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APPLYING APOS THEORY TO ENHANCE ALGEBRAIC LOGIC SKILLS: COMPARING TRADITIONAL TEACHING AND COMPUTER-ASSISTED INSTRUCTION

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ABSTRACT

This study aims to assess how effectively applied science students use equations to express connectivity in algebraic logic and understand their cognitive framework. This research measures students' understanding of mathematical logic connectors in line with the polytechnic curriculum and uses APOS theory as a basis. It compares the effectiveness of traditional teaching methods with the computer-assisted instruction (CAI) approach. The results show that students learning with CAI perform better in applying connectivity equations. These findings underscore the importance of students' cognitive schemes in understanding the relationship between algebra and logic. Students with a more mature conceptual understanding tend to overcome conceptual application difficulties better. For future research, it is suggested to explore the application of CAI methods in various learning contexts and other disciplines, as well as to assess its long-term impact on students' cognitive skills and mathematical understanding. Additionally, it is essential to compare the effectiveness of CAI with other innovative learning methods to determine the best approach to developing students' logic and algebra skills.

TEORI APOS DALAM MENINGKATKAN KETERAMPILAN LOGIKA ALJABAR: PERBANDINGAN ANTARA PENGAJARAN TRADISIONAL DAN INSTRUKSI BERBANTUAN KOMPUTER

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ABSTRAK

Penelitian ini bertujuan untuk menilai seberapa efektif siswa sains terapan dalam menggunakan persamaan untuk mengekspresikan konektivitas dalam logika aljabar, serta memahami kerangka pikir mereka. Kajian ini mengukur pemahaman siswa terhadap konektor logika matematis, sesuai dengan kurikulum politeknik dan menggunakan teori APOS sebagai dasar. Penelitian ini membandingkan efektivitas metode pengajaran tradisional dengan pendekatan berbantuan komputer (CAI). Hasilnya menunjukkan bahwa siswa yang belajar menggunakan CAI lebih baik dalam mengaplikasikan persamaan konektivitas. Temuan ini menggarisbawahi pentingnya skema kognitif siswa dalam memahami hubungan antara aljabar dan logika. Siswa dengan pemahaman konsep yang lebih matang cenderung mengatasi kesulitan aplikasi konsep dengan lebih baik. Untuk penelitian selanjutnya, disarankan untuk mengeksplorasi penerapan metode CAI dalam berbagai konteks pembelajaran dan disiplin ilmu lain, serta menilai dampak jangka panjangnya terhadap keterampilan kognitif dan pemahaman matematika siswa. Juga, penting untuk membandingkan efektivitas CAI dengan metode pembelajaran inovatif lainnya untuk menentukan pendekatan terbaik dalam mengembangkan keterampilan logika dan aljabar siswa.

1. INTRODUCTION

Logic is the science of reasoning and is the foundation of mathematics. Mathematics should be taught as a language that can be communicated precisely [1]. Logic is used in most science and engineering fields because it allows for modelling many natural phenomena and efficient computing with such models. To help students understand mathematics, researchers proposed that it be taught concerning real-life situations. There is a basic phenomenal analogy at the root of formal symbolic logic, where a central problem is establishing arithmetic consistency.

Takeuti pointed out that mathematical logic reflects mathematics, which questions its validity and effective calculability [2]. Cheng attempted an analysis of the understanding of mathematical concepts by experts (mathematicians) and non-experts in real life [3]. Cheng argued that there is a conspiracy theory among non-experts that there are no explanations for mathematical theories that can adequately justify the structural proof. In mathematics, a theory is a set of results proven true according to logic. Fletcher & Patty have confirmed that the activities of the earliest authors of mathematical logic have been recognized [4]. This research serves as a compelling example of the productive and powerful reasoning possible as students sense complex mathematics and what the APOS analysis of Students' Understanding of Symbolic Mathematical Logic reveals from the usage of conventional teaching and computer-assisted instruction as a tutorial tool. The APOS theory is a constructivist theory that postulates that a mathematical concept begins as one applies transformations to certain entities to obtain other entities. The APOS Theory was introduced by Dubinsky [5] and was first used by Cottrill [6]. Dubusky developed a theory or model in mathematical education, which focuses on how a theory can aid the learning process and understanding of mathematics by providing explanations of phenomena that researchers can observe in students who are trying to construct their knowledge of mathematical concepts [5]. According to Maharaj, it has been a valuable tool for researchers in analyzing students' understanding of various mathematical topics [7]. From the reviews on APOS theory, it is compelling to see the importance of APOS via the revealed analysis, as it helps recognize students' challenges in understanding a particular mathematical concept and provides an enhanced teaching pattern for an improved learning outcome. Bhatti stated that mathematics is the queen of sciences, a fact-driven discipline, and as such, requires augmentation in education to enrich learners with current skills for achieving maximum satisfaction [8]. Bayrak & Bayram stated that computer-aided teaching stimulates students' reasoning and produces more effective results than traditional teaching methods [9]. Chen noted that education must improve the thirst and yearning for learning under an encouraging situation [10]. This study used a combination of both the conventional lecture method and CAI. The latter included educational videos from YouTube on logic and abstract thinking and links shared via a WhatsApp platform for the students to have further insight into logic and abstract thinking. The researcher intended to use the WhatsApp platform and YouTube videos to reinforce what was learned through the conventional lecture method.

This study peculiarly seeks to apply APOS theory to get an insight into students' understanding and to help apply logic and linear algebra to real-life scenarios. This area has received little attention from authors and has been subjected to APOS theory analysis. This study aimed to understand what an APOS analysis of students' understanding of mathematical algebraic logic reveals and to expose the effect of using CAI to aid the conventional lecture method on the academic achievement of students taught logic and abstract thinking.

Numerous studies have been conducted on APOS theory in student understanding, including student comprehension based on APOS theory from a cognitive style perspective [11], concept understanding analysis based on APOS theory [12] and from a self-learning standpoint [13], as well as mathematical concept understanding ability based on APOS theory [14]. However, no research has yet examined student understanding based on APOS theory in relation to the connection between algebra and logic. This study describes and analyses students' understanding of algebraic operation concepts. The novelty of this research is an innovation in conducting an investigation of APOS theory on student understanding of the relationship between algebra and logic, where previous studies have examined student understanding, but not specifically the connection between algebra and logic. This research opens the door for further application of APOS theory in enhancing mathematics learning.

2. METHODS

The researchers set eight objective-type and three theoretical questions based on the concept of logic, taught in the classroom environment and tutorial classes linked to YouTube educational videos (as shown in Figure 1). A total of 173 first-year diploma students were registered for the course at the Federal Polytechnic, Ile-Oluji, where the topic of logic and abstract thinking is compulsory for students pursuing a National Diploma. One hundred five students participated in the CAI-tutorial-aided class via the WhatsApp platform, YouTube tutorials, and conventional classes, while all 173 registered students participated in the test. Eight students from 105 students were chosen at random to participate in the analysis of this research. The test was administered after students were exposed to formal lectures on the required concepts of logic and abstract thinking through combined conventional and CAI teaching methods (see Figure 2).

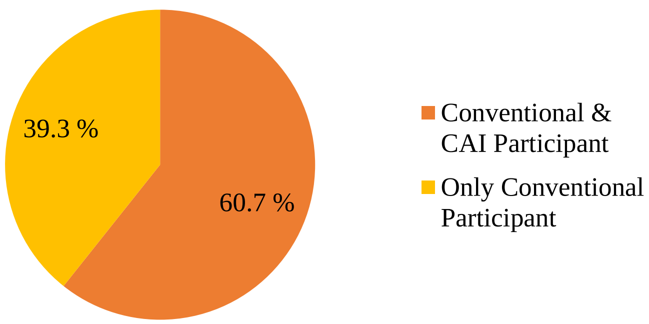

Figure 1. Demographic Representation of Participants

In Figure 1, conventional and CAI participants are students who consented to use the WhatsApp platform.

The insights from the test results were used to determine the student's level of understanding of logic and abstract thinking on the equal translation of the connectives and were also used by the tutor when revising the course with the students. The participants signed a consent form, and ethical clearance for the study was obtained from the institution.

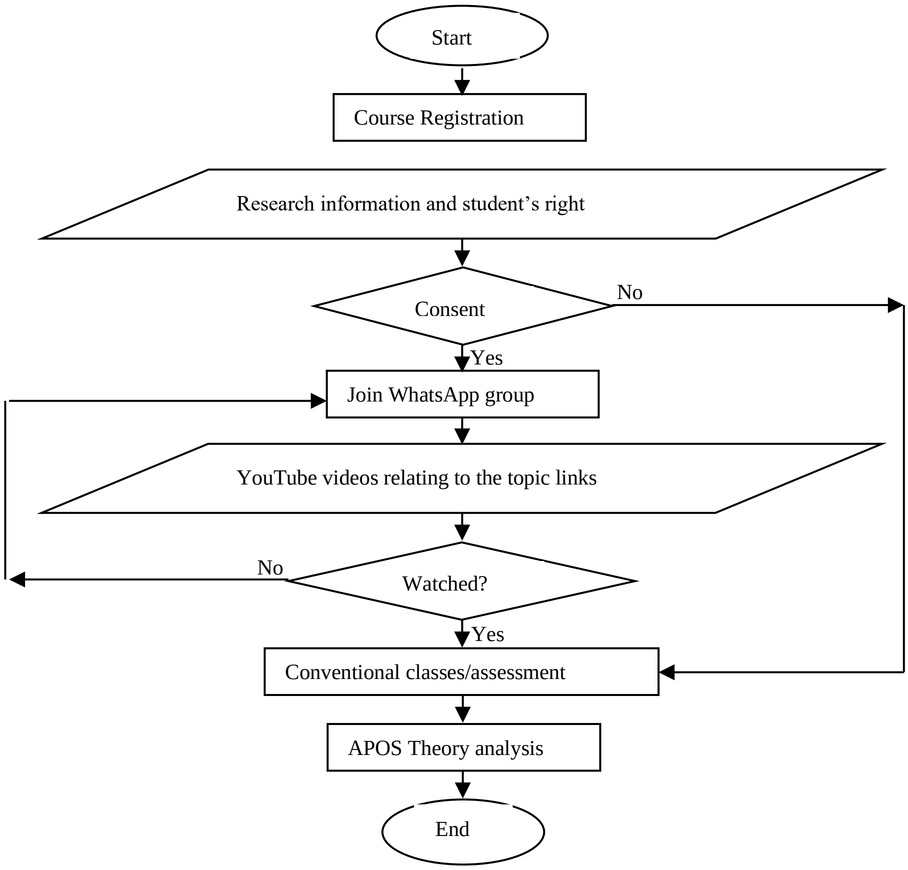

Figure 2. Flow Chat of Procedure and Research Analysis (Field Survey)

Suitable expressions were formulated based on Maharaj's APOS Genetic Decomposition [15], [16].

Action: a transformation is conceived as an action, if a student can recognize logic connectives such as (〖∧ 〗^' or^',∨ 'and^',⇒ 'implicatio 〖n, sufficient for, if..then, is neccessary〗^', 〖⇔ 〗^'') bi-implication or if and only if…then or if …then … and conversely) and propositions /statements (denoted p,q,r,..), which can either be true (T) or false (F) and cannot be both. These can be described as the action stage of understanding logic.

Process: The individual repeats and reflects on the action. Applying connectives to more than one proposition as a process is a mental structure that performs the same operation as the action but wholly in the individual's mind. Specifically, the individual can imagine performing the transformation without explicitly executing each step [15]. This involves the individual understanding that the truth table expression of the connectors and principles of the operator could apply to connectors. For example:

p := He puts too much salt in the food
q := The food is salty
$p \wedge q$:= He put too much salt in the food, and the food is salty
$p \vee q$:= He put too much salt in the food, or the food is salty
$p \Longrightarrow q$:= If he put too much salt in the food, then the food is salty
$p \Longleftrightarrow q$:= If and only if he puts too much salt in the food, then the food will be salty

Table 1. Truth Table Analysis or Explanation of Connector

1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\boldsymbol{p}$	$\sim p$	q	$\sim q$	$p \wedge q$	$p \vee q$	$\sim p \vee q$	$\left\{ \begin{matrix} (p \vee q) \\ \wedge (p \vee \sim q) \end{matrix} \right\} \vee \sim p$	$p \Rightarrow q$	$p \Leftrightarrow q$	$(p \rightarrow q) \wedge (q \rightarrow p)$	$(\sim p \vee q) \wedge (\sim q \vee p)$	$\sim (p \vee q)$	$\sim p \wedge \sim q$
$\boldsymbol{T}$	F	T	F	T	T	T	T	T	T	T	T	F	F
$\boldsymbol{T}$	F	F	T	F	T	F	T	F	F	F	F	F	F
$\boldsymbol{F}$	T	T	F	F	T	T	T	F	F	F	F	F	F
$\boldsymbol{F}$	T	F	T	F	F	T	T	T	T	T	T	T	T

Tautology: a compound statement (premise and conclusion) that always produces truth. No matter the individual parts, the result is a true statement (as shown in column 8).
Object: If one becomes aware of a process as a totality, realizing that transformations can act on that totality and can construct such transformations (explicitly or in one's imagination), it is the process into a cognitive object level of understanding of the algebraic properties of each connector, relating based on their properties and equality expressions based on its equality from the truth table (as shown in Table 1 column 10,11 and 12 which are equal) values for the relevant expression in Figure 3.

(1) $p \Rightarrow q \equiv \sim p \wedge q$ **(2)** $\sim (\sim p) \equiv p$ **(3)** $p \wedge \sim p \equiv F$ **(4)** $p \vee \sim p \equiv T$ **(5)** $p \wedge p \equiv p \vee p \equiv p$
(6)$\sim p \wedge \sim p \equiv \sim p \vee \sim p \equiv \sim p$ **(7)** $\sim (p \wedge q) \equiv (\sim p \vee \sim q)$[De morgan] **(8)** $\sim (p \vee q) \equiv (\sim p \wedge \sim q)$[De morgan]
(9)$p \wedge T \equiv T \wedge p \equiv P$ **(10)** $p \vee T \equiv T \vee p \equiv P$ **(11)** $p \wedge F \equiv F \wedge p \equiv F$ **(12)** $p \vee F \equiv F \vee p \equiv p$
(13)$\{p \vee (p \wedge q)\} \equiv p,\ p \wedge (p \vee q) \equiv p$ **(14)**$p \wedge q \equiv q \wedge p$ **(15)**$p \vee q \equiv q \vee p$ **(16)**$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ **(17)**$p \vee (q \vee r) \equiv (p \vee q) \vee r$
(18)$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ **(19)** $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ **(20)** $(p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$
(21) $(p \vee q) \wedge r \equiv (p \wedge r) \vee (p \wedge r)$ **(22)** $\{(p \vee q) \wedge (p \vee \sim q)\} \vee \sim p \equiv T$
Numbers **(14)** and **(15)** are commutative, **(16)** and **(17)** are associative, **(18)**,**(19)** and **(20)**,**(21)** are distributive (left and right, respectively), while **(22)** is a tautology and **(11)** is a fallacy.

Figure 3. Algebraic Representation of Logic Connector

Schema: A mathematical topic often involves many actions, processes and objects that need to be organized and linked into a coherent framework, called a schema. It is coherent because it allows an individual to decide whether the schema applies when presented with a particular mathematical situation. For this research, a schema includes the application of connectors to form statements and their accurate equality. For example,$\big((,q \Rightarrow p) \wedge q\big) \equiv \big((\sim q \vee p) \wedge q\big) \equiv \big((\sim q \wedge q) \vee (p \wedge q)\big) \equiv \big(F \vee (p \wedge q)\big) \equiv (p \wedge q)$, which can be expressed with or without the use of truth table values. The latter requires converting mathematical equivalent statements by accurately interpreting, manipulating, and using the properties of each connector to achieve the desired goal.

3. RESULTS AND DISCUSSION

For the benefit of the reader, we at this moment state and solve the questions asked below: Simplify the equation below without using the truth table. Answer any two (the expected result to show schema in logic concept is shown below, and they are to answer any two).

{p ∧(~p∨q)}∨{q∧~(p∧q)}
={p∧(~p∨q)}∨{q∧(~p∨~q)} De Morgan's theorem
={(p∧~p)∨(p∧q)}∨{(q∧~p)∨(q∧~q)} Distributive
={F∨(p∧q)} ∨{(q∧~p)∨F} Fallacy
=(p∧q)∨(q∧~p)
=(q∧p)∨(q∧~p) Commutative
=q∧(p∨~p) Distributive, Tautology
=q∧T=q Identity

{(p∨q)∧(p∨~p)}∨~p
=((p∨q)∨~p)∧((p∨~p)∨~p) Distributive
=((p∨~p)∨q)∧(T∨~p) Associative Tautology
=(T∨q)∧T
=T∧T=T Tautology

{(p∨q)∧(p∨~q)}∨~p
=((p∨q)∨~p)∧((p∨~q)∨~p) Distributive
=((p∨~p)∨q)∧((p∨~p)∨~q) Associative
=(T∨q)∧(T∨~q)
=T∧T=T Tautology

3.1 Analyzing S_1

L: What do you think about the course Math 111 (Logic)? The teaching method: the class teachings, WhatsApp platform, the note sent to you on a WhatsApp group, and YouTube videos. How do you think it helps your knowledge?
S_1: Very interesting compared to what I was taught in secondary school. I understand better here, especially the explanation of conjunction and disjunction.
L: I have something to show you here on your script. How do you see question 1?
S_1: this question is also part of the question you sent to our platform. I went through it, and I also saw the YouTube video. They are accommodating. If you understand the operator's properties and axioms, someone will be able to solve it.
L: looking at your question 2, how did you arrive at that answer?

This is p or q in bracket and p or negation q in the bracket and all in bracket (pointing to line 8) with this negation of p′meaning: $(((p \vee q) \wedge (p \vee\sim q)) \vee\sim p)'$ all in the bracket, I used distributive and associativity law to bring it together. Brought the negation of p to this side (pointing to line 9) and the negation of q to this side. That is how I solved it, I can even check it on the truth table.

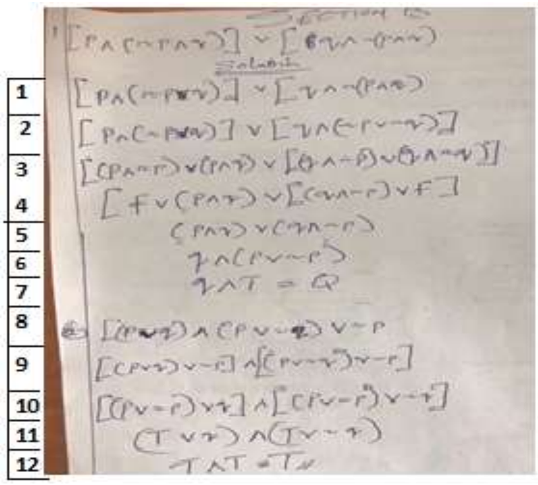

Figure 4. Field Survey

Observation: it was observed from S_1 responses and Fig. 3 that S_{18} had a suitable schema on the concepts of logic, connectors, and its algebraic representation based on the APOS genetic decomposition formulated for the study, see sub-section 2.1. He/she confirmed that the teaching method was beneficial in achieving his/her mental construction of the questions [9], [10]. Though he miswrote question 2 due to the bracket (line 8), this did not affect its solution due to the student's understanding. Still, precision in solving could help the student achieve clarity and a more coordinated and connected schema. Mathematical symbolism serves both an instrumental and communicative function [7]. The instrumental role helps one to keep track of one's thinking, while the communicative function serves the purpose of communicating or conveying logic and reasoning to others. Mathematical symbolism's instrumental role and communicative function are required to develop appropriate mental constructions that lead to suitable schema among students to deal with given problem situations. It was observed in the analyses of students' responses indicated below that many of the student difficulties could also be linked to a lack of understanding that mathematical symbolism serves both an instrumental role and a communicative function.
Teaching focus: precision in examination should be understood by students, and precision in teaching should adopt fluency (accuracy and speed) [17]. When properties are used, they should be indicated to show their coordinated schema.

3.2 Analyzing Student S_2

L: What do you think about the course Math 111 (Logic)? The teaching method: the class teachings, WhatsApp platform, the note sent to you on a WhatsApp group, and YouTube videos. How do you think it helps your knowledge?
S_2: the method is nice. Due to data, I did not see all the YouTube videos, but the class teaching was helpful.
L: can you explain how you got the second step in your number 1?
S_2: I made use of an idea. based on what we were taught in class. I am trying to group it (pointing at line 2)
L: what about question 2? How did you get that second step?
S_2: I derived it from opening the two brackets.
L: did you go through the video?
S_2: I did not go through the video.

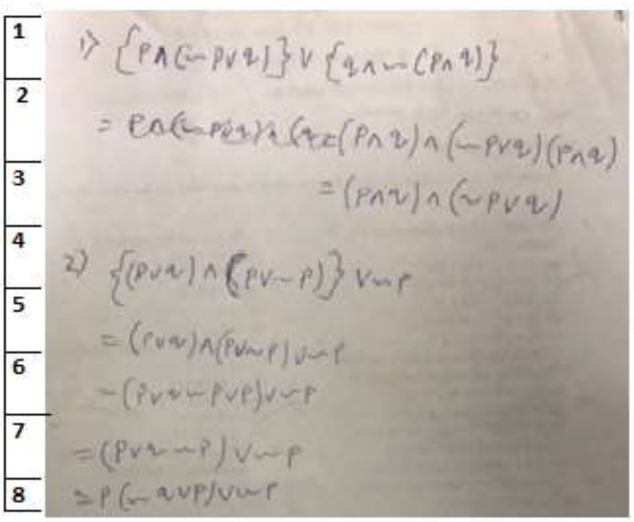

Figure 5. Field Survey

Observation: We observed that the response S_2 to the idea in Fig. 4 (question 1) does not correlate with the knowledge of the logic connectors by introducing multiplication $\left((\sim p \vee q)(p \wedge q)\right)$(in line 2), and on question 2, His/her method of opening the bracket (line 6) showed that he/she did not have the required mental construction in recognizing logic connectors. If this is accepted, then the student has an issue at the action stage of the APOS generic decomposition of logic. Although he/she admitted that the teaching method was good (agreeing with [18], [19], he/she did not watch the video (not following the methodology flow in Fig 2) even though he/she knew it was helpful. Hence, his/her attitude towards his study is questionable.
Teaching focus: students' attitude (self-responsibility) towards study at the tertiary level needs to be properly monitored to ensure proper schema and to encourage/motivate students for improved academic success [20], [21].

3.3 Analyzing S_3

L: What do you think about the course Math 111 (Logic)? The teaching method: the class teachings, WhatsApp platform, the note sent to you on a WhatsApp group, and YouTube videos. How do you think it helps your knowledge?
S_3: very explanatory, informative, helpful and interesting. I find the platform difficult to access because I sometimes do not have data. I watched the videos, but I could not watch them over and over again.
L: looking at your question 1, I realized you started well, but you had p as your answer. Please explain how you did everything.
S_3: (repeating the question) I used the distribution method there. I made a mistake here (pointing to line 2 of question 1). I should have introduced Morgan's theorem correctly, and then I will not use associative law. Hmmm, that is why I did get it, and this fallacy is from the truth table (pointing at 2)
L: what can you tell us about question 2?
S_3: hmm. My question 2, I was unable to finish that day. When I got to question 2, the time was already up. I was able to do the little because I was sitting in front, and the scripts were collected from the back.

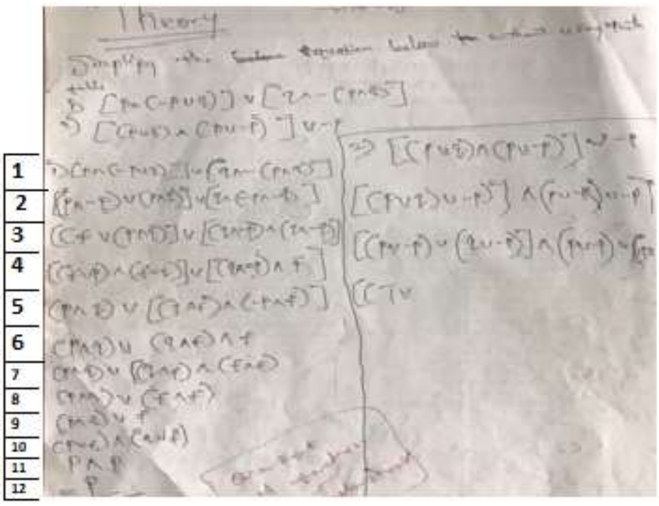

Figure 6. Field Survey

Observation: Question 1: S_3 correctly applied distributive property (line 2) but failed to accurately apply de Morgan principle (by not changing $(\wedge)$ to $(\vee)$(in line 2). This affected his/her solution, and on question 2, he/she correctly applied distributive and

associative properties, as indicated in Figure 5 (lines 2 and 3; question 2); also showed that $(p \vee \sim p) = T$ but could not finish due to poor time management by the student. When the student saw his/her script, he/she could point out and correct his/her errors, demonstrating a good understanding of the algebraic properties of each connector and its equality expressions but could not correctly apply them. If this is accepted, the student had some mental construction at the process stage of comprehension but needs some clarification on bracket usage. The student agreed that the teaching method was interesting and helpful for learning; not seeing the videos as much as expected, the student might have accepted his level of clarity. (Self-responsibility and a positive attitude toward learning are strongly advised [22].
Teaching focus: more explanations should be given on properties like De Morgan's law and the important use of brackets for proper clarity and to achieve the expected schema.

3.4 Analyzing S_4

L: Q: What do you think about the course Math 111 (Logic)? The teaching method: the class teachings, WhatsApp platform, the note being sent to you on a WhatsApp group, and YouTube videos. How do you think it helps your knowledge?
S_4: very explanatory and informative, sir. It aids learning
L: tell us how you were able to understand what to do.
S_4: it was through the video and the teaching.
L: explain what you did.
S_4: If you understand Morgan's theorem, other properties
L: Question 2: If you have that idea and understanding, how did you make this mistake?
S_4: I was rushing. So I can go and do the objectives
L: can you point out your error from the question?
S_4: around here (pointing to line 17).

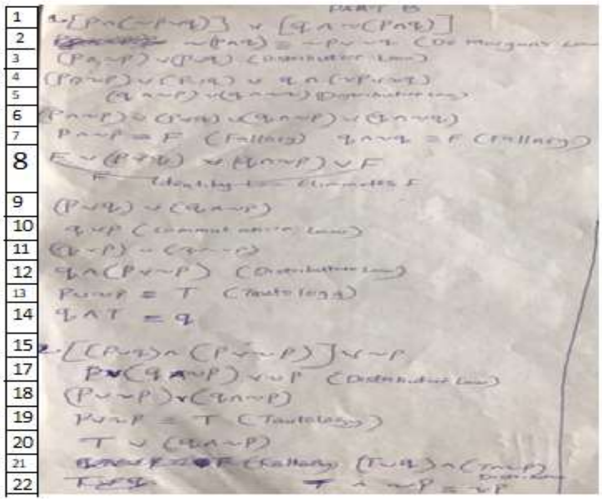

Figure 7. Field Survey

Observation: It was observed that there was some sort of schema in question 1 due to the manipulation of understanding and correctness in precision of the properties used [17], but an error was discovered in question 2 on his/her usage of brackets. However, his/her property precision was correct. The bracket error, however, impacted the manipulation idea in obtaining the correct answer (see Fig 6 (line 17)). Here again, we have a student who did not appreciate that mathematical symbolism serves both an instrumental role and a communicative function. If this is acceptable, his/her mental construction is still at the

object level according to APOS theory, and he/she requires more practice to achieve the perfect schema on the concept. The student also confirmed that the conventional method of teaching alongside CAI as a tutorial was helpful for his/her learning.
Teaching focus: An adequate orientation on time management and solving expressions with brackets in logic statements should be given when attempting mathematical questions, and students should practice enough exercises to improve their skills.

3.5 Analyzing S_5

L: Q: What do you think about the course Math 111 (Logic)? The teaching method: the class teachings, WhatsApp platform, the note being sent to you on a WhatsApp group, and YouTube videos. How do you think it helps your knowledge?
S_5: They are very nice. Everything is nice. I did not watch all the videos, though. I only watched one, but, I have seen it all.

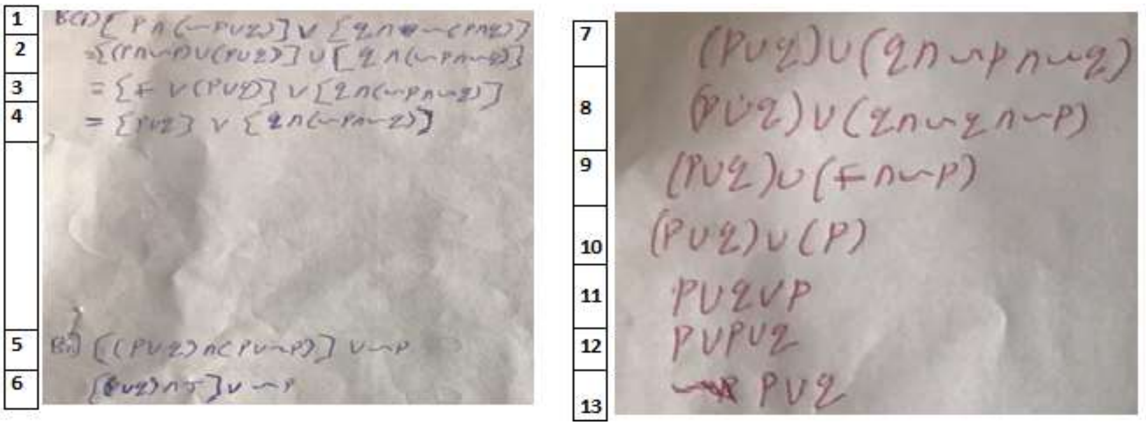

Figure 8. Field Survey

L: Can you explain what happened while solving question 1?
S_5: I started question 1 with distributive law and associative law. I opened the bracket. After that, I got fallacy '*F*' here (pointing at line 3), I should have opened this bracket after that, but, because of time, I could not. On this question 2, I was confused. First, there was no time; second, I was confused.
L: You had time to attempt Question 2, so why is the time your reason? If I asked you to continue now, what would you write?
S_5: I would open this bracket (was given a paper to solve)
Observation: It was discovered that S_5 did not fully comprehend the properties of connectors (operators) because he/she did not comprehend the significance of brackets in the statement (this is related to an understanding of relevant prerequisites or construction and reconstruction discussed [7]. He/she was also upset about De Morgan's property. The student had the knowledge of what to use but could not apply it correctly and accurately, even after collecting a blank sheet of paper. If accepted, he/she could be rated at the process stage of understanding. The student agreed with the teaching method, although he had issues seeing all the videos before the test. This might have contributed to his level of mental construction and why he was confused. This supports the findings of [20], [23], [24] that in order to achieve academic success, students must be motivated and have a sense of responsibility.
Teaching focus: When working with De Morgan's principle, focus on logic statement connector properties and the change in $(\wedge)$ to $(\vee)$ and vice versa, bracket applications, and motivations on attitude towards their study (since he/she may have a better mental

construction if he/she prepared himself/herself to see all the videos alongside classroom works).

3.6 Analyzing S_6

L: Q: What do you think about the course Math 111 (Logic)? The teaching method: the class teachings, WhatsApp platform, the note being sent to you on a WhatsApp group, and YouTube videos. How do you think it helps your knowledge?
S_6: I love the method, but other lecturers are not using it. I think I understand it better.
L: How did you go about answering question 1?
S_6: I solved it. The teaching and the videos were helpful, and I applied some properties like associative, etc.
L: If you look at your second line (pointing to line 2), applying De Morgan's law was incorrect, and this (pointing to lines 4-5) is incorrect, how did you get q?
S_6: I have seen the question before in the class. I knew that the answer was q
L: how about question 2
S_6: I used associative law (pointing to line 12), but when I got here (pointing to line 17 downward), it was no longer clear
L: what is the importance of brackets in Mathematics?
S_6: bracket like in BODMAS, we have to attend to it first or miss the answer

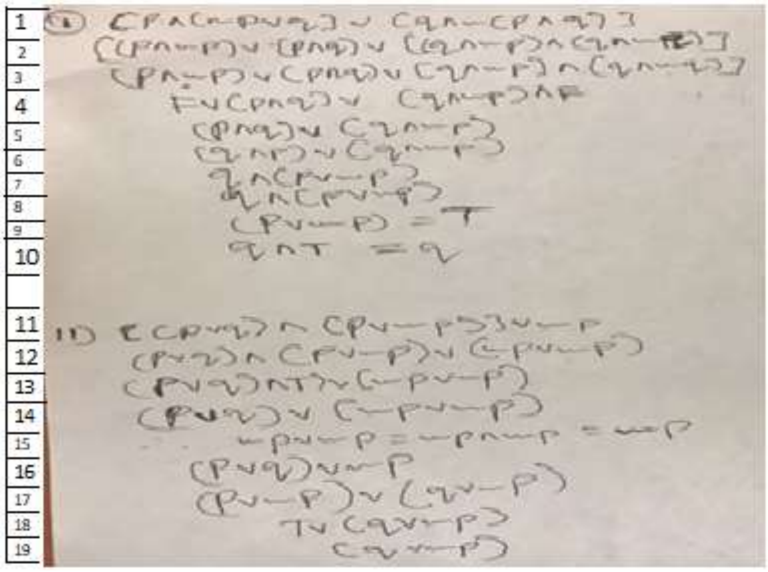

Figure 9. Field Survey

Observation: We deduced from the analysis that he/she could correctly apply associativity and some principles, as seen in Fig. 9, but could notice the question 1 solution was memorized, leaving no ability to explain. We observed that he made errors in placing his bracket. Although he understood brackets, he failed to apply them correctly. It was also observed that question 2 lacked precision and objectivity; this might be based on his/her use of brackets. Mental constructions of relevant prerequisites (in brackets) are needed for a better schema. If accepted, he or she could be placed at the object stage of understanding. S_6 agreed that the method of teaching is good and should be encouraged among other teachers (also discussed by [8]).
Teaching focus: A good schema of a concept should not be based on memorization but on understanding so it can be applied to any other concept or topic. Mathematical topics are prerequisites to other topics; therefore, a good understanding of schema is vital and

advised. If a student has a mental construction on a topic (bracket), it can be used to reconstruct other topics to gain a suitable schema [7].

3.7 Analyzing S_7

L : Q: What do you think about the course Math 111 (Logic)? The teaching method: the class teachings, WhatsApp platform, the note sent to you on a WhatsApp group, and YouTube videos. How do you think it helps your knowledge?
S_7: The classes and online videos are very helpful, but they consume data a lot of time, so I think it is better if someone downloads them and then shares them through the Xender app.
L: The part B question was for you to answer without using the table, so why did you solve it like this?
S_7: sir, I know that if I use a truth table to solve the equation, I can now use equality on the table to know the answer
L: how did you derive the truth value for column 5 '$(\sim p \vee q)$' in the question 1 solution
S_7: I got the column from disjunction 'or' of $\sim p$ and q. oh, I should have a column for $\sim p$,
L: Also, observe column 7 (lecturer point to column *7* $(q \wedge (p \wedge q))$
S_7: yes, I believe the answer is correct by using the truth table
L: the column you have there was supposed to be (lecturer wrote $q \wedge \sim (p \wedge q)$ or $q \wedge (\sim p \vee \sim q)$on a paper for the students) if you used De Morgan's theorem
L: what can you say about question 2

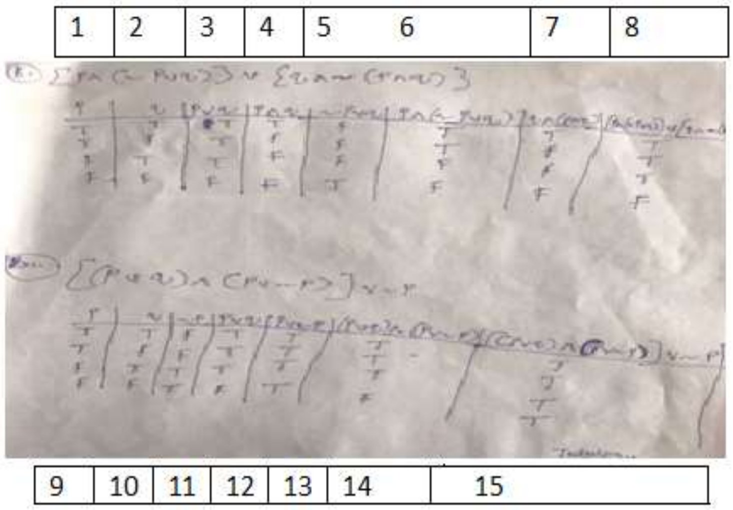

Figure 10. Field Survey

S_7: Sir, as I said, I don't really understand working without a table, but I am sure of this from what I learned from videos and class work.
Observation: We observed that users understood the operator definition and its applications on the truth table. Although he/she made some errors in question 1, which could be connected to impatience or a lack of coordination in his/her statement, the student was still corrected in question 2; he/she noted it was tautology and understood equality of logic on a truth table. If these are accepted, we may be able to reach the process level mental construction stage. S_7 agreed with the teaching method but advised that the videos should be available without snuffling out the internet due to data

consumption. S_{25} might need support (from parents or other sources) in procuring internet data to attain the required schema [24].
Teaching focus: there should be an emphasis on relating or linking the understanding of the truth table with algebraic representation to help the students attain a good schema. The student should be motivated by an attitude of personal responsibility in getting what is needed to achieve academic success [20]-[22].

3.8 Analyzing S_8

L: Q: What do you think about the course Math 111 (Logic)? The teaching method: the class teachings, WhatsApp platform, the note being sent to you on a WhatsApp group, and YouTube videos. How do you think it helps your knowledge?
S_8: It makes the course easier to study
L: How did you go about question 1?
S_8: in the first step, I used distributive (pointing line 2 from $\left(\mathrm{p} \wedge (\sim \mathrm{p} \vee \mathrm{q})\right)$ in line 1) and on the second bracket, I used associative principle since they have the same operator, and it got this (pointing line 3: $'(\mathrm{p} \wedge\sim \mathrm{p})'$) we are going to get false or fallacy that is 'F' if truth table is used, but when I get here (pointing to line 5) I was stuck, that why I stopped there.
L: what about question 2?
S_8: I know this (pointing toward lines 6 and 7) will give false, so I used distributive, then I got hanged here (pointing to line 10)

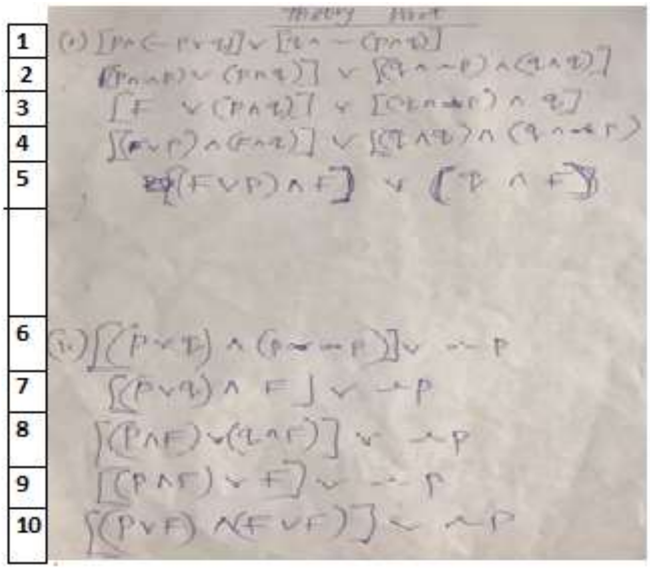

Figure 11. Field Survey

Observation: Question 1: S_8 forgot the relation of the negation sign on the bracket, where the proper property should be de Morgan's law$(\sim (p \wedge q) =\sim p \vee\sim q)$, even though he/she acquitted himself/herself with the application of associativity and distributive, and he/she understood the importance of the truth table in realizing the value, and on question 2, he/she reaffirmed his step '$p \vee\sim p = F$', in which he also confirmed this from the truth table. He or she claimed false (F) instead of true (T). With all the facts, we could see that the student cannot correctly process and apply the connector's properties. Even though they have become familiar with the connectors, they cannot correctly link the truth table to its algebraic form. If this is accepted, the student is at the process level of APOS analysis. Although the student correctly applied associative and distribution properties,

hence the application of [25] theory (which discussed some mathematical situations in which it is necessary to de-encapsulate an object and return to the process that produced it), He/she should have confirmed ($p \vee \sim p$) on the truth table if in doubt. The student's statement that "it makes the course easier to study" indicates that the teaching method benefits him or her.
Teaching focus: More explanations should be directed towards algebraic axiom definitions, and more exercises, examples, and explanations of De Morgan's law should be made so as to increase students' understanding.

This research work established the importance of logic and abstract thinking for the tested group of students and around the world, as seen through the research of various authors. The research looked into possible solutions for overcoming the identified challenges. It adopted computer-assisted instruction (CAI) as a tutorial. At the same time, the conventional teaching mode was used in testing the students to attain a desirable schema that was adaptive to their understanding of the topic. The research looked into possible solutions for overcoming the identified challenges. It adopted computer-assisted instruction (CAI) as a tutorial. At the same time, the conventional teaching mode was used in testing the students to attain a desirable schema that was adaptive to their understanding of the topic. The research found that most participants who followed the conventional teaching method and the CAI tutorial had a better schema than others. It also indicated areas of interest for teachers to intensify teaching or explanation while teaching the topic to achieve maximum success.

The findings also revealed that students find it hard to manage time when solving questions that showcase relevant understanding due to the unknown required mathematical methods and memorizing concepts that require proper understanding in their application. The implication of this study on students is for the proper interpretation of the connective properties of logical statements, and for teachers is to identify and understand students' difficulties and how to implement the solutions discovered. For future research, it is suggested to explore the application of CAI methods in various learning contexts and other disciplines, as well as to assess its long-term impact on students' cognitive skills and mathematical understanding. Additionally, it is important to compare the effectiveness of CAI with other innovative learning methods to determine the best approach to developing students' logic and algebra skills.

According to the research, APOS theory could be a useful way to analyze students' levels of understanding, not just in logic but in mathematics and the science field. However, the researchers intend to translate their findings into a formidable development for students to achieve maximum success as a form of positive learning outcome and assist teachers. The study's findings indicate that precision on properties used indicates a level of the schema, that constructed understanding of the application of brackets is critical to algebraic logic connectors, and that a suitable teaching method (CAI-tutorial enhanced conventional) that could help students achieve a suitable logic schema. One of the teaching implications of this study is that there should be a focus on using mathematical symbolism as an instrument for keeping track of one's thinking and serving a communicative function (as seen in S_1 and S_4). To be fully effective, students need to display good prerequisite mental structure, time management skills, attitude, and self-responsibility toward their study by investing quality time in going through the relevant materials (videos and reading materials) as provided for them, in addition to the classroom teachings. The findings indicate that students taught using the Computer-Assisted Instruction (CAI) method exhibit better proficiency in applying connective property equations to algebraic logic expressions. This suggests that a technology-based learning approach effectively enhances students' comprehension of the relationship between algebra and mathematical logic. Discussions can centre on the effectiveness of specific elements within the CAI method that support understanding these concepts. APOS analysis reveals students' challenges in connecting their understanding of truth tables to algebraic forms, particularly concerning connector properties like De Morgan. Discussions may explore teaching strategies, such as utilizing visual or manipulative representations, to help students overcome these hurdles.

The current research aligns with prior studies employing the APOS Theory to enhance students' understanding across various mathematical contexts [26]. Consistent outcomes indicate improved mathematical concept proficiency and abstract understanding through the application of the APOS Theory. It's important to note that this study focuses on content analysis of APOS Theory studies. At the same time, earlier research concentrates more on implementing the theory in the context of abstract algebra proof and students' conceptual mathematical abilities.

Acknowledging that students' understanding evolves over time is crucial. Research limitations may involve a lack of long-term assessments of learning impacts, and discussions should encompass suggestions for further research to address these limitations.

4. CONCLUSION

In conclusion, the study aimed to evaluate the effectiveness of Computer-Assisted Instruction (CAI) in enhancing applied science students' proficiency in applying connectivity equations in algebraic logic. The results demonstrated that students exposed to CAI exhibited superior performance, highlighting the technology-based approach's efficacy in improving their understanding of the relationship between algebra and mathematical logic. The significance of students' cognitive schemes in comprehending these concepts was underscored, with those possessing a more mature conceptual understanding better-navigating application difficulties.

Further discussions can delve into specific elements within the CAI method that contribute to its effectiveness, addressing challenges identified through APOS analysis, such as difficulties connecting truth tables to algebraic forms, especially regarding connector properties like De Morgan. This study aligns with prior APOS Theory research, emphasizing consistent positive outcomes in enhancing students' mathematical concept proficiency and abstract understanding across various mathematical contexts.

However, it's crucial to acknowledge the evolution of students' understanding over time. Research limitations, including a lack of long-term assessments, suggest the need for future investigations to explore the enduring impact of CAI on cognitive skills and mathematical understanding. Additionally, discussions should encompass recommendations for further research to address existing limitations and compare the effectiveness of CAI with other innovative learning methods in developing students' logic and algebra skills.

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