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RECORD OF SEMIOTIC REPRESENTATION USING GEOGEBRA: AN OLYMPIAD TRAINING ON BRAZILIAN STUDENTS

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ABSTRACT

This study aims to compile and develop an international didactic situtation presented with an approach to the center of an arbitrary triangle in visual exploration in 2D and 3D format using GeoGebra software as a technology for modification and construction of Olympiad questions. The methodology used in this study is an exploratory qualitative which is described from a didactic sequence of Mathematics Olympiad which were held in four online meetings using Google Meet. The results show that this exercise can be used as a resource that will provide a level of learning for students through command and visualization of images that serve a broad view of the international Olympiad situation. International didactic situations can be applied by mathematics teachers both in training for national and international Olympiad and for classroom teaching. In summary, it is noteworthy that the dynamic records of semiotic representations stimulated by the use of GeoGebra software has great potential to encourage the progress of students' representative geometric thinking, through the development of visualization, perception, and mathematical intuition. It is hoped that this work can serve as a pedagogical and methodological tool by teachers for various competencies.

REKAM SEMIOTIKA REPRESENTASI MENGGUNAKAN GEOGEBRA: SEBUAH LATIHAN OLIMPIADE PADA SISWA BRASIL

Kata Kunci:

Teknologi digital
GeoGebra
Pusat segitiga
Register representasi semiotik
Pengajaran matematika

ABSTRAK

Penelitian ini bertujuan untuk menyusun dan mengembangkan situasi didaktik internasional yang disajikan dengan pendekatan pada pusat segitiga sembarang dalam eksplorasi visual dalam format 2D dan 3D menggunakan perangkat lunak GeoGebra sebagai teknologi untuk modifikasi dan konstruksi soal Olimpiade. Metodologi yang digunakan dalam penelitian ini adalah kualitatif eksploratif yang dijabarkan dari rangkaian didaktik Olimpiade yang dilaksanakan dalam empat kali pertemuan *online* menggunakan Google Meet. Hasil penelitian menunjukkan bahwa latihan ini dapat dijadikan sebagai sumber daya yang akan memberikan tingkat pembelajaran bagi siswa melalui perintah dan visualisasi gambar yang memberikan pandangan luas tentang situasi Olimpiade internasional. Situasi didaktik internasional dapat diterapkan oleh guru matematika baik dalam pelatihan untuk Olimpiade nasional dan internasional maupun untuk mengajar di kelas. Disimpulkan bahwa rekam dinamis representasi semiotika yang dirangsang dari penggunaan perangkat lunak GeoGebra memiliki potensi besar untuk mendorong kemajuan pemikiran geometris representatif siswa, melalui pengembangan visualisasi, persepsi, dan intuisi matematis. Diharapkan karya ini dapat berfungsi sebagai alat pedagogis dan metodologis oleh guru untuk dalam berbagai kompetensi.

1. INTRODUCTION

Plane geometry is a field of Mathematics that studies plane geometric figures, being constantly used to answer everyday problems, in addition to contributing to the general development of the subject. According to Silva Junior [1], emphasizes that plane geometry develops "kills related to shape, space, distance, perception, among others, allowing a way to understand, describe and represent in an organized way, the world in which we live, as well as establish practical applications in everyday activities". Among the activities found, the inclusion of notable points of a triangle, namely: barycenter, circumcenter and orthocenter, in addition to the statements and demonstrations of some propositions, the Incenter of a Triangle becomes important for the description of applications and their direct relationship with the meeting point of the internal bisectors, and this is pointed out by authors as Powell [2], Bairral and Marques [3], Jesus [4] and Pereira [5].

Regarding the mathematical object under study in this research - the Incenter of a Triangle - there is an existing problem in the constructive process of teaching and learning, in which this content is presented in book IV of Euclides in a ready way, not explored the visualization of the notable points of a triangle [6], [7]. For this reason, the obstacles encountered in a single assertion are highlighted, which is admitted as positive through a good debate and explanation by the teacher, according to Lorenzato as it provides a more complete explanation of the world, a wider exchange of ideas, and a more balanced view of mathematics [8].

Based on this assumption and taking into account the relevance that the Incenter of a Triangle has for the student's understanding of the elements and notable points of a triangle, how the student can understand and develop geometric concepts about notable points using the Incenter of a Triangle, from a conception of visualization and geometric representation? For this, the GeoGebra tool was used because it is an easy access and manipulation software, being effective for the teaching of Geometry, among other areas, [9], plays a key role in teaching with the dynamics of mental structures responsible for concrete and experimental information to abstract and generalized parts.

The structuring of the Olympic didactic situation, according to Santiago e Alves, is defined as a set of problem-situations from questions taken from Olympic competitions whose purpose is to encourage a number of students, not just competitors, that is, those who are often among the classified among all participants in the International Mathematical Olympiad (IMO), and the GeoGebra software was used as a digital technological resource. According to Bortolossi describes that GeoGebra is software that includes numerical, symbolic, graphics and programming resources in algebra, arithmetic, geometry, functions, probability and statistics [10], [11]. Duval brings that "the diversity of representations of the same object originates from the variety of physical or semiotic systems that can produce representations" [12].

Thus, GeoGebra has the potential to encourage the student's intuition and representative geometric visualization, providing deduction and interaction through experimentation with representative content in dynamic geometry. Furthermore, according to Leivas, Bettin and Pretto, GeoGebra 3D serves as an initial stimulus and motivation to provoke student curiosity, confrontation of ideas and new strategies to find representations of spatial objects [13].

The objective of this work is to develop an Olympic didactic sequence to support the learning of the Incenter of any Triangle in a teaching practice in mathematics, with the support of GeoGebra software, as a way of helping the student in the construction of the geometric object, through visualization, perception and mathematical intuition. Therefore, this research brings the Theory of Semiotic Representation Records, by Raymond Duval, which guides through activities, taking into account the student's prior knowledge and use of operational materials.

Research on the use of GeoGebra in learning mathematics has been carried out several times, including: GeoGebgra to teach logarithmic functions through didactic sequences by exploring their representations [14], GeoGebra has also been used for the needs of the olympics but only for visualization [15], GeoGebra in analyzing the mobilization of recordings of semiotic representations of pyramid rods by students on concrete material and everyday life [13], and GeoGebra for learning to visualize mathematics [16]-[19]. However, no research has been found on the development of international didactic situations presented with an approach to the center of an arbitrary triangle in visual exploration in 2D and 3D formats using GeoGebra software.

The purpose of this study is to compile and develop an international didactic situation presented with an arbitrary triangle center approach in visual exploration in 2D and 3D format using GeoGebra software as a technology for modification and construction of math Olympiad questions. Therefore, our study seeks, through historical, epistemological and didactic aspects, a proposal to minimize the learning difficulties of the Incenter of a Triangle in the context of the IMO, the Olympic Didactic Situation (ODS), based on in the international Olympic Problems (OP), the Theory of Semiotic Representation Records, as a guide for the didactic teaching session associated with the GeoGebra software, from the manipulation of the parameters established in this learning object, the students visualize the movement of the points and understand how they work in an Olympic situation related to Plane Geometry.

2. METHOD

For this research, an exploratory research methodology was adopted, based on a case study, observing the applied experiments and providing relevant data that allow the presented hypotheses. In order to make the descriptions of the article viable and to include itself in a long and tiring study of one or a few objects, so that its extensive and detailed knowledge is available [15].

The research was applied to a group of twenty-five students, aged between 16 and 18, attending the 3rd year of high school and coming from a state public school in Quixeramobim - CE, Brazil. The application took place in four out-of-class meetings, in which students were invited to participate in a moment of experimentation with dynamic geometry through the use of GeoGebra. The classes took place in the remote format, due to the current scenario of Covid-19, using the videoconferencing platform (Google Meet). The steps to carry out this research can be illustrated in the following flowchart.

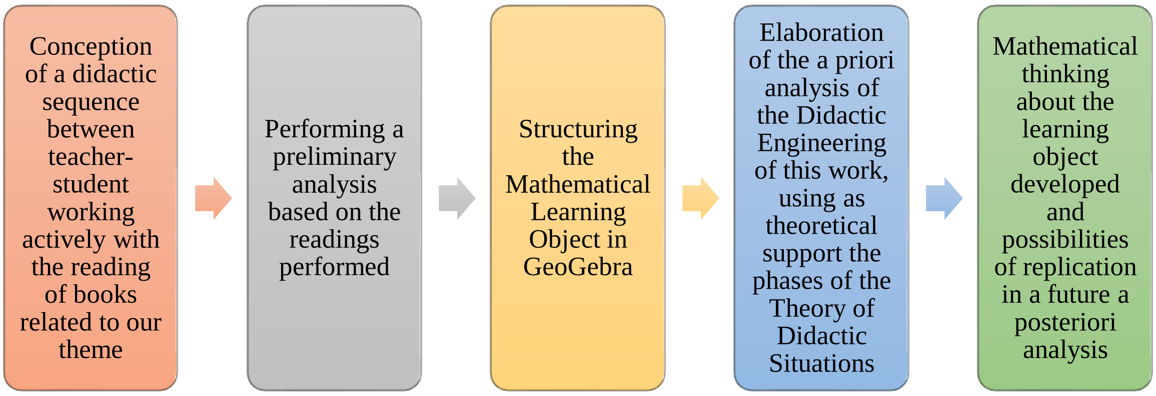


Figure 1. Flowchart with the Research Phases

For data collection, a virtual form on the Google Forms platform was used, the questionnaire is used to survey the class about their knowledge of the GeoGebra software and reflections on their learning in remote classes, audio and video recording file of the application of the interactive moment and photographic record. To preserve the identity of the subjects of this application, students have their names represented by Participant 1, Participant 2 and so on.

3. RESULTS AND DISCUSSION

The results presented in this topic were collected from the application of the international Olympic didactic sequence carried out in the virtual meetings from a perspective of development of the role of the Mathematics teacher. Thus, the classes were designed in the teacher's planning, including tasks related to teaching and thinking.

Data collection took place in four virtual meetings. For each class, students had access to activities that described plane geometry using the notable points of the incenter. In addition, debates took place through the messaging application regarding the structure of activities. In this way, the work had the participation of four students from the 3rd year of high school from a school in the countryside, in the discipline of Mathematics, developed in four meetings remotely, due to the scenario of the Covid-19 pandemic that occurred around the world.

3.1 Application of the Olympic Didactic Sequence using GeoGebra

The use of digital technologies as a tool for Mathematics classes is a method that differs from the traditional one summarized in pencil and paper or from the use of this support only for research. The various possibilities of practices of these technologies can be included for studies, depending on the moment exposed by the teacher in the structuring and observation of theories, leading students to build their strategies to the point of developing hypotheses [16].

The selected problem is a question extracted from the IMO evaluation carried out in 2006 for high school students, this OP should address concepts such as incenter of any triangle and triangle and circumference relationships. Further on, question 1 will be presented with the topic of geometry, taken from the IMO proof and, soon after, the problem will be transposed in GeoGebra.

Question 1: (IMO/2006) - Let ABC be a triangle with incenter I. A point P inside the triangle verifies: $\angle PBA + \angle PCA = \angle PBC + \angle PCB$. Prove that $AP \geq AI$, with equality if and only if $P = I$. The competition was held in the Slovenian city, Ljubljana, and the problem was proposed by the South Korean delegation (Korea).

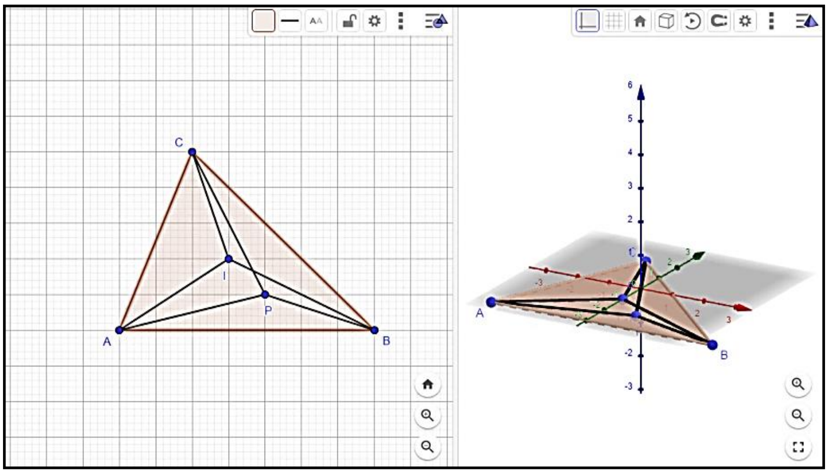


Figure 2. International Didactic Situation Selected for Student Viewing

The Olympic didactic sequence sought to work with the students on the context of the problem and adjust, if necessary, the ODS to the GeoGebra software. With this questioning, it is expected that students interpret the problem to search for previous contents, characterizing the condition of connecting the points (ABC) of the triangle to the point P visualized in Figure 2 and having as an element the point I.

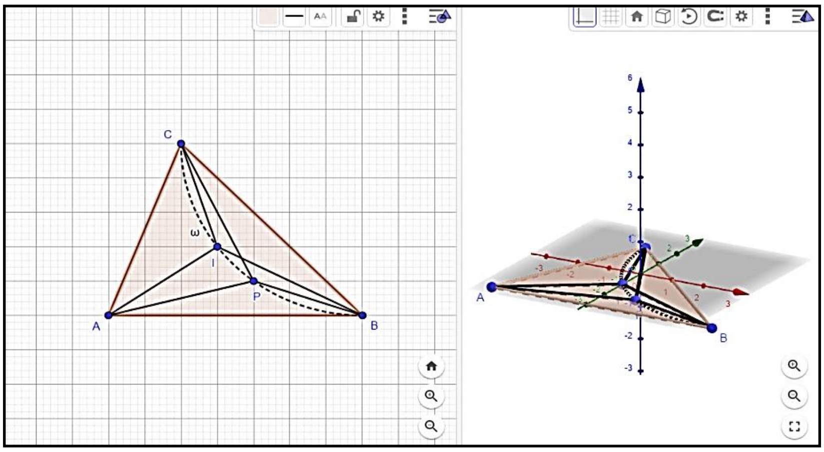


Figure 3. First Step Expected by the Student in the GeoGebra Software

The information extracted from the question statement reiterates some information about the proposed problem, they are: points I and P have the following description of the points ∠IBP = ∠IBC - ∠PBC = 1/2 ∠ABC - ∠PBC = 1/2 (∠PCB - ∠PCA) (1) and, in the same way, ∠ICP = ∠PCB - ∠ICB = ∠PCB - 1/2 ∠ACB = 1/2 (∠PBA - ∠PBC) (2), as illustrated in the previous image (Figure 3).

At first, the variables are identified from the description of the problem, and the students reach the conclusion of the incenter (meeting of the rays) of the triangle ΔABC, finding an internal angle of 180º. From this perspective, students describe that ∠PBA + ∠PCA - ∠PBC + ∠PCB, connecting the points ∠PBA - ∠PBC = ∠PCB - ∠PCA. Then the descriptions (1) and (2) are developed the following points ∠IBP = ∠ICP, therefore, B, I, P, C are concyclic (those that belong to the same circle).

The explanation expected by the student is that the area of the triangle (ABC) is divided by the incenter connected to points A, B and C.

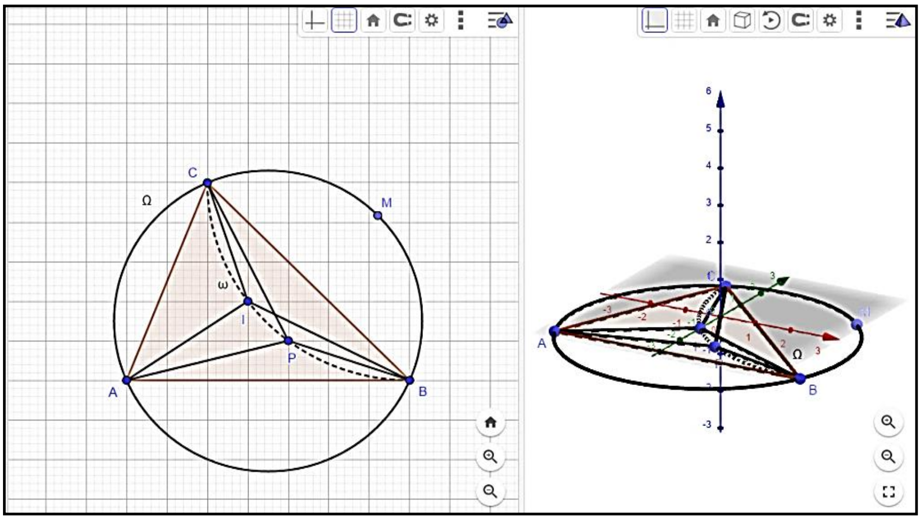


Figure 4. Any Concyclic Triangle on the Circumference of the Didactic Situation

Thus, the radius AI is observed, finding the circumference of ΔABC transcribed at point M. Then, the segment of the incenter can be visualized at points MB = MC = MI = MP of the same exposed line segment (Figure 4). Thus, the points AP + MP ≥ AM = AI + IM become MI = MP and, at the end of the question, we have the solution of the lines AP ≥ AI.

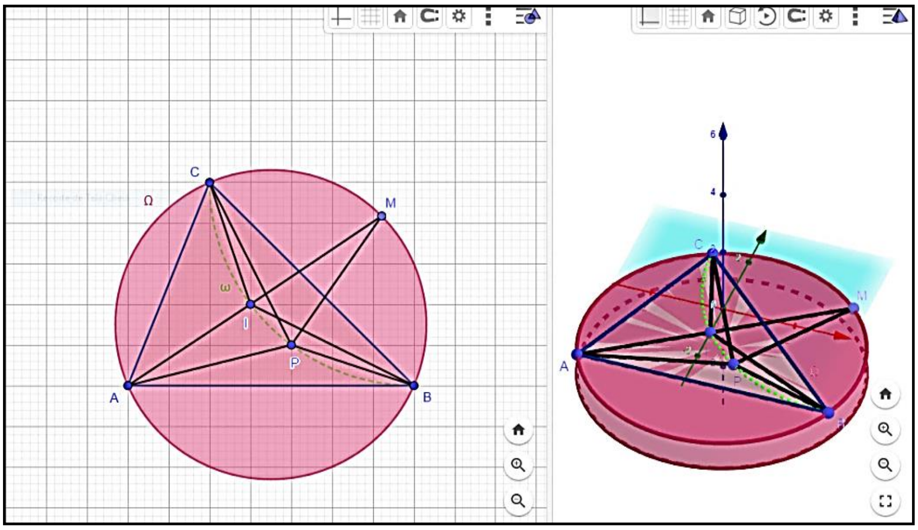


Figure 5. Final Solution of the International Olympic Problem

In this way, the student arrives at the final resolution of the problem presented on the blackboard with the possible justifications that his mathematical reasoning concluded. GeoGebra becomes useful to arrive at the result of the question. Given the assumptions, the resolution performed by the students is presented, concluding that the lines AP ≥ AI have an equality at point P, being with the line segment AI, which occurs at the points P = I (Figure 5).

In this case, the professor justifies himself, describing that the bisectors (of the sides) of a triangle interconnect at a single point equidistant from their vertices. Finally, the teacher shows the construction protocol in GeoGebra, data provided by the software throughout the structuring process of the proposed problem (Figure 6).

1	Ponto A			
2	Ponto B			
3	Ponto C			
4	Ponto I			
5	Triângulo t1		Polígono A, B, C	Polígono(A, B, C)
5	Segmento c		Segmento A, B	Segmento(A, B, t1)
5	Segmento a		Segmento B, C	Segmento(B, C, t1)
5	Segmento b		Segmento C, A	Segmento(C, A, t1)
6	Ponto P			

Figure 6. Construction Protocol Olympic Didactic Situation of the First Part

In the observation of Figure 7, there is the rest of the construction protocol developed for students to understand each command of the figure exposed to the Olympic problem extracted from the IMO.

7	Segmento f		Segmento A, P	Segmento(A, P)
8	Segmento g		Segmento B, P	Segmento(B, P)
9	Segmento h		Segmento C, P	Segmento(C, P)
10	Segmento i		Segmento A, I	Segmento(A, I)
11	Segmento j		Segmento B, I	Segmento(B, I)
12	Segmento k		Segmento C, I	Segmento(C, I)

Figure 7. Protocol of the Didactic Situation Second Part

In the visualization of Figure 6 and Figure 7, the points and segments created during the construction of the ODS of the IMO of the year 2006 can be observed. Therefore, it is expected that the students perceive, from the resolution of this OP, the use of the concepts of plane geometry addressed for other future questions. Furthermore, it is intended that future teachers stimulate their students, providing meaningful learning from discussions with their class/group. GeoGebra makes it possible to work with dynamic geometry in the classroom and have that moment of observation of the figure built in the problem proposed by the teacher, leading him to use more digital technology in his school planning.

These questions presented in the IMO assessments require a significant level of knowledge from the student, so few are chosen to represent the country of origin. In this way, there is a priority to make available methods and technological instruments that provide the teacher with a way to insert them in the classroom. In this regard, Alves emphasizes the dissemination of mathematical culture worked on in international and national competitions as an essential component for competitors and participants in these competitions. In the same way, ODS, with the technological support of the GeoGebra software, contributes to the structuring of the figures addressed in the IMO questions, allowing teachers a choice for their daily planning so that, consequently, they can use them for the teaching of geometry flat in the classroom context, thus motivating student interaction and learning [17].

3.2 Preliminary Analysis

3.2.1 International Mathematics Olympiad

In this work, we used a problem from the International Mathematical Olympiad (IMO), which was created in 1959, in Romania, and has been held annually since then, with the exception of 1980. The IMO has been in existence for over 50 years, generating an amazing legacy for the participating countries and has universally established itself as the best-known international mathematics competition in which a high school student can aspire to participate.

According to Kenderov, the International Mathematical Olympiads hold their tournaments with formal rules that regulate each moment of the competition: interaction, selection of questions, access to test resolutions, delivery of medals and other important details [18]. An efficient way to develop student's Olympic preparation is to build mathematical thinking and logical-mathematical reasoning ability to solve problems, which provides the development of new strategies that can be applied during the tests.

Every year, around 600 students take the test of the international competition, which serves an average of 105 countries each year. These data make international competition an important factor for educational institutions in each country, as it ensures moments of interaction between participants from all countries, generating training opportunities for skilled students and disseminating mathematics around the world [15]. In the same way that mathematical problems are challenging and different, IMO is also a time to interact and socialize, creating bonds of friendships with similar interests and being able to get acquainted with your future friends in this journey of professional and scientific mathematics [20].

Still on the mathematics olympiads, Alves states that they depend on the specific nature and practice of the issue involved, providing moments of challenges between the participants or that, in other situations, reveal a favored interaction with the new distinct mathematical skills [21]. The urge to compete in a tournament is immensely strong in human nature and has been included for centuries to promote skills and develop your thinking ability in various activities [22].

In addition to what was mentioned above, the participating country has the right to send up to six members and each one competes individually, without any collaboration [20]. Each country chooses a leader for the Olympic team, who participates in the selection of math problems and is isolated from the rest of the team for a period until the conclusion of the competition, and a vice-leader, who is responsible for the participants.

Preparation for IMO takes place in a way that each delegation (except the host country) sends the questions to create an initial database, called a long list (LongList, LL). Such questions must not have been used in previous competitions, nor exposed and must contain pre-university mathematics content. The host country of the Mathematics Olympiad forms a Selection Committee that determines which LL problems will be used to structure the short list (ShortList, SL). Team lead faculty receive the SL on the first day of the meeting and indicate the six SL issues that will be kept secret until the next competition [23].

The questions addressed in the Olympiad are provocative, fascinating and present various mathematical concepts and forms that lead to the final solution. The Olympiad seeks to motivate the student to solve problem situations through their own knowledge and strategies, not always referring to the curricular contents worked in high school [24]. The international competition lasts for two days and the assessment consists of solving six problems, involving the contents of mathematics (geometry, number theory, algebra and combinatorics). Thus, the assessment takes four and a half hours to resolve three questions each day. Still on the application of the evaluation, the first question is usually selected to be the easy one of the day and the last one, the most difficult, are rare exceptions to be included in the competition. Each problem has the value of seven points, that is, the highest score reaches 42 points. The value of points obtained by a student in each question is the result of the agreement between the coordinators of the problem, selected by the host country and by the leaders and vice-leaders of the class, who protect the interests of their participants. Considering these aspects, it was decided to structure an Olympic didactic sequence to work the student's geometric thinking based on the visualization and perception of the Incenter of a Triangle, using the dynamics of the GeoGebra software.

3.2.2 Olympic Didactic Situations

An Olympic Didactic Situation (ODS), according to Alves, is used as a teaching methodology, bringing to an interactive classroom environment moments characteristic of exercises related to mathematical investigation, included in preparations for national or international mathematics Olympiads. In this way, the Olympic Problem (OP) is adapted, with the aid of the GeoGebra software, to provide interaction with more students and, as a result, better learning focused on Olympic mathematics occurs [17].

From this perspective, the methodology arises from a characteristic equation, ODS = OP + TSD, in which the Olympic problems are issues extracted from the International Mathematical Olympiad, and the Theory of Didactic Situations (TSD) is adopted as a teaching theory allowing the teacher to systematize the accesses and barriers that students may face, for through a conjecture/construction. Thus, it must be based on the phases of action, formulation, validation and institutionalization, at the time of development of the ODS, allowing the educator to have better control over the procedures performed by the students [21], [25].

The TSD was developed by Guy Brousseau, aiming to study the interactive moments in the search for knowledge and the relationships between teacher, student and mathematical knowledge to be worked in the classroom. Didactic situations are used to teach and therefore include the entire environment that surrounds the student, teacher and the education system [26].

In the works of Santos and Alves [27], Silva, Alves and Menezes [28], Alves [17], ODS is applied as a didactic proposal for teachers to build in the classroom in the mathematics discipline, by inserting OP, as they are used in preparations for the IMO. Based on the studies of Silva, Alves and Menezes [28], whose research evidenced the teaching of these Olympic issues through a didactic sequence, on the TSD conjectures, it is understood "[...] the dynamization and visualization of the figure presented in the problem situation, which provides a greater exploration of mathematical properties". In addition, it was structured based on the work of Santos and Alves [26], who mentions TSD as an opportunity to build "[...] an environment that encourages research in mathematics in such a way that students can reproduce, even in an elementary way [...]", the process similar to that of a mathematician in developing his hypotheses.

In the research developed by Alves [16], two ODS were shown to differentiate common traditional methods used in the work of the teacher in this international Olympiad, as well as in the classroom environment. In this way, the author emphasizes the role of the teacher, "[...] so that the group of students understands the scientific mathematical concepts of interest and the formal mathematical theory that governs them, as a consequence of the formal game and the choice of a strategy victorious". Together, the previously mentioned articles have the GeoGebra software as technological support, which allowed them to observe, construct the figures and act as a digital tool for understanding new teaching and learning strategies.

The questions in this article will be based on Olympic Problems, which, according to Santos and Alves [27], are problem situations worked on in mathematics tournaments, involving Olympic mathematics students. The so-called Olympic Didactic Situation (ODS) permeates the dialectical stages of TSD with the OP which [27], are applications determined between the student or group of students, a medium (mathematical knowledge of the Olympiads) and a educational system, aiming to improve student's knowledge for Olympic mathematics competitions and problems.

In this sense, the teacher observes the issues that are included in this Olympic context, through research and studies, to be organized and provide control of the student's actions, analyzing at what moment the development of learning can happen. This structuring of the ODS makes it possible to improve their training and adapt their planning. GeoGebra is used as a digital technological support to unite the TSD, through the contact, visualization and movement of the figures, promoting learning methods for the student and also for the understanding of new strategies for Olympic mathematical problems, confronting each other in the course of solving the problem ODS.

In the following topic, the development of an overview of semiotic representation records will be described and an equalization of their mobilization from the use of GeoGebra as a tool for the student's geometric thinking from the visual field will be described.

3.2.3 International Mathematics Olympiad

Euclidean geometry is the part of mathematics that focuses on idealized objects. In this way, they do not exist in the physical world, but only in the intellectual world, with the following geometric shapes: square triangle, circles, spheres are idealizations of physical shapes that exist in the surrounding world. The spontaneous idealization of any child is based on the shared qualities of some objects that exist only in the visual impression associated with certain names. Understanding axiomatic theories, understanding the meaning and necessity of arguments, authoring the proof is neither simple nor spontaneous [29], describes the understanding of meaning in a hypothetical deductive structure, a sense of coherence and consistency, competence in the propositional thinking independent of practical constraints, are not spontaneous acquisitions.

Turning to the specific question of the theory of semiotic representation registers, Duval collaborates in the understanding of the development of the construction of geometric thinking [30]. Duval introduces the concept of representation registration with the aim of explaining how representation systems, in addition to transmitting mathematical knowledge, can be tools that help in the creation and development of new ideas and concepts [30]. In this sense, Duval identified four types of records: natural language; writing system numbers; algebra and symbols; and figures and graphs [30].

It describes the role of representation registers in the process of understanding and generating mathematical ideas through the concept of transformation, classified into two types: a) processing is the transformation of one symbolic representation into another, registered in the same type (for example, register Image); b) conversion is a transformation between symbolic representations belonging to registers of different nature for example natural language and drawing registers. One of its differentials in relation to conventional manipulative resources is the possibility of clicking, dragging and transforming a figure, keeping or not its Euclidean properties. Visualizing a geometric object is another unique feature of a Dynamic Geometry Environment (DGA): it allows the user to observe the constructed figure in different perspectives (sizes, positions, etc.) on the screen [3].

When learning geometry, it is necessary either to register in natural language and/or to register symbols and drawings. By introducing the concept of image concepts (in italics), Fischbein, shows the relevance of the close relationship between these two registers [29]. The image concept (in italics) has two components: a concept and a figure. The concept component presents natural and/or symbolic languages, characterizes a certain idealizing class (for example, the definition of a square). Image components, on the other hand, are visual (shape, position, size) and expressed through geometric design (for example square design).

Identifying relationships between geometric elements is the essence of geometry - that's why proofs dealing with theorems explain it - and, for that, an adequate symbiosis between conceptual and figurative components becomes important, which is important for students. Thus, according to Pereira, it is timely "the fruit of the evolution of knowledge, new concepts, definitions and nomenclatures were inserted in such a way that ancient geometers worked with the notable points of triangles without assigning specific names to them" [5].

Observing beginners in the use of GeoGebra software, very articulates about our way of thinking is influenced by representational systems that transmit knowledge and that people are familiar with. Our experience with 3rd year high school students doing Olympic mathematics training shows that it is not natural for beginners to take advantage of the dynamism of dynamic registers. Still with regard to dynamic intuition, Powell, states that "dialogue is something that an individual does with himself and with others [2]. It has content composed of perceptions of three possible specific elements: objects, relationships between objects and dynamics that link the relationships".

In the presence of dynamic figures, their initial performance is like a static image drawn on paper; so, even using the movement of points, they still cannot simply identify the relationship between the elements that make up the dynamic figure, identify the dynamic type in the properties of the revealed figures.

Jesus, argues that the GeoGebra software also allows you to use points, lines, line segments, polygons, and more, as well as allowing you to enter functions and perform subsequent changes dynamically to all objects. In dynamism GeoGebra already offers a variety of tool sets, including perpendicular line, parallel line, bisector, conic by five points; many more can be incorporated into your knowledge base through the create new tool feature but it is by analyzing two other properties of dynamic geometry software that we can really answer the problem of the work done earlier [4]. The first feature is the availability of symbolic representation, known in the literature as dynamic figures [31]. Once the construction is completed, applying displacements to the initial elements (usually points), the drawing on the computer screen - an instance of the representation of the construction - is transformed, but maintains the geometric relationships imposed on the construction with the resulting relationships of the image.

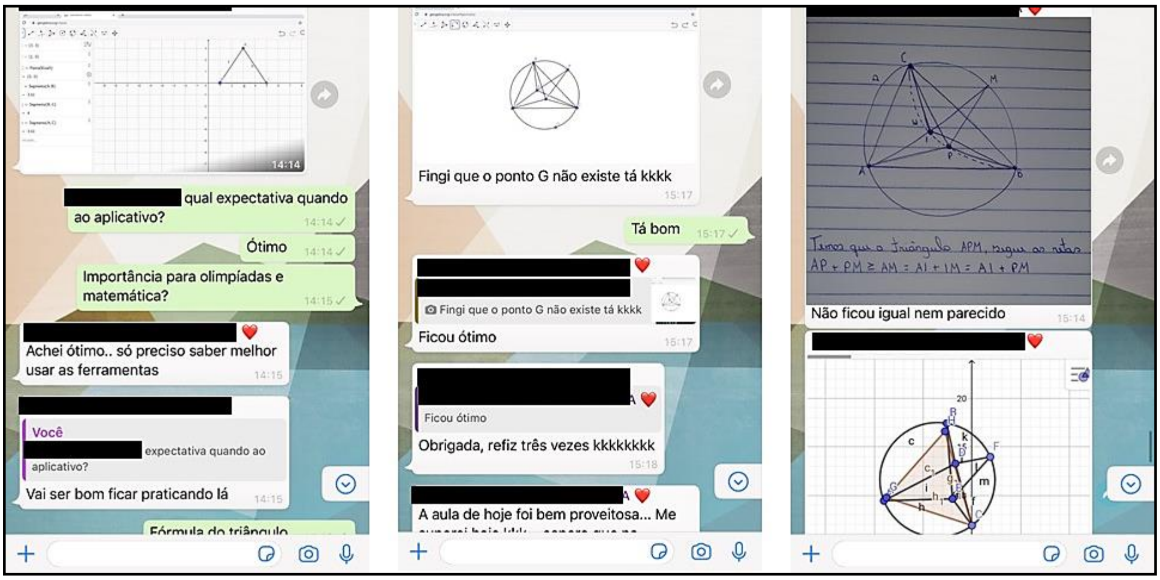
From what has been described in this topic, this study is based on the relationship of dynamic registers and plane geometry, and that the dynamism of geometry can be used to discover a new relationship in the teaching of incentro, to build mathematical models suitable for its cognitive evolution. The next topic describes the practical form and development of the Olympic didactic proposal. The virtual meetings took place on four different days, from 01/18/2021 to 02/20/2021, divided into 1 h/a remote, plus 2 h/a of weekly monitoring on the day of the remote meeting, totaling a total of 12 h/a. The schedule developed during the period of application of the ODS via the Google Meet platform can be seen in Table 1.

Table 1. Schedules of virtuais classes for the olympic team

Contents	Task	Date	Class time
Right triangle, geometric mean, circle.	Introduction, concepts of plane geometry aligned to the notable incenter point	01/19/2021	3 hours/classes
Circumference, incenter, triangle notable points.	Troubleshooting and explanation of GeoGebra's basic commands	01/262021	3 hours/classes
Right triangle, circumcenter, incenter, midpoint of the segment.	Olympic Didactic Situation	02/02/2021	3 hours/classes
Study on the subject addressed in the ODS.	Reading preparatory books	02/09/2021	3 hours/classes

The student's resolutions show us that a reasonable number were able to describe the solution to the problem, with the exchange of information between students through the Whatsapp application. Thus, there is a discourse among the participants, in the search for a solution to the problem proposed by the teacher in a remote classroom.

In view of this, the following question was asked to the group of students: what is the relevance of these constructions in the GeoGebra software for the math olympiads? With the purpose related to the first structuring of the figure of the geometric object (Figure 8 (a)), follows the description of the audio, made available by the students (Participant 8 and 3) in the WhatsApp message application, in an attempt to find new strategies to solve the problem (Figure 8). The GeoGebra software, made available by the teacher during his remote math classes (Figure 8 (b)), provided better learning in solving IMO problems, a construction that had the ability to visualize the geometric shapes in 2D and 3D and interact with the dynamics of the activity (Participant 8). At the moment, I started to place the points and segments without exact measurements, I found some difficulties at first, but then I managed to get as close to the structuring visualized by the teacher, so I tried to improve in the procedures until I reached the result of the task (Participant 3).



(a) (b) (c)

Figure 8. Formulation of the International Olympic Didactic Situation

Then, two students developed a mathematical model with pencil, pen and paper, performing a geometric record and formulated the OP answer (Figure 7 (c)). Therefore, it can be reported that the information, pragmatic and cognitive knowledge among the students were well explained in Figure 9.

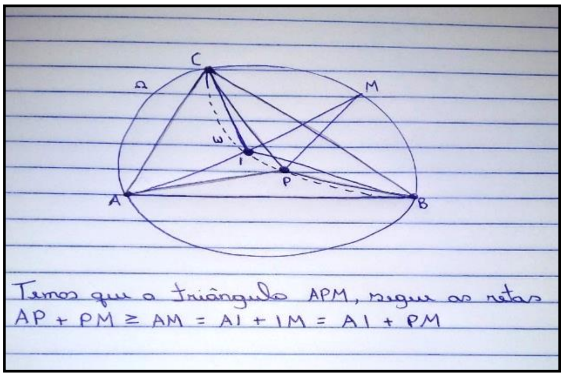


Figure 9. Formulation of the International Olympic Didactic Situation

Finally, in the next topic, the students answered a virtual questionnaire with questions directed about the international Olympic didactic situation and its main questions found during the virtuais meetings, as well as the improvement of the practice and in relation to the constructions of the incenter of a triangle in the visualizations. 2D and 3D dynamics.

In view of the data collection, it can be seen that GeoGebra enabled the cognitive development of the student, based on the information seen during the development of the Olympic problem, in which some important points are highlighted for ideas and conjectures of their own perceptions. First, students were asked to complete the virtual questionnaire available in the virtual messaging application, containing three questions (Table 2):

Table 2. Virtual Questionnaire Proposed to Olympic Students

Questions	Assertives
P1	Did you learn something that you feel is relevant in using GeoGebra software?
P2	Has your interest in the topic grown as a result of the course on solving Olympic Didactic Situations (ODS)?
P3	Were the course materials well-prepared and carefully conveyed to the understanding of GeoGebra software tools?

The answer referring to question 1: “the student was able to better understand the contents of plane geometry using the GeoGebra software” (Participant 1). “Because they have several commands, making the classes increasingly attractive and easy to understand” (Participant 2). In question 2: “students describe that GeoGebra greatly influenced their learning” (Participant 3). “Mainly to resolve dynamic issues” (Participant 4). “In addition to having several graphical views that improve the interpretation of the questions” (Participant 5). Finally, in the answer to question 3: “students reported on the material used in remote classes” (Participant 6). “Which was passed on to the virtual platform for download” (Participant 7). “Continuing with the reading and then with the construction code to visualize the commands and have a north in the modeling of the geometric object” (Participant 8).

Even in the face of the difficulties of accessing the internet encountered during the meetings, it is clear that the students were able to understand the didactic situation proposed by the teacher, solving the problem with the knowledge acquired from the concept of Incenter of any Triangle, as well as the use of some GeoGebra tools, being interactive in all remote classes and generating importance for the subject

4. CONCLUSION

This work aimed to structure and develop an international didactic situation presented with an approach in the Incenter of any Triangle in visual exploration in 2D and 3D format, using the GeoGebra software as a technological tool for modification and construction of the Olympic problem, as a resource that will provide a level of learning for students, through commands and visualization of the figure, providing a broad look at the international Olympic situation. The explored questions allowed students to evolve their geometric perception of the mathematical content in question through visualization, reaching objective research, as students are able to build new knowledge from the exploration and association between GeoGebra and the questions presented.

In addition, the student's speeches justify the contribution of the GeoGebra software, through the manipulation of geometric structures that allow experimentation and exploration in plane geometry and concepts of notable points, presenting itself as a dynamic and interactive tool. It is noteworthy that the dynamic records of semiotic representation are mobilized from a mediation in a directed way, where the support of the GeoGebra software allows the visualization and construction of operations for the students to build their conjectures and explore the knowledge that constitutes the recognition of insights in the development of the educational digital app.

That said, it is pointed out, in the Olympic didactic situation discussed above, the insertion of these technological tools together with the methodological change of the teacher. In fact, considering the lack of specific instruction for the development of tasks that aim to use questions from the Olympic mathematics tournaments in the classroom, the lack of computers in educational institutions (educational computer lab), the lack of time in teacher's school planning and the didactic transposition of these Olympic issues to the GeoGebra technological resource.

However, we emphasize the need for more possibilities of analysis and reflection, which can consider a more critical look at the potential and limitations of the GeoGebra software concomitant with the planning and direction at any level of teaching in the discipline of Mathematics.

Finally, the international didactic situation can be applied by mathematics teachers both in training for national and international Olympiads and for teaching in the classroom. Likewise, it is expected that this work will serve as a pedagogical and methodological tool and, consequently, lead more teachers to use the problem situations of IMO and other competitions.

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REFERENCES

[1] R. S. Silva Junior, "A proposal for teaching flat geometry in elementary school: mathematics present in student's daily lives," *Research, Society and Development*, vol. 9, no. 7, pp. 1-12, 2020.

[2] A. B. Powell, "Construção colaborativa do conhecimento tecnológico, pedagógico e do conteúdo de professores de Matemática," *Boletim Gepem*, vol. 64, pp. 1–19,

2014.
[3] M. A. Bairral and F. de J. R. Marques, "Onde se localizam os pontos notáveis de um triângulo? Futuros professores de matemática interagindo no ambiente vmt com GeoGebra," *Educação Matemática Pesquisa*, vol. 18, no.1, pp. 111–130, 2016.
[4] V. R. de. Jesus, "A utilização do software GeoGebra no estudo dos pontos notáveis do triângulo," Dissertation (Professional Master's in Mathematics in National Network - PROFMAT) – Institute of Mathematics and Statistics, Federal University of Bahia, Salvador, Brazil, 2018.
[5] L. F. Pereira, "O equilíbrio dos planos e os pontos notáveis do Triângulo: Arquimedes, Euclides, Huzita e Hatori trabalhando juntos," Dissertation (Professional Master's in Mathematics in National Network – PROFMAT) – Center for Natural and Exact Sciences, Federal University of Santa Maria, Santa Maria, Rio Grande do Sul, Brazil, 2021.
[6] T. L. Heath, *The Thirteen Books of Euclid's Elements*. 2 ed. 2. Cambridge: Dover Publications, 1908.
[7] F. Commandino, *Euclides – Elementos de Geometria*. Edição revisada e ampliada. São Paulo: Cultura, 1994.
[8] S. A. Lorenzato, "Por que não ensinar Geometria?," In: Educação Matemática em *Revista São Paulo-SP*, Blumenau: SBEM, vol. 4, pp. 3–13, 1995.
[9] E. K. Fainguelernt, "O Ensino de Geometria no 1º e 2º graus," *Revista da Sociedade Brasileira de Educação Matemática*, São Paulo, year III, no. 4, pp. 45–53, 1995.
[10] P. V. da S. Santiago and F. R. V. Alves, "Teoria das Situações Didáticas no Ensino de Geometria Plana: o caso da Olimpíada Internacional de Matemática e o auxílio do software GeoGebra," *UNION*, vol. 17, no 61. pp. 1-17, 2021.
[11] H. J. Bortolossi, "O uso do software gratuito GeoGebra no ensino e na aprendizagem de estatística e probabilidade," *VIDYA*, vol. 36, no. 2, pp. 429–440, 2016.
[12] R. Duval, "Understanding the Mathematical Way of Thinking – The Registers of Semiotic Representations," Preface of Bruno D'Amore. Cham, Switzerland: Springer, 2017.
[13] J. C. P. Leivas, A. D. H. Bettin and V. Pretto, "O GeoGebra 3d na construção da pirâmide a partir de seu tronco: registros de representação semiótica," *Didasc@lia:* Didáctica Y educación, vol. 9, no. 1, pp. 241–258, 2018.
[14] A.T.C. Santos ," O Ensino da Função Logarítmica por meio de uma sequencia didática ao explorar suas representações com o uso do software GeoGebra, Dissertation Programa de Estudos Pós-Graduados em Educação Matemática Salvador, Brazil, 2011.
[15] F. R. V. Alves, "Situação Didática Olímpica (SDO): aplicações da teoria das situações didáticas para o ensino de olimpíadas," *Revista Contexto e Educação*, vol. 36, no. 113, pp. 116–142, 2021.
[16] A. C. Gil, *Métodos e Técnicas de Pesquisa Social*. 5 ed. São Paulo: Atlas, 2008.
[17] M. C. Borba and M. G, Penteado. *Informática e Educação Matemática*. Coleção Tendências em Educação Matemática. 5ª ed. Belo Horizonte: Autêntica, 2012.
[18] P. Kenderov, A. Rejali and M. G. Bartolini, "Challenges Beyond the Classroom - Sources and Organizational Issues," In Barbeau, E. J; Taylor, P. J. *Challenging Mathematics in and Beyond the Classroom*. The 16th ICMI Study. New York: Springer, 2011.

[19] P. V. da S. Santiago, "Olimpíada Internacional de Matemática: Situações Didáticas Olímpicas no Ensino de Geometria Plana," Dissertation (Professional Master's in Science and Mathematics Teaching) - Dean of Research and Graduate Studies, Federal University of Ceará, Fortaleza, Brazil, 2021.
[20] D. Djukić, V. Jankovic, I. Matic and N. Petrovic, "The IMO Compendium: a collection of problems suggested for the International Mathematical Olympiads," ed. New York: Springer, 2011.
[21] F. R. V. Alves, "Visualizing the olympic didactic situation (ODS): teaching mathematics with support of the geogebra software," *Acta Didactica Napocensia*, România, vol. 12, no. 2, pp. 97–116, 2019.
[22] P. Kenderov, "World Federation of National Mathematics Competitions - A Short History," *Mathematics Competitions*, vol. 22, no. 2, pp. 14–31, 2009.
[23] J. López Linares, J. P. M. dos Santos and A. F. de Jesus, "Baricentro ou centroide: cinco problemas resolvidos das listas da Olimpíada Internacional de Matemática," *Revista de Matemática de Ouro Preto*, vol. 2, pp. 46–69, 2021.
[24] L. C. Fernandes and R. M. F. Mota, "O pensar matemático nas olimpíadas da matemática," *Rede Amazônica de Educação em Ciências e Matemática*, vol. 5, no. 1, pp. 115–130, 2017.
[25] F. R. V. Alves, "Situações Didáticas Olímpicas (SDOs): ensino de Olimpíadas de Matemática com arrimo no software Geogebra como recurso na visualização," *Alexandria:* Revista de Educação em Ciência e Tecnologia, vol.13, no.1, pp.1–30, 2020.
[26] G. Brousseau, "La théorie des situations didactiques Le cours de Montréal," 1997, [Online]. Available: http://guy-brousseau.com/1694/la-theorie-des-situations-didactiques-le-cours-de-montreal-1997/. [Accessed: 10-Jul-2021]
[27] A. P. R. A. Santos and F. R. V. Alves, "A teoria das Situações Didáticas no ensino das Olimpíadas de Matemática: Uma Aplicação do Teorema de Pitot," *Revista Indagatio Didactica*, vol. 9, no. 4, pp. 279–296, 2017.
[28] J. G. A. da Silva, F. R. V. Alves and D. B. Menezes (2021), "Situação Didática Olímpica sob a perspectiva da Teoria das Situações Didáticas: uma aplicação com professores em formação inicial no Brasil," *Revista Binacional Brasil-Argentina:* Diálogo entre as ciências, vol. 10, no. 1, pp. 380–411, 2021.
[29] E. Fischbein, "The Theory of Figural Concepts. Educational Studies in Mathematics," *Educational Studies in Mathematics*, vol. 24, no. 2, pp. 139–162, 1993.
[30] R. Duval, "A cognitive analysis of problemas of comprehension in a learning of mathematics," *Educational Studies in Mathematics*, vol. 61, pp. 103–131, 2006.
[31] C. Laborde and B. Capponi, "Cabri-géomètre constituant d'un Milieu pour l'Apprentissage de la notion de figure, em Balacheff, N. e Vivet," M. *Didactique et Intelligence Artificielle*, Grenoble, France: La pensée Sauvage, 1994.