includes the elementary row operation.<br>*Procedural Nature of the Solution.* (This essential theme emerged in steps 3, 4, and 6)<br><br>Incorrect and incomplete answers in the previous steps will affect the accuracy of the following steps. This theme emerged in step 3 since steps 1 and 2 are requisites in step 3. It's also evident in step 4 since the incorrect matrix in step 3 affects step 4, and all the steps are requisite in step 6. | Connecting and Merging (Confirmation) |

The approach used in this study in data integration is merging for convergent design since this study uses the explanatory design [23]. This study uses an explanatory sequential design, so a connecting approach will be utilized in data integration.

Using the simplex method, the quantitative data showed us the students' procedural knowledge levels in each step of the solution in linear programming. This means that the students' procedural knowledge level has a decreasing trajectory; the farther the step is from the beginning, the lower the level of procedural knowledge is, and the more difficult it is for the students. The overall average of the steps is 38.31, categorized as low, which means that the student's level of procedural knowledge is poor. The means of the steps from step 1 to step 6 are 60.61, 54.55, 53.03, 43.94, 33.33, 15.15, and 7.58, respectively. The first four steps were described as moderate, 5a is low, and 5b and 6 are very low. These procedural knowledge levels were further investigated by asking the participants about salient points of the quantitative results. The decreasing trend of the means from 60.61 in step 1 to 7.58 in step is generally attributed to the procedural nature of the solution. The mistakes in the preceding steps, 1 to 5, were accumulated, which explains the rapid 53.03 mean decrease. This is explained in the study of Bautista et al. [4], wherein they stated that the mistakes in the first few stages affect the accuracy of the latter steps, i.e., wrong answers in the previous steps also mean wrong answers in the following steps wherein he stated that solving matrix operations consist of a series of steps and procedures and making a mistake in one step will have a domino effect on the next steps.

Noticeably, steps 1, 2, and 3 got high mean, even moderate, which means that the student's procedural knowledge is satisfactory. The quantitative data confirmed this quantitative result, wherein an essential theme emerged regarding the manageable steps. Participants have claimed that the step is easy, not challenging, and not confusing.

The themes of computational errors, poor retention, and lack of mastery explain the low averages in the later steps. These themes emerge three to five times out of the six steps, which means the participants repetitively experience them. The low scores and mistakes committed in general can be traced to computational errors primarily due to the lack of requisite knowledge. Lack of retention and mastery also explains the low average of the later steps, especially step 5b. The lack of mastery was caused by the lengthy process of finding the solution. In addition, students lack concentration when operating, lack symbolic understanding of numerical coefficients and variables within an equation, and are confused concerning the prescribed technique using row operation to transform the augmented matrix to echelon form [3].

The student's dropping score in the quantitative phase is attributed to the internal factor, also known as the students' thought process. As an information processor, the student's memory connections decline through lengthy steps [24]. With that, more students commit mistakes as the process continues. Thus, students get a high mean percentage of 60.61 in step 1 and a very low mean percentage of 7.58 in the last step.

The qualitative result shows four essential themes in the standpoints of the participants in the salient points of their answer per step. The essential themes are the manageable steps, computation error, poor retention and lack of mastery, and procedural nature of the solution. The manageable step made sense of the satisfactory procedural knowledge of the students in steps 1, 2, 3, and 4. The computation error, poor retention lac,k of mastery and the procedural nature of the solution explain the low mean scores in the later steps of the solution. Generally, the decreasing trend of the quantitative result is explained by the last essential theme about the procedural nature of the solution. The diminishing mean is caused by the accumulated mistakes in the preceding steps, 1 to 5, that later manifested in step 6.

The essential themes of this research, which are student computational errors, poor retention and lack of mastery, and the procedural nature of the solution, can be explained from the point of view of cognitivism. The computational error, primarily caused by the lack of requisite knowledge of the students and the procedural nature of this solution, can be solved by emphasizing the student's mental process. The student's mental process should be focused on the pre-requisite knowledge to guarantee that all the steps in the solution are understood and that all the necessary concepts are acquired. The problem of poor retention and lack of mastery can be surpassed by providing practice to students [24].

Since the students said they struggled to answer the step because they forgot the process, they will be given more opportunities to practice. These learning components emphasize the learning process and the importance of drills or practices, highlighted in Piaget's Cognitivist classroom [2].

## 4. CONCLUSION

This study explores students' procedural knowledge of the Simplex method in linear programming through an explanatory sequential mixed design, integrating quantitative and qualitative analysis. The results reveal that, in general, students have a low level of procedural knowledge, with increased difficulties in advanced steps of the solution process. These findings highlight the importance of a teaching approach that focuses more on students' procedural understanding and offers significant insights for developing curriculum and teaching strategies that can enhance students' understanding of linear programming, particularly through the Simplex method. This research contributes to the academic literature and provides practical recommendations for educational practitioners in designing and implementing more effective teaching.

## AUTHOR CONTRIBUTION STATEMENT

LRSR contributed to the conceptualization of the research framework, design of the quantitative study, analysis of the quantitative data, and drafting of the initial manuscript, and JCC was primarily involved in the design of the qualitative study, data collection and analysis, integration of quantitative and qualitative results, and significant revisions of the manuscript.

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