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ETHNOMATHEMATICS: EXPLORATION OF FRACTAL GEOMETRY IN GATE ORNAMENTS OF THE SUMENEP JAMIK MOSQUE USING THE LINDENMAYER SYSTEM

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ABSTRACT

Jamik Sumenep Mosque, a heritage site of the Sumenep Kingdom, was built to emphasize the acculturation of Chinese, Islamic, and Madurese cultures. This ethnomathematics research aims to reveal and explore the fractal geometry forms in the gateway of Jamik Sumenep Mosque. This study employed qualitative methods with an ethnographic approach. Research data were gathered through observation, field notes, documentation, interviews, and literature studies. Field observations were computationally analyzed using the Lindenmayer system through the L-Studio application to examine fractal shapes. The study found that the length and angle size of the ornamental parts form the basis of fractal geometry in the mosque gateway's ornamentation, thereby confirming the presence of fractal geometry concepts. These findings can be utilized in teaching fractal geometry, applied mathematics, and computational geometry. Further research could explore non-Euclidean geometry methods, such as stochastic L-system methods.

ETNOMATEMATIKA: EKSPLORASI GEOMETRI FRAKTAL PADA ORNAMEN GAPURA MASJID JAMIK SUMENEP MENGGUNAKAN *LINDENMAYER SYSTEM*

Kata Kunci:

Etnomatematika
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Masjid Jamik Sumenep
Sistem lindenmayer

ABSTRAK

Masjid Jamik Sumenep menjadi salah satu masjid peninggalan Kerajaan Sumenep yang dibangun dengan menitikberatkan pada akulturasi budaya Tionghoa, Islam, dan Madura. Tujuan dari penelitian ethnomatematika ini yaitu untuk mengungkap dan mengeksplorasi bentuk geometri fraktal pada gapura Masjid Jamik Sumenep. Penelitian ini menggunakan metode kualitatif dengan pendekatan etnografi. Data penelitian diperoleh dengan observasi, catatan lapangan, dokumentasi, wawancara, dan studi literatur. Hasil observasi lapangan dianalisis secara komputasi menggunakan lindenmayer system melalui aplikasi L-Studio untuk melihat bentuk fraktal. Hasil penelitian menunjukkan bahwa ukuran panjang serta besar sudut pada bagian ornamen menjadi dasar pembentuk geometri fraktal dalam ornamen gapura masjid. Dengan demikian terdapat adanya konsep geometri fraktal pada ornamen gapura masjid. Temuan ini dapat dimanfaatkan dalam pembelajaran geometri fraktal, matematika terapan, dan geometri komputasi. Penelitian lanjutan dapat dilakukan dengan metode geometri non euclid, seperti metode stokastik L-system.

1. INTRODUCTION

One of the objectives of learning mathematics is for students to be able to use mathematics to solve various problems in everyday life [1]–[3]. Various complicated issues in life can be solved logically and practically with mathematics. Therefore, learning mathematics presents multiple problem situations, problems, and tasks that are always related to the context of everyday life [4], [5]. That is to prepare and familiarise students to become problem solvers in life.

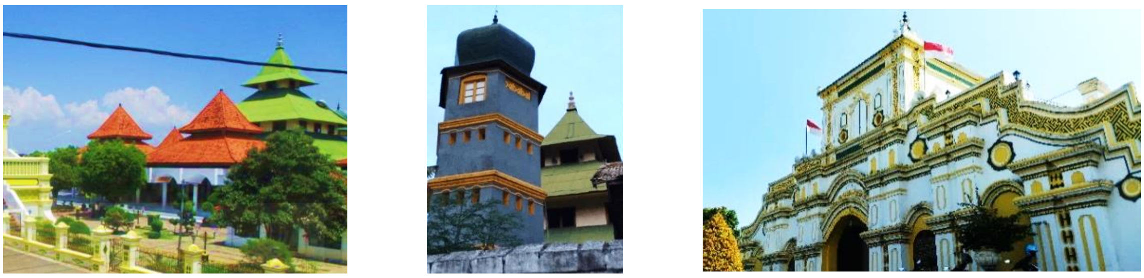
However, the contextual approach only provides students with mathematical problems related to everyday life to solve [6]. In other words, the contextual approach has not offered 'good values' in mathematics learning [7], [8]. In comparison, one of the principles of education is to make students have good attitudes and love for the country [1], [9], [10]. Therefore, there is a need for a different approach that relates mathematical problems to everyday life and teaches the good values contained in mathematics.

One alternative that can provide 'value' in learning mathematics is through a culture-based mathematics approach or what is known as ethnomathematics [7], [11]. Ethnomathematics not only plays a role in teaching mathematics in a culture but also provides students with cultural values and the meaning of a culture [7], [12]. That means the relationship between mathematics and culture in Ethnomathematics is bi-implicative [13]. In other words, Ethnomathematics has additional benefits for students in introducing knowledge from a culture, instilling a love for the country, and fostering a sense of tolerance and appreciation between cultures [7], [14], [15].

Ethnomathematics is the application of mathematics in cultural groups [11]. In addition, Ethnomathematics is interpreted as mathematical ideas found in culture [12]. Ethnomathematics shows that mathematics concepts and applications can be found in everyday social and cultural life [8], [16]. Furthermore, Ethnomathematics explores 'hidden mathematics' and uncovers mathematical concepts in culture and society [17], [18]. Thus, Ethnomathematics can be a 'path' that connects mathematics with culture.

Cultural diversity gives Indonesia much potential for ethnomathematics studies, including Ethnomathematics on artefacts [19]. Artefacts are cultural heritage and human works from the past that have historical value [20]. Artefacts are also interpreted as objects of historical and cultural importance [21], [22]. One of Indonesia's artefacts with historical and cultural value is the Sumenep Jamik Mosque.

One of the heritage mosques of the Sumenep Kingdom that is still standing and functioning is the Jamik Sumenep Mosque [23]. This mosque is hundreds of years old and still stands strong, working correctly [24]. This mosque was built in 1779 AD during the Panembahan Sumolo kingdom by a Chinese architect named Lauw Piango [23]. Lauw Piango is the grandson of Lauw Kun-Thing, a Chinese who fled from Semarang during the 'Chinese riots' in 1740 AD [25], [26]. Parts of this mosque show acculturation between Madurese and Chinese culture, such as the main building, towers and gates [27]. The characteristics of the building at the Sumenep Jamik Mosque are shown in Figure 1.



(a) (b) (c)

Figure 1. Parts of the Sumenep Jamik Mosque (a) The Mosque, (b) The tower, and (c) The Gate

The gate at the Sumenep Jamik Mosque is the main gate to the main building of the Sumenep Jamik Mosque. This gate has two floors and three special rooms [27]. The two rooms on the first floor have iron bars that function as prisons for criminal offenders [23]. Meanwhile, one more room on the second floor has a large drum. This room is where the drum is sounded, a marker of prayer time [23], [28]. The room at the gate at the Sumenep Jamik Mosque is shown in Figure 2.

(a)

(b)

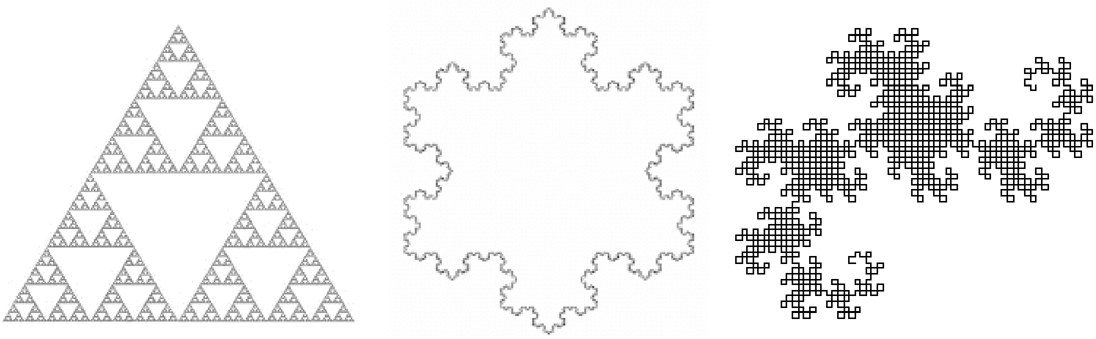
Figure 2. The Room at the Gate of the Sumenep Jamik Mosque (a) First-Floor Room and (2) Second-Floor Room

In addition to having a social and religious function at its time, the gate at the Sumenep Jamik Mosque is decorated with unique and aesthetic ornaments [23]. The ornaments are decorated with yellow, white and green colours and have geometric shapes [27]. These ornaments are found around the mosque's gate, giving the nuance of the beauty of Madurese-Chinese acculturation [28], [29]. The shape of the ornament is not only regular but also seems complicated and repetitive, as shown in Figure 3.

Figure 3. Ornament on the Sumenep Jamik Mosque Gate

Based on preliminary observations, the shape of the ornament shown in Figure 3 is similar to that of a fractal. Fractal geometry is a new part of maths that can represent natural phenomena with non-integer dimensions (fractal dimensions). It can be very helpful [30]. Fractal shapes are not fixed, including them in the non-Euclidean geometry section [31]–[33]. Fractal shapes have regular repetition and self-similarity at each level [34]. Several fractal shapes can be shown in the Sierpinski triangle, the Mandelbrot set, the Julia set, the Hilbert curve, and the dragon curve [31], [35].

Furthermore, fractal forms can be found in nature, such as in clouds, lightning, plant branches, and others [36], [37]. The repeated fractal shape visually gives it its aesthetic value [38]. An example of a fractal shape is shown in Figure 4.



(a) (b) (c)

Figure 4. Some Examples of Fractal Visualization (a) the Sierpinski Triangle, (b) the Snowflake Koch Curve, and (c) the Dragon Curve

Complicated fractal shapes are difficult to formulate using Euclidean geometric concepts [38], [39]. Therefore, a special method is needed to reveal the mathematical form of fractal geometry, one of which uses the Lindenmayer system method. The Lindemayer system, also known as the l-system, is a geometric rewriting method pioneered by Astrid Lindenmayer. This method uses a computationally assisted geometric transformation base to produce visual objects [34], [40]. Computationally, the L-system is an iterative and recursive rewriting of 'strings' using certain functions. The function built is repetitive and aims to return to the initial function [36].

The L-system is a tuple $G = (V, \omega, P)$. V is defined as a finite set, ω as the initial function on the string V, and P as a production set on a finite set or production rule. The deterministic L-system or D0L-system has the rule $A \rightarrow x$, for $A \in V$, which is interpreted as the initiator. At the same time, x is a string in V, called the generator of A. The generator maps x to exactly one in the production rules. Each A is a member of V as an initiator [40].

For example, given an l-system with the string $\omega_i = A_1 ... A_m$ for each $A_i \in V, 1 \leq i \leq m$, then the → symbol is defined as the mapping $A_1 ... A_m \rightarrow x_1 ... x_m$, where $A_i \rightarrow x_i$ is in P for every $i, 1 \leq i \leq m$. When the process repetition is done $n - 1$ times, the process runs from the initiator $(\omega = \omega_1 \rightarrow \omega_2 \rightarrow \cdots \rightarrow \omega_n)$. The initiator sequence $\omega_1, \omega_2, ... \omega_n$ has n iterations. Then, each initiator and generator will repeat according to the rules, which produce images in two-dimensional and three-dimensional space [40].

In the l-system, the symbol F means to draw a straight line in one step. The + or – symbol will direct F to rotate clockwise or counterclockwise in two-dimensional space. The symbol & or ^ controls three-dimensional space's up and down rotation. Whereas the symbol / or \ controls rotation to the right and left in three-dimensional space [34], [36], [37].

Several previous studies have examined Ethnomathematics in historical mosques. For example, revealed the concept of geometric transformation in the Great Mosque of Al-Barkah Bekasi [19], revealed mathematical concepts in the Great Mosque of Demak [41], and explained the mathematical concepts in the Historic Ulu' Mosque in Turkey [42]. The three studies are only limited to finding mathematical concepts that are Euclid geometry, such as finding the shape of a flat shape, a space, finding the area of a shape, finding the volume of a space, and geometric transformations. These studies also have not explored the concept of non-Euclid geometry. Meanwhile, the study of geometry in mathematics is limited to Euclid spaces and non-Euclid spaces, such as fractal geometry.

On the other hand, several studies have also been conducted on non-Euclid geometry objects in historic mosques. The research examined the recurring patterns in the Ijami Mosque in Iran [43], and other research looked at group symmetry in the Sultan Qaboos

Grand Mosque in Oman [44]. Both studies only show various types of ornamental patterns that decorate the walls of mosques. Furthermore, the research looked at these forms from an algebraic perspective using group theory [44], while other research reconstructed the pattern using the tessellation method [43]. Although both can see some ornaments are non-Euclid geometry, neither has been able to construct the ornaments into mathematical language. Thus, very little research still examines non-Euclid geometry in mosque ornaments.

Based on previous studies, researchers must explore non-euclidean mathematical concepts such as fractal geometry. Furthermore, fractal geometry can be constructed using special methods, including the Lindenmayer system. On the other hand, fractal objects that have irregular shapes are self-similar and recursive. That is shown by the first observation of the ornament on the gate of Sumenep Jamik Mosque, which fulfils these characteristics. In addition, ornaments on the gate at Sumenep Jamik Mosque have a long history and acculturation values of Chinese culture, Islam, and Madura. Therefore, this study uses the Lindenmayer System method to explore the concept of fractal geometry in the ornament of the gate of Sumenep Jamik Mosque.

This study aims to uncover and explore the fractal geometry forms in the Jamik Mosque of Sumenep. Previous research has not conducted a similar study. Specifically, this study explores the concept of fractal geometry, which has not been mathematically constructed in previous research.

2. METHOD

This study uses a qualitative method with an ethnographic approach. The research stages are described in Figure 5.

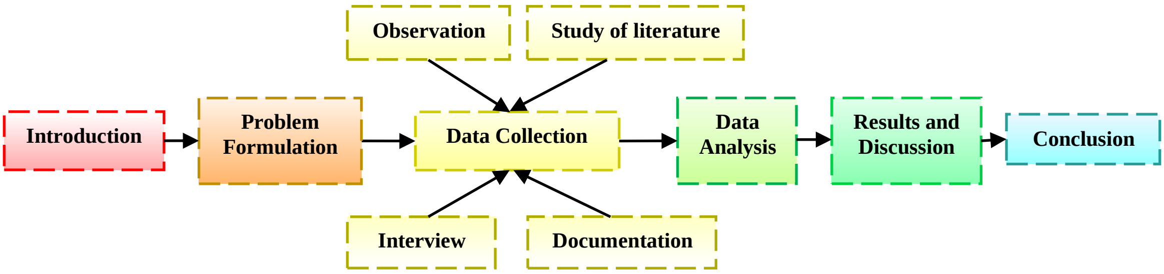


Figure 5. Research Stages

The research process is carried out through various steps. The research topic, object, and location are determined in the preliminary stage, focusing on Ethnomathematics at the Jamik Sumenep Mosque, particularly its ornamentation. This is followed by formulating the research problem. The data collection stage involves field observations to identify the parts of the ornament to be analyzed for fractal structures, measurements of ornament dimensions, photography for documentation, and interviews with a historian of the Jamik Sumenep Mosque. A lecture on Ethnomathematics validated the interview guide. Researchers conducted literature studies through journals, articles, and documents discussing Chinese ornaments, the Sumenep kingdom, and fractals. The data analysis involves analyzing observations, interviews, fractal visualization, and literature studies. The analysis of fractals involves reducing observation results and processing data computationally with the Lindenmayer system in the L-studio application, followed by visual presentation and descriptive explanation.

3. RESULTS AND DISCUSSION

3.1 Results of Interviews with Cultural Experts and Management of the Sumenep Jamik Mosque

This section explains the results of interviews with cultural experts and administrators of the Sumenep Jamik Mosque. The researcher interviewed a cultural expert and the caretaker of the Sumenep Jamik Mosque. The researchers reduced the results of the interviews, and two main points were obtained, namely regarding the history and function of the gate and ornaments at the entrance of the Sumenep Jamik Mosque. The ornaments on the gate of the Sumenep Jamik Mosque were the work of the architect Lauw Piango. Furthermore, the shape of the ornament remains the same as in 1779 AD when the mosque was first built. Lauw Piango built the ornament at the request of the King of Sumenep, Panembahan Sumolo. Panembahan Sumolo asked Lauw Piango to build the Great Mosque of Sumenep to represent the acculturation of Islamic, Chinese, and Madurese cultures. That is because during the leadership of Panembahan Sumolo, the Sumenep region was not only populated by residents with Madurese tribes, but several Chinese, Arab and Dutch tribes inhabited the Sumenep region. Therefore, Lauw Piango made Chinese ornaments on the mosque gate as a form of cultural acculturation requested by Panembahan Sumolo. The ornaments on the gate of the Jamik Sumenep mosque were painted white when it was first built. However, the shape that adorns the gate is still the same today. Then, in 2013, the painting was carried out on the gate using the typical Madura colours: white, yellow, and green. In addition, on the north and south walls, the gate was changed into a fence. In other words, the main form of the gate and the ornaments that adorn the entrance are still preserved as the original.

Furthermore, the ornaments that adorn the gate of the Sumenep Jamik Mosque have the same shape and size. He added that the slope, distance, and angles that adorn the ornaments have something in common. These ornaments are also connected so that the shape surrounds the gate. On the other hand, the shapes of the ornaments appear to be the complete opposite of one another. Even though they are reversed, the forms of the ornament have order and neatness.

The gate ornaments of the Sumenep Jamik Mosque contain swastika symbols and Nazi symbols. He explained that the swastika symbol is a symbol that comes from China. The existence of the swastika symbol is evidence of a blend of Chinese culture and Islamic culture in Sumenep. This symbol also adds to the aesthetic value and attractiveness of the community to revive the Sumenep Jamik Mosque.

Thus, the ornaments on the gate of the Jamik Sumenep Mosque retain their shape the same as when they were first built. The ornament's shape has similarities and uniqueness, which characterizes the blend of Chinese and Islamic cultures in Sumenep. In addition, certain symbols on the gate ornaments provide aesthetic value, thereby increasing the attractiveness of the community to worship at the mosque.

The research results from interviews show that the ornaments on the Jamik Mosque Sumenep are symbols of cultural acculturation between Chinese, Islamic, and Madurese cultures. This cultural amalgamation holds educational significance and should be emphasized. Educators must teach the values of balance, tolerance, and unity in diversity in all teaching practices [24], [45]. Additionally, the decorative elements of the Sumenep Jamik Mosque possess aesthetic qualities and hold significant historical and cultural value, which make them suitable for employment as STEM learning materials that incorporate cultural and religious elements [7], [46], [47].

3.2 Measurement results of Sumenep Jamik Mosque Ornaments

This section describes the data obtained from measuring the length and angles of each part of the ornament. The ornament length was measured manually using a raffia rope, ruler, wooden bow, paper and markers. Measurement of the angle of the ornament is done using a protractor. The process of measuring ornaments is shown in Figure 6.

Figure 6. The Process of Measuring Ornaments

The results of measuring the length and angle of the ornament are written down on an observation sheet and then sketched in millimetre blocks. Sketching is needed to see the similarities between the measurement results and the shape of the ornament. The results of the sketch on the gate ornaments of the Sumenep Jamik Mosque are shown in Figure 7.

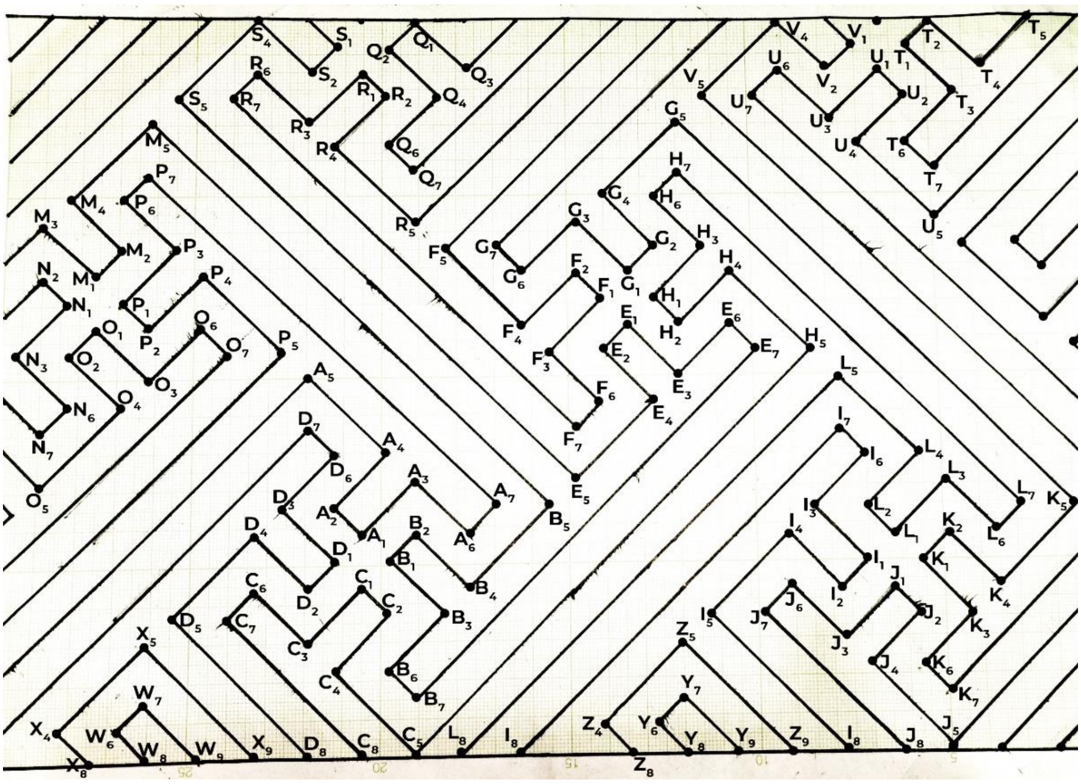


Figure 7. The Result of the Sketch on the Ornament

The results of measuring the length of the ornament are represented in sketch form, as shown in Figure 7. Then, the researcher gave each corner a name with letters of the alphabet and numbers. This is done to make it easier for researchers to see the similarity of the measurement results with the shape of the ornament. The length of each part of the ornament is in multiples of 5 cm. So, the researcher uses 5 cm as the ratio of the ornament length. Giving the ratio aims to facilitate the processing of computer performance when visualizing. In addition, giving the ratio makes the form of l-system coding simpler. The results of the length measurements are shown in Table 1.

Table 1. Measurement Results on the Gate Ornaments of the Sumenep Jamik Mosque

The Part of the Ornament Represented in Figure 7	Size	Ratio (5 cm)
$\lvert\overline{A_1A_2}\rvert, \lvert\overline{A_6A_7}\rvert, \lvert\overline{B_1B_2}\rvert, \lvert\overline{B_6B_7}\rvert, \lvert\overline{C_1C_2}\rvert, \lvert\overline{C_6C_7}\rvert, \lvert\overline{D_1D_2}\rvert, \lvert\overline{D_6D_7}\rvert, \lvert\overline{E_1E_2}\rvert, \lvert\overline{E_6E_7}\rvert, \lvert\overline{F_1F_2}\rvert, \lvert\overline{F_6F_7}\rvert, \lvert\overline{G_1G_2}\rvert, \lvert\overline{G_6G_7}\rvert, \lvert\overline{H_1H_2}\rvert, \lvert\overline{H_6H_7}\rvert, \lvert\overline{I_1I_2}\rvert, \lvert\overline{I_6I_7}\rvert, \lvert\overline{J_1J_2}\rvert, \lvert\overline{J_6J_7}\rvert, \lvert\overline{K_1K_2}\rvert, \lvert\overline{K_6K_7}\rvert, \lvert\overline{L_1L_2}\rvert, \lvert\overline{L_6L_7}\rvert, \lvert\overline{M_1M_2}\rvert, \lvert\overline{N_1N_2}\rvert, \lvert\overline{N_6N_7}\rvert, \lvert\overline{O_1O_2}\rvert, \lvert\overline{O_6O_7}\rvert, \lvert\overline{P_1P_2}\rvert, \lvert\overline{P_6P_7}\rvert, \lvert\overline{Q_1Q_2}\rvert, \lvert\overline{Q_6Q_7}\rvert, \lvert\overline{R_1R_2}\rvert, \lvert\overline{R_6R_7}\rvert, \lvert\overline{S_1S_2}\rvert, \lvert\overline{T_1T_2}\rvert, \lvert\overline{T_6T_7}\rvert, \lvert\overline{U_1U_2}\rvert, \lvert\overline{U_6U_7}\rvert, \lvert\overline{V_1V_2}\rvert, \lvert\overline{W_6W_7}\rvert, \lvert\overline{Y_6Y_7}\rvert$	5 cm	1:1
$\lvert\overline{A_1A_3}\rvert, \lvert\overline{A_2A_4}\rvert, \lvert\overline{A_3A_6}\rvert, \lvert\overline{B_1B_3}\rvert, \lvert\overline{B_2B_4}\rvert, \lvert\overline{B_3B_6}\rvert, \lvert\overline{C_1C_3}\rvert, \lvert\overline{C_2C_4}\rvert, \lvert\overline{C_3C_6}\rvert, \lvert\overline{D_1D_3}\rvert, \lvert\overline{D_2D_4}\rvert, \lvert\overline{D_3D_6}\rvert, \lvert\overline{E_1E_3}\rvert, \lvert\overline{E_2E_4}\rvert, \lvert\overline{E_3E_6}\rvert, \lvert\overline{F_1F_3}\rvert, \lvert\overline{F_2F_4}\rvert, \lvert\overline{F_3F_6}\rvert, \lvert\overline{G_1G_3}\rvert, \lvert\overline{G_2G_4}\rvert, \lvert\overline{G_3G_6}\rvert, \lvert\overline{H_1H_3}\rvert, \lvert\overline{H_2H_4}\rvert, \lvert\overline{H_3H_6}\rvert, \lvert\overline{I_1I_3}\rvert, \lvert\overline{I_2I_4}\rvert, \lvert\overline{I_3I_6}\rvert, \lvert\overline{J_1J_3}\rvert, \lvert\overline{J_2J_4}\rvert, \lvert\overline{J_3J_6}\rvert, \lvert\overline{K_1K_3}\rvert, \lvert\overline{K_2K_4}\rvert, \lvert\overline{K_3K_6}\rvert, \lvert\overline{L_1L_3}\rvert, \lvert\overline{L_2L_4}\rvert, \lvert\overline{L_3L_6}\rvert, \lvert\overline{M_1M_3}\rvert, \lvert\overline{M_2M_4}\rvert, \lvert\overline{N_1N_3}\rvert, \lvert\overline{N_3N_6}\rvert, \lvert\overline{O_1O_3}\rvert, \lvert\overline{O_2O_4}\rvert, \lvert\overline{O_3O_6}\rvert, \lvert\overline{P_1P_3}\rvert, \lvert\overline{P_2P_4}\rvert, \lvert\overline{P_3P_6}\rvert, \lvert\overline{Q_1Q_3}\rvert, \lvert\overline{Q_2Q_4}\rvert, \lvert\overline{Q_4Q_6}\rvert, \lvert\overline{R_1R_3}\rvert, \lvert\overline{R_2R_4}\rvert, \lvert\overline{R_3R_6}\rvert, \lvert\overline{S_2S_4}\rvert, \lvert\overline{T_1T_3}\rvert, \lvert\overline{T_2T_4}\rvert, \lvert\overline{T_3T_6}\rvert, \lvert\overline{U_1U_3}\rvert, \lvert\overline{U_2U_4}\rvert, \lvert\overline{U_3U_6}\rvert, \lvert\overline{V_2V_4}\rvert$	10 cm	1:2
$\lvert\overline{A_4A_5}\rvert, \lvert\overline{B_4B_5}\rvert, \lvert\overline{C_4C_5}\rvert, \lvert\overline{D_4D_5}\rvert, \lvert\overline{E_4E_5}\rvert, \lvert\overline{F_4F_5}\rvert, \lvert\overline{G_4G_5}\rvert, \lvert\overline{H_4H_5}\rvert, \lvert\overline{I_4I_5}\rvert, \lvert\overline{J_4J_5}\rvert, \lvert\overline{K_4K_5}\rvert, \lvert\overline{L_4L_5}\rvert, \lvert\overline{M_4M_5}\rvert, \lvert\overline{O_4O_5}\rvert, \lvert\overline{P_4P_5}\rvert, \lvert\overline{R_4R_5}\rvert, \lvert\overline{S_4S_5}\rvert, \lvert\overline{T_4T_5}\rvert, \lvert\overline{U_4U_5}\rvert, \lvert\overline{V_4V_5}\rvert, \lvert\overline{X_4X_5}\rvert, \lvert\overline{Z_4Z_5}\rvert$	15 cm	1:3
$\lvert\overline{A_7A_8}\rvert, \lvert\overline{D_7D_8}\rvert, \lvert\overline{F_7F_8}\rvert, \lvert\overline{G_7G_8}\rvert, \lvert\overline{J_7J_8}\rvert, \lvert\overline{K_7K_8}\rvert, \lvert\overline{P_7P_8}\rvert, \lvert\overline{O_7O_8}\rvert$	30 cm	1:6
$\lvert\overline{A_7P_7}\rvert, \lvert\overline{B_7E_7}\rvert, \lvert\overline{F_7R_7}\rvert, \lvert\overline{H_7L_7}\rvert, \lvert\overline{I_7I_8}\rvert$	65 cm	1:13
$\lvert\overline{B_5M_5}\rvert, \lvert\overline{C_5H_5}\rvert, \lvert\overline{D_5O_5}\rvert, \lvert\overline{E_5S_5}\rvert, \lvert\overline{G_5K_5}\rvert$	75 cm	1:15

Based on the measurement results and Figure 7, the researchers found that the angles on each part of the ornament were 90^o or right angles. The researcher also found that the slope of the ornament with the long part of the ornament is 45^o. The results of the angle measurements are shown in Table 2.

Table 2. The Results of Measuring the Angles in the Ornaments of the Sumenep Jamik Mosque

The Part of the Ornament Represented in Figure 7	Angle Size
$\angle A_1, \angle A_2, \angle A_3, \angle A_4, \angle A_5, \angle A_6, \angle A_7, \angle B_1, \angle B_2, \angle B_3, \angle B, \angle B_5, \angle B_6, \angle B_7, \angle C_1, \angle C_2, \angle C_3, \angle C_4, \angle C_5, \angle C_6, \angle D_1, \angle D_2, \angle D_3, \angle D_4, \angle D_5, \angle D_6, \angle D_7, \angle E_1, \angle E_2, \angle E_3, \angle E_5, \angle E_6, \angle E_7, \angle E_8, \angle F_1, \angle F_2, \angle F_3, \angle F_4, \angle F_5, \angle F_6, \angle F_7, \angle G_1, \angle G_2, \angle G_3, \angle G_4, \angle G_5, \angle G_6, \angle G_7, \angle H_1, \angle H_2, \angle H_3, \angle H_4, \angle H_5, \angle H_6, \angle H_7, \angle I_1, \angle I_2, \angle I_3, \angle I_4, \angle I_5, \angle I_6, \angle I_7, \angle J_1, \angle J_2, \angle J_3, \angle J_4, \angle J_5, \angle J_6, \angle J_7, \angle K_1, \angle K_2, \angle K_3, \angle K_4, \angle K_5, \angle K_6, \angle K_7, \angle L_1, \angle L_2, \angle L_3, \angle L_4, \angle L_5, \angle L_6, \angle L_7, \angle M_1, \angle M_2, \angle M_3, \angle M_4, \angle M_5, \angle M_7, \angle N_1, \angle N_2, \angle N_3, \angle N_6, \angle N_7, \angle O_1, \angle O_2, \angle O_3, \angle O_4, \angle O_5, \angle O_6, \angle O_7, \angle P_1, \angle P_2 \angle P_3, \angle P_4, \angle P_5, \angle P_6, \angle P_7, \angle Q_1, \angle Q_2, \angle Q_3, \angle Q_4, \angle Q_6, \angle Q_7, \angle R_1, \angle R_2, \angle R_3, \angle R_4, \angle R_5, \angle R_6, \angle R_7, \angle S_1, \angle S_2, \angle S_4, \angle S_5, \angle T_1, \angle T_2, \angle T_3, \angle T_4, \angle T_6, \angle T_7, \angle U_1, \angle U_2, \angle U_3, \angle U_4, \angle U_5, \angle U_6, \angle U_7, \angle V_1, \angle V_2, \angle V_4, \angle V_5, \angle W_6, \angle W_7, \angle X_4, \angle X_5, \angle Y_6, \angle Y_7, \angle Z_4, \angle Z_5$	90^o
$\angle C_7C_8D_8, \angle C_4C_5C_8, \angle D_5D_8X_9, \angle D_5D_8X_9, \angle H_5C_5L_8, \angle I_5I_8Z_9, \angle J_7J_8I_8, \angle J_4J_5J_8, \angle L_5L_8I_8, \angle W_6W_8X_8, \angle W_7W_9W_8, \angle X_5X_9W_9, \angle Y_6Y_8Z_8, \angle Y_7Y_9Y_8, \angle Z_5Z_9Y_9$	45^o

The measurements obtained from the gate ornaments of the Sumenep Jamik Mosque, as displayed in Figure 7, Table 1, and Table 2, indicate the presence of mathematical concepts and activities that can be executed. The concepts that can be built upon include similarity, congruence, geometric transformation, and angles. On the other hand, mathematical learning can involve measuring, calculating, and exploring patterns [48], [49]. This follows the explanation that mathematical activities carried out on a cultural element form part of Ethnomathematics within the context of learning [7], [50], [51].

3.3 Fractal Shape Coding Results with the L-system

The fractal model of the Sumenep Jamik Mosque ornament is implemented in a two-dimensional plane using the Lindenmayer system method. Researchers make parameters based on the results of measuring the length and angle of the ornament. Parameters are used to shorten the strings in coding and lighten the computer's workload when visualizing from L-studio. The definitions of symbols and parameters in the L-system are described in Table 3.

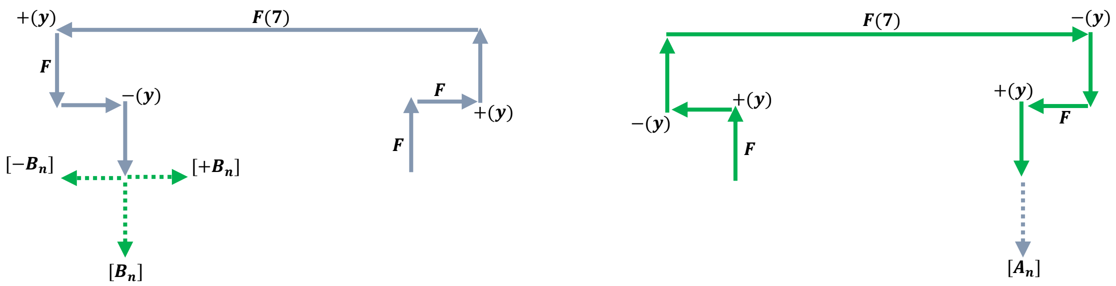
Table 3. The Symbol and L-system Parameter

Parameter	Symbol	Meaning
Direction	F	Move forward 1 unit (5 cm).
	$+(\alpha)$	Rotate counterclockwise by (α) degrees.
	$-(\alpha)$	Rotate clockwise by (α) degrees.
	$\rightarrow$	Mapping
Angle	x	Rotate by 45°
	y	Rotate by 90°
Length	$*$	Multiplication operation
	A	A set A that has specific production rules.
	B	A set B that has specific production rules.

After determining the symbols and parameters of the l-system, the researcher compiled the l-system coding based on the observations, parameters and symbols in Table 3. Researchers coded with the help of the L-studio application. The following is the result of the coding formed from the gate ornaments of the Sumenep Jamik Mosque**.**

Parameter: Define $x = 45$ dan $y = 90$
Initiator (w)*:* $[-(x)A_{n+1}][+(x)A_{n+1}][-(3*x)A_{n+1}][+(3*x)A_{n+1}]$
The number of iterations( ): 6
Production Rules (P)
$P_1 : A_{n+1} \rightarrow [F-(y)F+(y)F+(y)F(7)+(y)F+(y)F-(y)F[[B_n][+B_n][-B_n]]]$
$P_2 : B_{n+1} \rightarrow [F+(y)F-(y)F-(y)F(7)-(y)F-(y)F+(y)F[A_n]]$

Two production rules are built. Namely, the first production rule (P_1) is the mapping A_{n+1}, and the second production rule (P_2) is the mapping B_{n+1}. A_{n+1} contains several F's ending with a mapping to B_n. At the same time, B_{n+1} contains several F's ending with a mapping to A_n. The graphical interpretation of the P_1 and P_2 production rules is shown in Figure 8.



(a) (b)

Figure 8. Graphic Interpretation of Production (a) Rule A_{n+1} and (b) Rule B_{n+1}

The screen display on the production rules for the gate ornaments of the Sumenep Jamik Mosque in the L-studio application is shown in Figure 9.

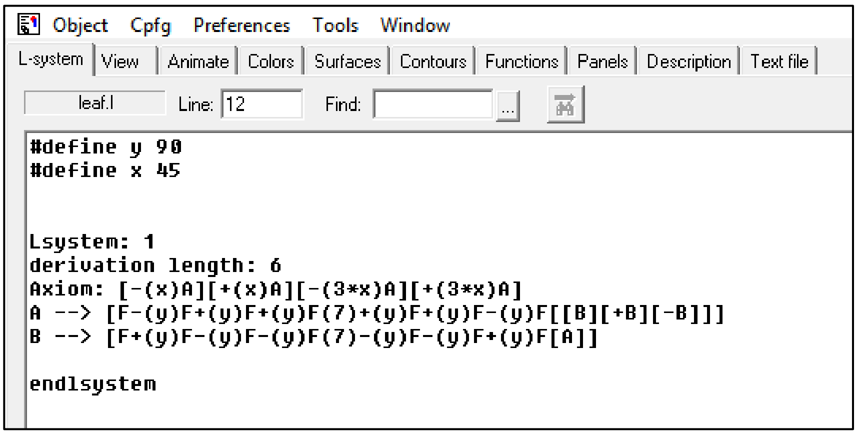
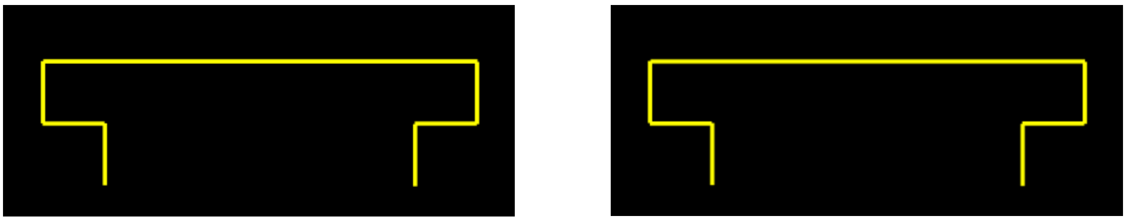


Figure 9. Display of Production Rules on the L-studio Application

3.4 The Results of the L-System Visualization on Sumenep Jamik Mosque Ornaments

The results of the visualization of the ornaments of the Sumenep Jamik Mosque are based on the results of measurements of length and angles. The angles 90° and 45° are used as x and y parameters. Meanwhile, the length measurement with a ratio of 5 cm is used as the length of one F unit. That means that each visualization result of one F has a meaning of 5 cm. In addition, the direction parameters used are only $+(a)$ and $-(a)$, which are limited to the Cartesian plane. That is, the visualization results can only be seen from one perspective.

The basic shape of the ornament is formulated through the first production rule. (P_1), which is in the set A_{n+1}. Then, the set A_{n+1} maps the second production rule (P_2), the set B_n. Then, the set B_{n+1} maps back to the set A_n. And so on, until many mappings are repeated up to n times (iterative). The visualization results for the first and second production rules are shown in Figure 10.



(a) (b)

Figure 10. Results Visualization of (a) the First Production Rules P_1 and (b) Second Production Rule P_2

Based on Figure 8, the directions of the two graphical interpretations of the production rules P_1 and P_2 have differences and are opposites. However, the results of running the production rules produce the same form, as shown in Figures 10a and 10b. The string from P_1 will end with B_n mapping in three different directions. The string from P_2 will end with the mapping A_n. Even though P_1 and P_2 have different strings, the visualization results from the two production rules are the same, as shown in Figure 10. Thus, the visualization results from P_1 and P_2 become the basic shapes that construct fractal shapes in the Sumenep Jamik Mosque's arch ornaments. The visualization results in the first to sixth iterations are shown in Figure 11.

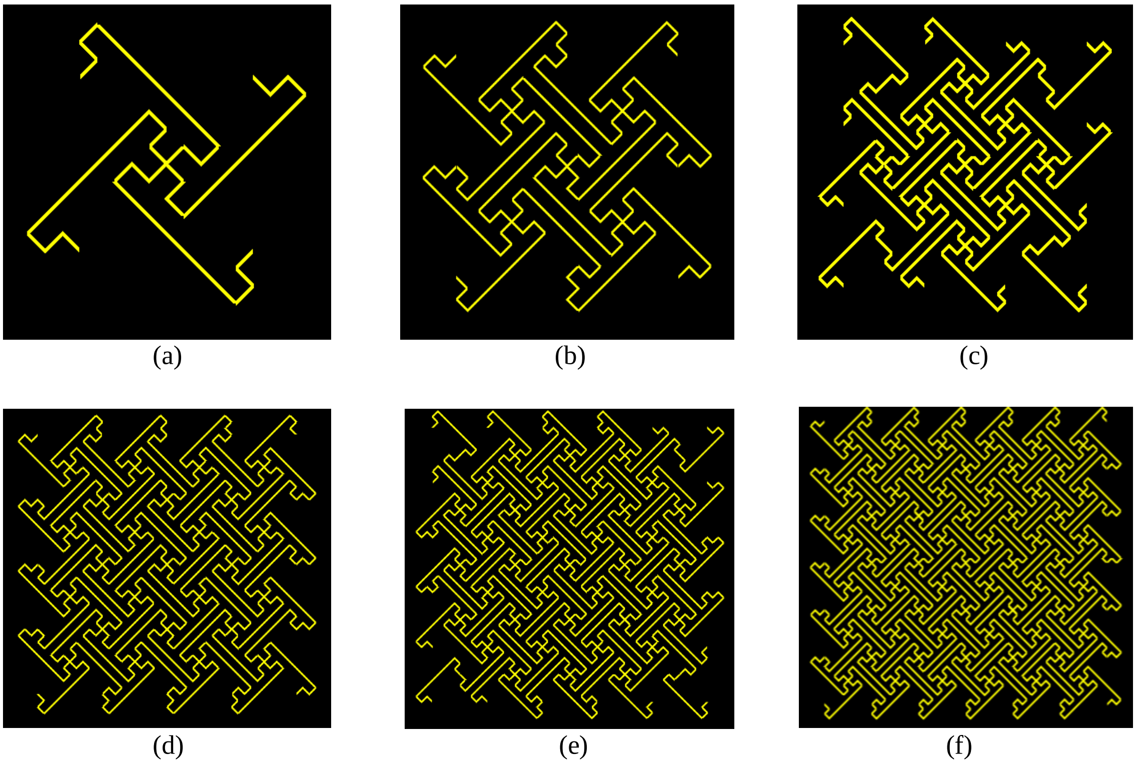


Figure 11. The Results of the Visualization of the L-system on the Gate Ornaments of the Sumenep Jamik Mosque (a) First Iteration, (b) Second Iteration, (c) Third Iteration, (d) Fourth Iteration, (e) Fifth Iteration, and (f) Sixth Iteration

Based on Figure 11a, the first iteration produces a shape similar to a combination of the letter 'T'. At first glance, the image copies Figure 10, displayed in 4 different directions. That is caused by the first iteration being able to map the set. A_{n+1} to the set B_n and B_{n+1} to map back to the set A_n With the number of iterations (n), namely 1. In addition, the mapping is carried out on A_{n+1} in four different directions, namely 45°, 135°, 225° and 315°, as directed by the initiator (w) with the string $[-(x)An+1][+(x)An+1][-(\,3*x)An+1][+(3*x)An+1]$. In other words, the shape of the letter 'T' in Figure 10 has built itself according to the direction of the initiator (w).

Figure 11b, Figure 11c, Figure 11d, Figure 11e, and Figure 11f show that the visualization results from the running program in Figure 9 have formed fractals. That is caused by repeated iterations so that the mapping of sets. A_{n+1} and B_{n+1} are carried out repeatedly. Furthermore, this repetition also occurs in the initiator (w). The more iterations that are done, the more repetitions on the initiator. That also makes the branching direction of the first production rule. (P_1) and the second production rule (P_2) Wider. In other words, the more iterations are carried out, the greater the visualization results and the wider the repetition in Figure 11a.

After visualizing through the L-studio application, the visualization results are checked again. Checking is done by comparing the visualization results with the original image of the Sumenep Jamik Mosque gate ornaments. That aims to see whether errors are made in the visualization results. If an error occurs, it is necessary to re-evaluate the components in the coding and the production rules that were made. The comparison of the visualization results with the gate ornaments of the Sumenep Jamik Mosque is presented in Figures 12 and 13.

(a) (b)

Figure 12. Comparison of Visualization Results with the Ornaments of the Sumenep Jamik Mosque (a) Second Iteration Result and (b) Sumenep Jamik Mosque Ornament

Figure 13. Comparison of Visualization Results with the Ornaments of the Sumenep Jamik Mosque (a) Sixth Iteration Result and (b) Sumenep Jamik Mosque Ornament

3.5 Geometric Value in Sumenep Jamik Mosque Ornaments

Based on the L-system production rules in Figure 9 and the visualization results in Figure 11, Figure 12, and Figure 13, the researcher found that the Sumenep Jamik Mosque gate ornaments could be studied mathematically. That is because the shape of the ornament has a *self-similarity*. In addition, Figure 11 shows that the ornament is recursive in 6 iterations. Thus, the ornament on the gate of Masjid Jamik Sumenep fulfils the characteristics to be studied with fractal geometry. That aligns with the research result that Chinese-style ornaments can be studied mathematically through lattice theory [52]. An analysis showed fractal geometric shapes in the ornaments of the Tian Ti Pagoda [53]. In addition, another research used the meander theory to examine the twists and turns in ornaments left by Greece and China [46].

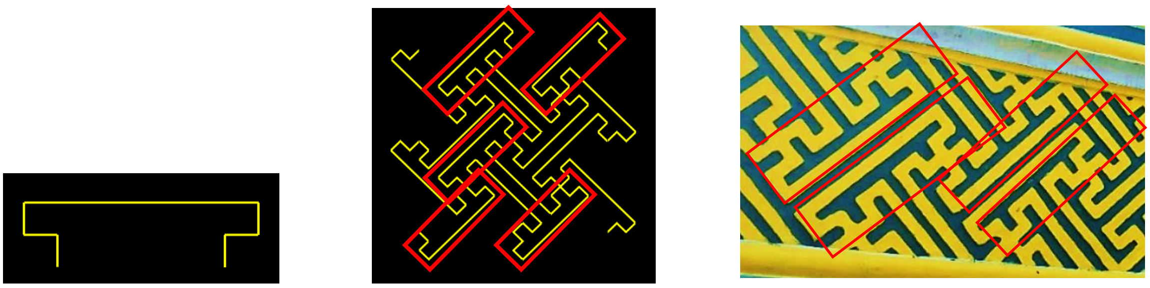
Some literature on Chinese ornaments shows that several types have recursive, repetitive, geometric, and opposite characteristics [54]–[56]. Furthermore, some Chinese ornaments have dynamic, balance-oriented, and symmetrical shapes [55], [56]. Dynamic forms make ornaments look opposite and centrifugal and can be combined with the same or other elements. The symmetrical shape makes the ornaments have the same shape, size, and tilt. The ornament's orientation on balance gives it a centre point to balance its elements.

The statement made by a historian of the Jamik Sumenep Mosque in the interview results is consistent with the properties of fractal geometry. Mathematically, shapes that do not end and resemble themselves (self-similarity) are characteristics of fractal geometric shapes [34], [57]. The recursive nature of fractals always makes the results of repetitions the same [53], [58]. In addition, fractals have complex but regular shapes, so they are classified in non-Euclidean geometry [31], [39].

Based on this description, the Sumenep Jamik Mosque's arch ornament geometrically fulfils the properties of fractal geometry. In addition, these ornaments are in harmony with the geometric nature of Chinese ornaments. That was reinforced by the results of interviews, which explained that the form of the ornament had not been renovated since the beginning of construction. Thus, the ornaments at the Sumenep Jamik Mosque can be classified into Chinese ornaments in the form of fractal geometry.

3.6 Key Motif 'T-Shape' on Sumenep Jamik Mosque Ornaments

Based on the visualization of production rules P_1 and P_2, the results of the second iteration, and the initial observations, it was found that the ornament has a T-shaped key motif. The indentation motif in Figure 14 is often called a T-shaped or T-shaped pattern [54], [59]. The T-shape is one of the forms that has become a key pattern artists have used since ancient Greece [56]. The T shape adapted to several world civilizations, such as Western, Islamic, and Chinese [59]. The T-shape motif is widely applied as a border art on an object.



(a) (b) (c)

Figure 14. T-form Motifs (a) Display of P_1 and P_2, (b) The second iteration, and (c) The ornaments on the Mosque

Chinese people feel empty or dislike empty spaces without decoration in their art [54]. Some Chinese craftsmen and artists often give certain motifs to empty narrow spaces to eliminate the innocence of an object [56], [59]. One of the motifs often used is the striped motif in the shape of the letter T, as shown in Figure 15.

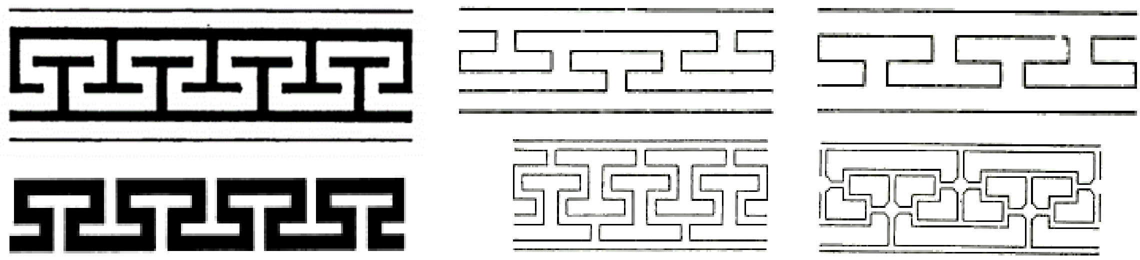
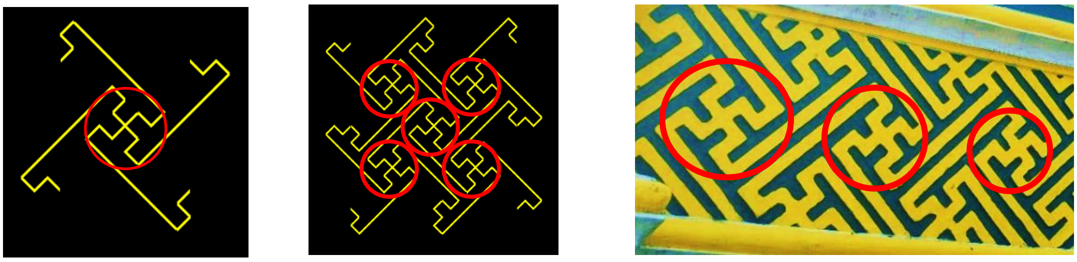


Figure 15. The T-shaped Key Motif on Chinese Ornaments [54], [59]

In Chinese carving, the T shape is combined with several other shapes, such as thunder or cloud patterns [54]. Chinese artists often use the T-shape as the basic pattern for constructing the swastika [59], [60]. A T-shape that repeats and is continuous with one another has the meaning of 'longevity' for the object's owner with that motif [54], [59], [60].

3.7 The Shape of the Swastika in the Sumenep Jamik Mosque Ornament

Based on the visualization of the production rules P_1 and P_2, the results of the second iteration and initial observations show that the ornament had a swastika symbol, as shown in Figure 16. The swastika symbolized as (卍), is a sacred and ancient symbol that was once created by humans [61], [62]. The word swastika comes from Sanskrit, namely 'svastika' or 'sv-asti', which means luck, happiness, and prosperity [63], [64]. The general form of the swastika is two intersecting lines perpendicular to each other or branching with a 90 angle. The four arms of the swastika have the same size and length and are bent to the right or left [59], [61].



(a) (b) (c)

Figure 16. The Swastika Shapes (a) The First Iteration, (b) The Second Iteration, and (c) The Ornaments on the Mosque

The swastika symbol will have an exemplary meaning if the rotation is clockwise and a lousy meaning if it is counterclockwise [64], [65]. However, the shape of the swastika rotating clockwise symbolizes the masculine nature of God, while the swastika that rotates counterclockwise represents the feminine nature of the goddess [59]. In Buddhist belief, the swastika is a sacred symbol representing the heart of the Buddha [54], [66]. The swastika is interpreted as 10,000 blessings, longevity, and unlimited goodness [29], [60]. The basic form of the swastika symbol is shown in Figure 17.

(a) (b)

Figure 17. The Swastika Shapes (a) Basic Form and (b) Variation of the Shape of the Swastika [59], [61]

Based on the interview results, the gate ornaments of the Sumenep Jamik Mosque contain a swastika symbol. The swastika symbol on the ornament proves the connection between Chinese culture and the construction of the Sumenep Jamik Mosque. That symbol is found in several parts of the Sumenep Palace. In addition, the swastika symbol adds aesthetic value to the Sumenep Jamik Mosque.

The statement made by a historian of the Jamik Sumenep Mosque aligns with what was explained by previous research regarding the swastika symbol [54], [67]. Chinese ornaments on walls, windows, and vents usually feature a stylized swastika and waves on the edges [54], [67]. Apart from symbolizing beauty, the swastika symbol in this section means purity, grace, and hope for goodness [59], [63]. Swastika-style ornaments are also expected to bring a thousand good luck [60]. Furthermore, swastika ornaments were found in the Sumenep Palace carvings [29].

Based on this description, it can be concluded that there is a swastika symbol on the gate ornaments of the Jamik Sumenep Mosque. Aside from being a gate decoration, this symbol has a religious meaning and a message from Chinese culture about goodness. That was also reinforced by the results of interviews, which explained that the shape of the swastika was one of the evidence of acculturation of Chinese-Islamic culture at the Sumenep Jamik Mosque.

3.8 The Contribution of Cultural Values in The Gate Ornament of Sumenep Jamik Mosque to Learning

The production rules in Figure 9 and the visualization results in Figure 11 show that the Lindenmayer system method can be used to construct fractals on the gate ornaments of Sumenep Jamik Mosque. That will provide more benefits if applied to learning computational geometry, mathematical modelling, and fractal modelling in universities [39]. If learning fractal geometry is usually dominated by fractal shapes derived from books, then this study's results give students new colours that the construction of fractals can be found in Indonesian cultures.

In addition, the results of this study can be associated with realistic mathematic education learning (RME) models in computational geometry lectures. By making the gate ornament of the Sumenep Jamik Mosque a contextual problem, students will be faced with constructing fractal models in their own way and method. That will train students' understanding and reasoning in building fractal models [39], [58]. That aligns with [4] and [5] that the ethnomathematics approach with a realistic mathematics learning model positively impacts students' reasoning and understanding of a mathematical problem.

On the other hand, historical and cultural values in the gate ornament of the Sumenep Jamik Mosque can add value to learning. As conveyed by a historian of the Jamik Sumenep Mosquekey, the ornaments on the Sumenep Jamik Mosque are the result and evidence of acculturation from Chinese, Islamic and Madurese cultures. In other words, there are values of acculturation, love of culture, and tolerance that can be used as moral values to be instilled in students. That is in line with [24] and [27], which explained that the religious values, tolerance, hard work, nationalism, and unity emanating from the Jamik Sumenep Mosque ornaments must be conveyed to students to build moral character. Further research could explore non-Euclidean geometry methods, such as stochastic L-system methods.

4. CONCLUSION

Based on the results and discussion, the ornaments on the gate of the Sumenep Jamik Mosque contain the concept of fractal geometry. The size of the length and the angles of each part of the ornament affect the shape of the fractal. The formulation of fractal shapes is carried out using the Lindenmayer system method. With angle parameters of 45^o and 90^o, a ratio of 5 cm, and iterations of 6 times, a fractal model resembles the ornaments on the gate of the Sumenep Jamik Mosque.

This research is still limited to the ornaments of the Sumenep Jamik Mosque. So that the exploration of fractal shapes can be done in other Chinese ornaments, this study also does not provide a thickness variable in the visualization results, so similar research is expected to be able to give this variable. In addition, the results of this research can be applied in lectures, especially in learning computational geometry, dynamic systems, and mathematical modelling. So that students can be introduced to fractal forms contained in cultured buildings.

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