



DIDACTIC ENGINEERING AND LEARNING OBJECTS: A PROPOSAL FOR TEACHING PARABOLAS IN ANALYTICAL GEOMETRY

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Article Info

Article history:

Received: January 20, 2022

Accepted: March 20, 2022

Published: March 31, 2022

Keywords:

Analytical geometry
Didactic engineering
Educational technologies
GeoGebra
Mathematics teaching

ABSTRACT

This work aims to investigate the feasibility of using a Learning Object built in GeoGebra software and its potential for teaching parabolas in Analytical Geometry, having as support for its replication in a teaching session the Theory of Didactic Situations. The methodology adopted was Didactic Engineering, in its first two phases – preliminary analysis and a priori analysis. In the preliminary analysis, some epistemological and didactic aspects that permeate the teaching of parabolas, the concept of Learning Objects and the Theory of Didactic Situations were raised. In the a priori analysis, we present the Learning Object called Suspension Bridge and its manipulation in GeoGebra for the exploration of the parabola, as well as a student's attitudinal prediction. Thus, we seek to collaborate with the development of new approaches to teaching this topic, contributing to the advancement of the use of educational technologies integrated into the teaching of mathematics.

REKAYASA DIDAKTIK DAN OBJEK BELAJAR: PROPOSAL PENGAJARAN PARABOLA DALAM GEOMETRI ANALITIS

Kata Kunci:

Geometri analitis
Rekayasa didaktik
Teknologi pendidikan
GeoGebra
Pembelajaran matematika

ABSTRAK

Penelitian ini bertujuan untuk menyelidiki kelayakan penggunaan objek pembelajaran dengan perangkat lunak GeoGebra untuk pembelajaran parabola dalam Geometri Analitik, yang mendukung replikasinya dalam sesi pengajaran Teori Situasi Didaktik. Metodologi yang diadopsi adalah Rekayasa Didaktik, dalam dua fase pertama—analisis pendahuluan dan analisis apriori. Dalam analisis pendahuluan, diangkat beberapa aspek epistemologis dan didaktik yang meresapi pengajaran parabola, konsep Objek Pembelajaran dan Teori Situasi Didaktik. Dalam analisis apriori, kami menyajikan Objek Pembelajaran yang disebut Jembatan Gantung dan manipulasinya di GeoGebra untuk eksplorasi parabola, serta prediksi sikap siswa. Jadi, kami berusaha untuk berkolaborasi dengan pengembangan pendekatan baru untuk mengajar topik ini, berkontribusi pada kemajuan penggunaan teknologi pendidikan yang terintegrasi ke dalam pengajaran matematika.

1. INTRODUCTION

Analytical Geometry is a branch of Mathematics that makes it possible to solve Geometry problems, however through the use of algebraic methods. However, the student is commonly required to have a level of mathematical abstraction for their understanding, which sometimes causes difficulties in their understanding. Nasser [1] points out that abstraction is an essential part of the evolutionary process of mathematical thinking, however, for this evolution to occur, it is necessary to reconstruct cognitive structures that allow for a favorable understanding.

Regarding the subject of parabola within Analytical Geometry, its teaching is still treated in a theoretical way and developed only through formulas in the classroom, being an approach impregnated with formalism and with loss of meaning for the student shown in surveys such as Cerqueira [2], Macedo [3] and Otero e Llanos [4]. However, there are several applications of the subject that would bring meaning to the student, from the understanding of the use of the parabola, which motivates us to proceed with this study. This work comes from research developed in the master's course in Science and Mathematics Teaching, at the Federal Institute of Education, Science and Technology of Ceara, in Brazil. According to Dantas [5], it is important for students to develop a repertoire of experiences regarding the treatment of mathematical problems, building meanings for them. In this way, students can exercise their skills in building mathematical models and designing processes to solve such problems.

In the case of the parabola in Analytical Geometry, it is suggested that the teacher incorporate technology in their classes, as "they are alternatives of varied experiences and learning facilitators that reinforce the ability to reason logically, formulate and test conjectures, assess the validity of reasoning and build arguments", according to Brazil [6]. In other words, in addition to technology having the potential to make classes more dynamic and attractive to students, it can also provide a more concrete visualization of what is abstract, making their learning meaningful. Thus, for this work we bring the GeoGebra software as technological support.

GeoGebra is a free access software, which allows manipulations in a Dynamic Geometry computational environment, being used in several other areas of knowledge. According to Alves and Borges Neto [7], the exploration of GeoGebra as a technological instrument enables the visualization of unimaginable situations, when restricted to pencil and paper. This software integrates Algebra and Geometry in a dynamic environment, capable of creating favorable conditions for the student to develop when handling the tool, facilitating their understanding of the subject.

Based on the above, we draw as an objective for this work to investigate the viability of using a Learning Object built in GeoGebra software and its potential for teaching parabolas in Analytical Geometry, having as support for the teaching session the Theory of Didactic Situations. To structure this article, we adopted Didactic Engineering (DE) as a methodology in its first two phases – preliminary analysis and a priori analysis – given the character of theoretical research in the format of the didactic proposal of this work. According to Alves and Dias [8], DE is a method that brings alternatives from a systematic perspective of preparation, conception, planning, modelling, in addition to the possibility of executing and/or replicating structured teaching sequences.

In Analytical Geometry there is a relationship between algebra and geometry, in which we can analyze algebraic questions geometrically and vice versa. In the case of parabolas, these are commonly studied in basic education in a way related to the graph of a quadratic function, however their study within the field of analytical geometry usually occurs in a way that is dissociated from the study of functions, as the authors Cerqueira

[2], and Macedo [3] say. Many methods for constructing the graph of a quadratic function, for example, derive from algebraic procedures, being sometimes insufficient for the geometric interpretation of the parabola, making it difficult to understand through the prism of Analytical Geometry.

As a result of this fact, there is great difficulty for the student to understand graphically what the parabola equation represents and what is the relationship between its variables to solve problems, according to Halberstadt [12]. Understanding the 2nd degree polynomial function in canonical form, where the equation can be written from its vertices, is a way to understand the parabola equation within Analytical Geometry, as well as its focus, vertex and directrix elements. However, according to the author, this approach is little explored in the classroom, causing difficulties for the student to enter higher education courses in the area of exact sciences and to come across disciplines that demand this knowledge.

Therefore, our study seeks, through this Didactic Engineering, a proposal to minimize the difficulties of learning the parabola in the context of Analytical Geometry, with the contribution of GeoGebra dynamic geometry software. We hope that by replicating this study for a posteriori analysis and validation, from the manipulation of the parameters established in this learning object, students will visualize the movement of the parabola and understand how it works and how it can be related to Analytical Geometry.

2. METHOD

According to Artigue [9], the Didactic Engineering nomenclature was idealized for didactic work compared to the work of an engineer who, when carrying out a project, is supported by scientific knowledge in his area, accepts to submit to a scientific control, but, at the same time, he feels impelled to develop objects that are much more complex than the objects refined by science and, consequently, to face up to all the means available to situations that science does not aspire to or cannot take into account.

According to Artigue [9],[10], DE is characterized as an experimental scheme based on didactic achievements in the classroom, that is, on the conception, realization, observation and analysis of teaching sequences. The author also reiterates that "The methodology of Didactic Engineering is also characterized, in comparison with other types of research based on experimentation in the classroom, by the register in which it is located and by the forms of validation to which it is associated".

Leivas e Gobbi [11] explains that "this methodology can be understood both as a specific research methodology and as a sequence of classes or activities designed and organized in a coherent way". Systematically following this methodology in its planning and execution, the following phases should be guided: i) Preliminary analyses, ii) Conception and a priori analysis of teaching situations, iii) Experimentation and iv) A posteriori analysis and validation. In summary, each of these phases can be described as: (1) Preliminary analysis: It consists of the epistemological study of the research object, through a bibliographic research regarding the theoretical framework of this content, as well as the way in which it is approached in the classroom. To this end, a study is carried out on the students' prior knowledge, their barriers to learning and an analysis of the research locus; (2) Conception and a priori analysis: Stage in which the variables (global and local) on which teaching can act will be delimited, as a way of guiding the research and drawing up an action plan. Global variables are intended to guide research choices, while local variables are aimed at predicting possible student behaviors and obstacles, through didactic situations; (3) Experimentation: Phase in which the didactic situations or didactic sequences constructed in the previous phase are applied, where the didactic

contract is signed and data related to the research are collected; and (4) A posteriori analysis and validation: This phase is based on all the data collected during the experiment. After analyzing the collected data, it is necessary to compare them with the a priori analysis initially carried out in order to validate (or not) the hypotheses formulated in the investigation.

In this work, we will present the development of the first two phases of DE- preliminary analysis and a priori analysis - as a theoretical approach to the development of the aforementioned Master's research. The steps to carry out this research can be illustrated in the following flowchart:

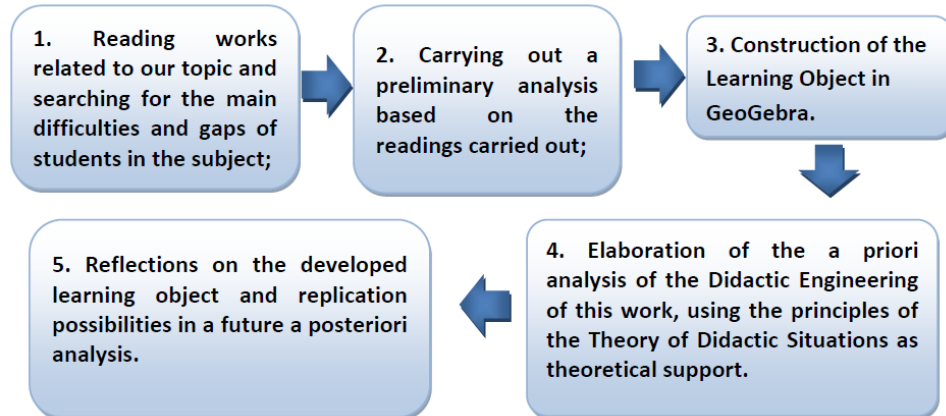


Figure 1. Flowchart with The Research Steps

At first, this work is configured in a didactic proposal to the mathematics teacher, being a possibility of work with this theme. Our data collection should be carried out for a posteriori analysis and validation of this learning object, in a later study, in which we will replicate this learning object in the classroom and collect empirical information for analysis. We leave this preliminary study as a support for the mathematics teacher to enrich and reproduce it.

3. RESULTS AND DISCUSSION

The results and discussions of this research revolve around the preliminary analysis of the topic of parables and the a priori analysis, with the elaboration of the learning object presented in the subsequent parts. For the preliminary analysis, we seek to present the epistemological and didactic aspects that permeate the teaching of parabola in basic education, relating them to the importance of using Learning Objects to enable the teaching of this theme and the Theory of Didactic Situations, as a guide for teaching practice.

3.1 Preliminary Analysis

3.1.1 Epistemological and Didactic Aspects about Teaching of Parabolas

Analytical Geometry is a theme present in the last year of basic education in Brazilian schools, in which students are faced with the relationship between Geometry and Algebra, in search of geometric meaning for algebraic situations, represented by equations, learning these two mathematical aspects in an articulate and harmonious way. Thus, the student needs to graphically understand what the equations represent and the relationship between their variables to solve problems, and for that, it is necessary to recall topics previously studied, which serve as a basis for this understanding. However, according to Halberstadt [12], possibly the association and articulation of previous concepts between Algebra and Geometry can be considered an epistemological difficulty in the construction of Analytical Geometry concepts.

The parabola permeates the daily lives of students, for example, in the trajectory of a ball going to the goal, in satellite dishes, car headlights, parabolic lanterns and mirrors, in addition to constructions in the field of Architecture and Engineering. A concept from an analytical view of the parabola, according to Venturi [13] is: “Consider in a plane, a point F and a line d that does not contain F . We call the focus parabola F and directrix d the locus of the points on the plane that are equidistant from F ”, as shown in Figure 2:

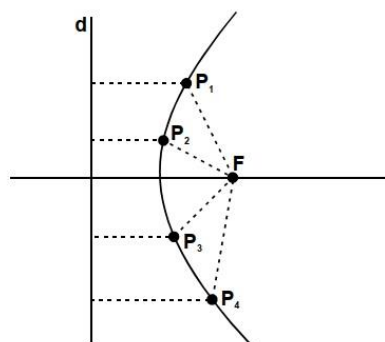


Figure 2. The Concept of a Parabola in a Geometric View

In Figure 2 we have the representation of some points that belong to the parabola with focus F and directrix d , equidistant from the point F and the line d . This definition pointed out by Venturi [13] is commonly presented in textbooks within the study of Analytical Geometry, but without geometrically exploring the meaning of each element. Cerqueira [2], and Macedo [3] point out in their research that the parabola is approached in a simplified way, in an analytical/algebraic view, with the use of determined equations, in which this topic is not explored from a geometric perspective, nor with use of technology for the understanding of its elements.

Another way to present the 2nd degree polynomial function and even extend it to the parabola equation is its canonical form, in which the equation can be written from its vertices. This format is little discussed in Mathematics classes in basic education in Brazilian schools, as well as some textbooks do not cover this topic. According to Lima et al. [14], we have that the coordinates of the vertex (x_v, y_v) of a parabola that is associated with a quadratic function can be obtained in different ways. The function is in canonical form (1):

$$f(x) = a \cdot (x - h)^2 + k, \text{ with } a \neq 0 \tag{1}$$

And in this way, the vertex can be determined in a simple way, as it only corresponds to the coordinate point (h, k) . In this way, using the method of completing squares you can transform the general shape (2):

$$f(x) = ax^2 + bx + c, \text{ with } a \neq 0 \tag{2}$$

for canonical form (3):

$$f(x) = a \cdot (x - m)^2 + k \tag{3}$$

Knowing that the coordinates of the vertex of the parabola are $V(x_v, y_v) = (-\frac{b}{2a}, -\frac{\Delta}{4a})$, with $m = -\frac{b}{2a}$ and $k = -\frac{\Delta}{4a}$, which $\Delta = b^2 - 4ac$, we have that the canonical form is related to the equation of the parabola, because “the graph of the quadratic function $f(x) = a \cdot (x - m)^2 + k$ is obtained from the graph of $g(x) = a \cdot (x - m)^2$ through the vertical translation $(x, y) \mapsto (x, y + k)$, which takes the OX on the $y = k$ and the line $y = -1/4a$ on the line $y = k - 1/4a$ ” by Lima et al. [14]. Thus, we

can infer that the graph of any quadratic function $f(x) = a \cdot x^2 + bx + c$ is a parabola, whose directrix is the horizontal line (4):

$$y = \frac{4ac - b^2 - 1}{4a} \quad (4)$$

and whose focus is the point (5):

$$F = \left(-\frac{b}{2a}, \frac{4ac - b^2 - 1}{4a} \right) \quad (5)$$

Where the parameter a determines whether the concavity of the parabola faces up ($a > 0$) or down ($a < 0$). This brief demonstration shows us the relationship between these two subjects (quadratic function and equation of the parabola), which makes the articulation between them robust.

For the learning of this subject to occur properly, it is necessary for the student to have the ability to make deductions and build essentially abstract elements, using their previous knowledge in a combined way. According to Silva [15], to explore geometric ideas using manipulable didactic resources, the student needs to be stimulated to discover and build new perceptions, where autonomous strategies for solving problem situations are provided, using these perceptions whenever possible to the establishment of existing geometric relationships.

Alves and Pereira [16] reinforce that the language presented in textbooks and used in the transmission of concepts in Analytical Geometry causes difficulties in understanding the subject, as the treatment given to the topic brings a fragmented approach, without relating Analytical Geometry to other topics in the field. Mathematics, with a loss of meaning for the student. However, the efficient understanding of these concepts is necessary for the cognitive development of the student in the field of Geometry, not only during the school stage, but also for higher education. This fact substantiates the relevance of the teacher's professional skill in the epistemic sphere, in which he/she must seek means for a clear presentation of the content, with real mathematical meaning, application possibilities, as well as a better resourcefulness of their work, reflecting on the learning of their students.

The authors Paula and Bittar [17] emphasize the articulation between Geometry and Algebra, with the use of technology in the construction of mathematical meanings for the student, reinforcing those equations should not be treated in a way that is disconnected from their geometric interpretation. Borba and Penteadó [18] reiterate that working with technology favors the construction of knowledge and student development, emphasizing their learning. In the document of the Common National Curriculum Base, which guides the curriculum of Basic Education in Brazil, we can find one of the skills that reinforce the importance of knowledge about polynomial functions of the 2nd degree using technologies and that can be extended to the study of parabola. "Convert algebraic representations of 2nd degree functions to geometric representations in the Cartesian plane, distinguishing the cases in which one variable is directly proportional to the square of the other, using or not algebra and dynamic geometry software or applications" according to Brazil [6].

Thus, we understand the importance of relating Algebra and Geometry within the subject of parabola in the context of Analytical Geometry, in search of a comprehensible articulation for the student. From this perspective, we emphasize the relevance of developing different didactic proposals to work on this theme, in support of teachers working in Basic Education, given the fact that this theme is little explored in their initial training, according to Siqueira [19]. Brazil [6] suggests the use of technology through software or applications as a way to make the methodology used by the teacher viable.

Thus, this work seeks to link the use of technology through Learning Objects for the teaching of parabolas.

3.1.2 Learning Objects

Learning Objects (LOs) are digital resources whose purpose is to support the teaching of a wide range of subjects. LOs are didactic resources created due to the need for a new teaching and learning strategy for a specific content, supported by digital technology to facilitate understanding of the subject studied, as say Azevedo et al. [20]. Tarus et al. [21] points out a definition for LO as being a term that emerged at the beginning of the 21st century to indicate digital resources (video, animation, simulation, etc.) which allow teachers and students to explore specific concepts in mathematics, science, language, etc.

Thus, LOs can be created in different media and formats, as long as they are methodologically designed to achieve an objective within the learning proposal of the selected content. For Ramalho [22], when manipulated in an educational context with knowledge at its core, LO serves to mediate and facilitate the formation and consolidation of new knowledge. Also, according to Azevedo et al. [20], and Tarus et al. [22], the use of LOs to teach Mathematics helps students in their learning process in stages such as the relationship between previous and new concepts, hypothesis analysis, relationships between content and their respective applications, among other benefits. Aguiar and Flôres [23] state that LOs can help students to understand more elaborate concepts, with the possibility of handling and manipulating parameters allowing the analysis of cause-and-effect relationships of phenomena.

Thus, the idea of using GeoGebra software as a resource for the elaboration of a Learning Object for this work arises, considering the possibility of a dynamic experience in the construction of knowledge about parabolas in a visual way, going beyond abstraction and mathematical formulas. GeoGebra was chosen because it is free software and easy to manipulate, having features that explore the student's cognition. Abar [24] states that the use of digital technologies through GeoGebra can support the teacher's methodological strategies, modernizing school knowledge and transforming this knowledge into exercises and problems capable of making concepts more understandable, as the software offers resources visual and manipulable. Thus, the methodologies made possible by GeoGebra, in addition to using technology as an ally in the classroom, contribute to making the class more attractive for the student, instigating curiosity and openness to the new. Based on this, it is intended to work on this theme guided by the dialectics of the Theory of Didactic Situations.

3.1.3 Theory of Didactic Situations

The Theory of Didactic Situations (TDS) was conceived in France in the 1980s by researcher Guy Brousseau, one of the pioneers of Didactics of French Mathematics. TDS seeks to discuss the different ways to present a certain mathematical content to students, whenever there is a clear intention of the teacher to enable the student to learn (acquisition of knowledge) through a planned didactic sequence, as say Brousseau and Warfield [25]. In this way, TSD offers a possibility to understand the existing relationship between student, teacher, and knowledge, as well as the environment (*milieu*) in which the conjuncture of a specific didactic situation occurs.

Brousseau [26] argues in his theory that each knowledge is linked, through the interaction between two or more people, to a type of situation and these interactions occur through a game, problem/challenge, or device with its own interaction rules, where the

development of the situation causes the apprehension of knowledge by the student. Brousseau [26] defines that “a 'situation' is a model of interaction between a subject and a given environment”. The creation of a didactic situation involves the elaboration of circumstances in which the student really develops his/her skills and builds knowledge. In order for him to reach this construction, there is an articulation with a final purpose, giving him a notion of the activity's objectives and possible obstacles. Encouraging cooperation between the group – students and teacher – is a relationship that promotes integration and develops important skills for the apprehension of knowledge.

TDS is organized into four dialectics or phases, which are action, formulation, validation, and institutionalization, the first three being considered adidactic situations (without the direct intervention of the teacher) and can be described, according to Brousseau [26], summarily, by: (1) Action: the student is faced with the problem and seeks, in their previous knowledge and in their interaction with the environment, elements that help them in the search for the correct solution. A position is taken on the proposed problem; (2) Formulation: there is an exchange of information between the student and the milieu. It is time to expose ideas in a clear and verbalized way, however without the obligation to use a formalized mathematical language. Thus, the student draws strategies and begins to appropriate knowledge; (3) Validation: the student presents his solution strategy to the class and seeks to argue based on his reasoning, verifying whether what he conjectured is, in fact, valid; and (4) Institutionalization: the figure of the teacher enters the scene to synthesize everything that was exposed and discussed by the students in the previous stages, in a formal way and with appropriate mathematical language.

When analyzing the dialectics/didactic situations from the perspective of Brousseau and Warfield [25], and Brousseau [26], we have that the moment when the student builds knowledge occurs in the adidactic situation, which is constructed for the student to interact with an environment without the interference of the teacher, such as mentioned before. Alves [27] states that the dialectics pointed out demand a detailed and careful expedient of attention and analysis, as we are interested in the perspective and understanding of the teacher's action, mediated, and supported by fundamentals that subsidize and indicate the adoption of a methodology of teaching for Mathematics. Based on what was exposed, in the next sub-section we deal with the a priori analysis based on the Learning Object to be presented as a didactic proposal.

3.2 *A Priori Analysis*

In this a priori analysis, we seek to describe the didactic variables involved in the process, analyze the importance of this didactic situation for the student and predict possible behaviors, showing how this analysis allows for a control of meaning, ensuring that the expected behaviors come from the learning of the subject. worked with the LO in question.

3.2.1 *Learning Object Conception*

This LO, called “Suspension Bridge” built in GeoGebra software to work with parabolas can be found at the following website: <https://www.geogebra.org/m/yvgywvnn>. A proposed statement to explore this LO as a didactic situation would be: “Observe the format adopted by this rope suspended at two points, when supporting a uniformly distributed load. The shape of the cables of this suspension bridge is identified with which mathematical figure? What mathematical elements made you reach this conclusion? Explore the construction from the manipulation of parameters a , x_v and y_v ”. Your initial GeoGebra interface is shown in Figure 3.

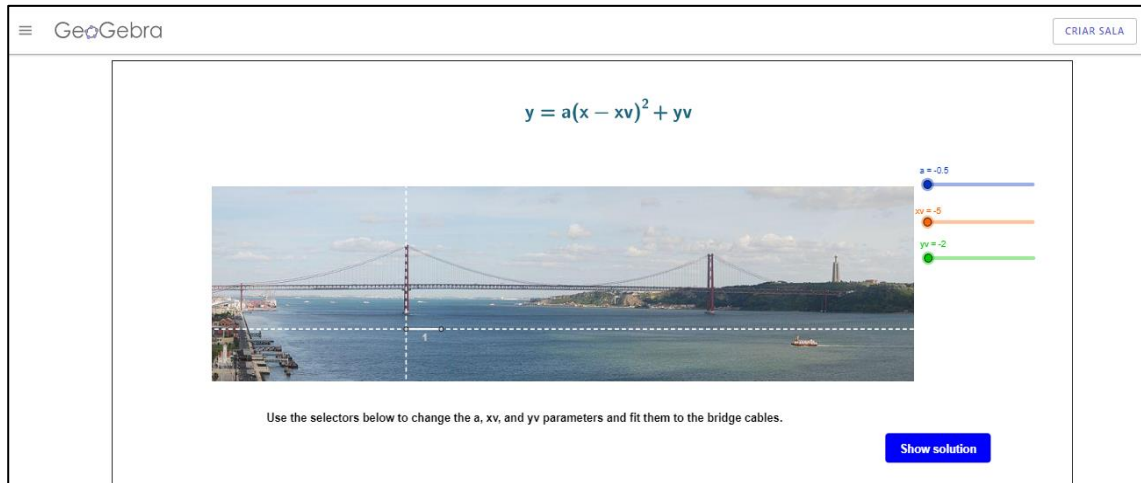


Figure 3. Initial LO Interface Built in GeoGebra

This construction can be considered an LO, as according to the studies by Diaz-Urdanetta et al. [28] this is justified by the following characteristics: (1) It is a virtual resource, built in GeoGebra software; (2) Its conception was based on the need to help with a specific content of Mathematics, in this case, the parabola; (3) It is configured in an interactive activity, as the student can explore the construction by manipulating its values in the created sliders; and (4) It presents a simulation of a bridge cable, which is a real situation, promoting the visualization and geometric perception of the parabola.

In this case, the didactic variables to be addressed are micro didactic, or local, that is, related to the local organization of Engineering, that is, to the organization of a sequence or a phase, according to Artigue [9]. This choice aims to use the change of the parabola by an algebraic approach to its geometric representation, from the visualization and manipulation of its elements, seeking to raise the student's level of understanding based on an interaction carried out in a teaching session.

3.2.2 Possibilities of using the LO "Suspension Bridge" in the Classroom

At this stage of the a priori analysis, an attitudinal prediction of possible behaviors/reactions from the student was carried out from the manipulation of the LO "Suspension bridge" in the classroom, guided by the dialectics of the Theory of Didactic Situations (TDS) - action, formulation, validation, and institutionalization.

Action dialectic: In this step, it is expected that the student, in possession of the construction previously made available by the teacher, pays attention to the statement suggested in the previous section and carefully observes all the elements available in the GeoGebra visualization window. The aim is for the student to seek in their previous knowledge the mathematical concepts necessary for the proper analysis and solution of the proposed didactic situation.

Formulation dialectic: For this step, it is expected that the student, from his initial position, explores the construction from the available elements, that is, that he manipulates the sliders a , x_v and y_v , and observing the changes that occur in the geometric shape of the graph of the parabola in question. This step can be simulated as illustrated in the manipulations described in Figure 4, Figure 5, and Figure 6:

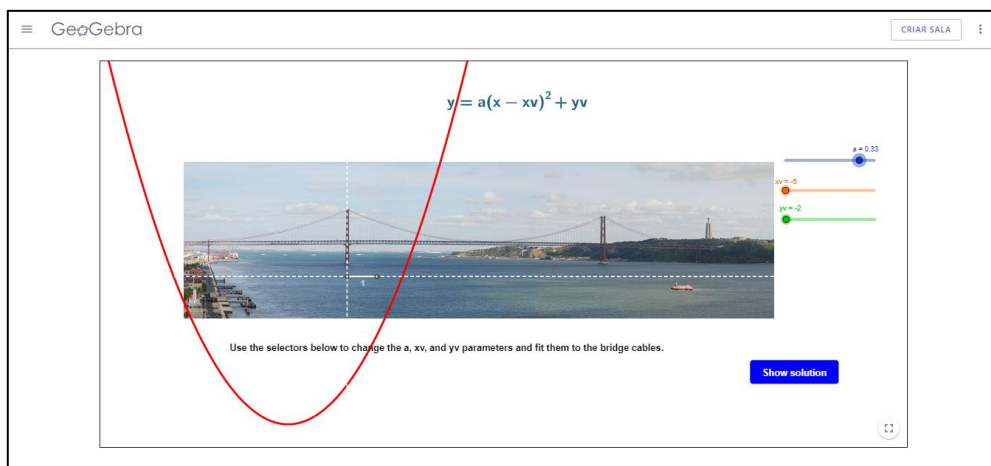


Figure 4. Slider Movement “a”

In Figure 4, the student can see that when moving the slider a , referring to the parameter a of the parabola equation, the concavity has an opening facing upwards, if the sign of a is positive ($a > 0$), or facing downwards, if the sign of a to be negative ($a < 0$). Following the construction exploration, in Figure 5, we have the x_v slider movement:

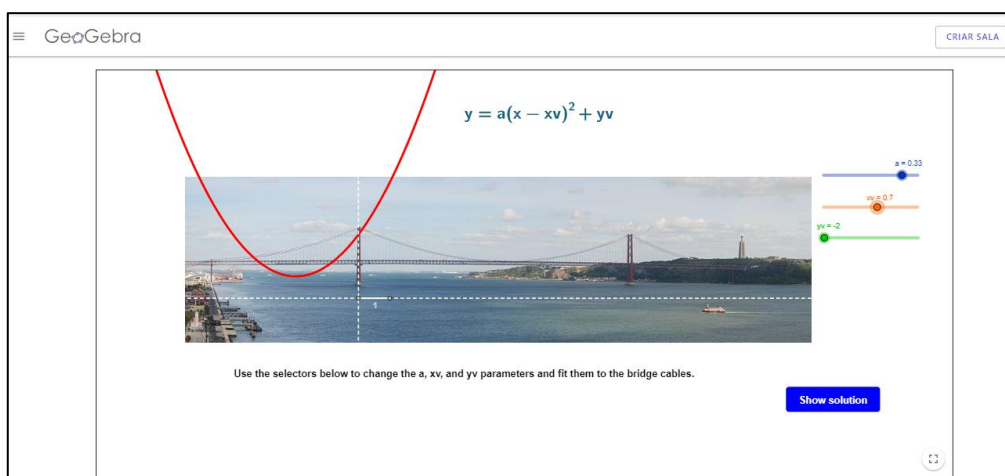


Figure 5. Slider Movement “x_v”

In Figure 5, the student can notice that by manipulating the x_v slider, referring to the x_v parameter of the equation, the parabola is translated vertically, up, or down, in the y -axis. In Figure 6, we have the movement of the y_v slider:

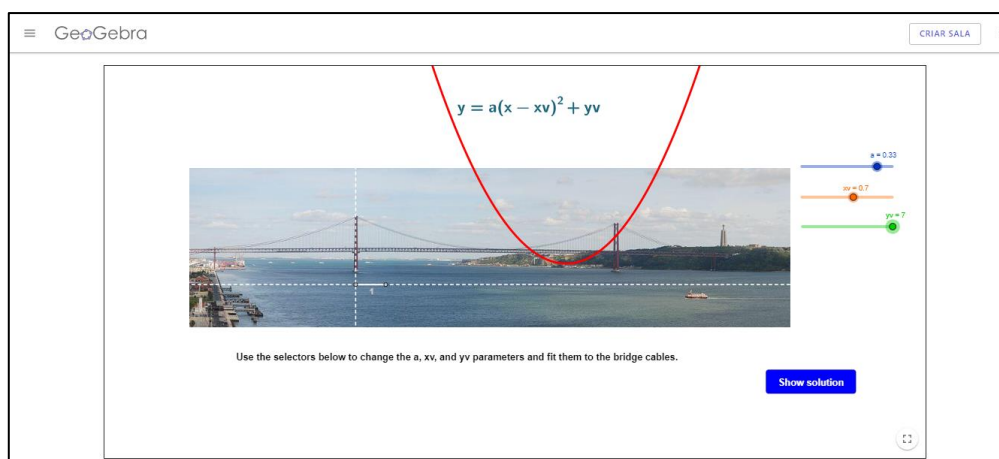


Figure 6. Slider Movement “y_v”

In Figure 6, we expect the student to notice that by moving the y_v slider, the parabola is translated horizontally to the right or to the left, along the x -axis. Based on these movements and observations, the student must elaborate his conjectures, proceeding to the validation dialectic.

Validation dialectic: At this stage, it is important for the student to verbalize everything he could observe in the construction, from the movement of the a , x_v and y_v sliders, relating the changes in position, opening of the concavity and translation of the parabola, as it increased or decreasing the values of parameters a , x_v and y_v . Thus, the student is expected to present arguments about the parabola from the visualization, such as "the parabola has the concavity facing upwards when the slider/parameter a has a positive sign" or "the parabola is translated upwards to the as the value of x_v increases and down as the value of x_v decreases", for example.

It is also important that the student discovers the equation of the parabola in question by manipulating the sliders, and that he uses the "show solution" button as a last resort, just to check the legitimacy of the values pointed out by him as a solution. In Figure 7 we have an illustration of the solution to be found by the student in GeoGebra:

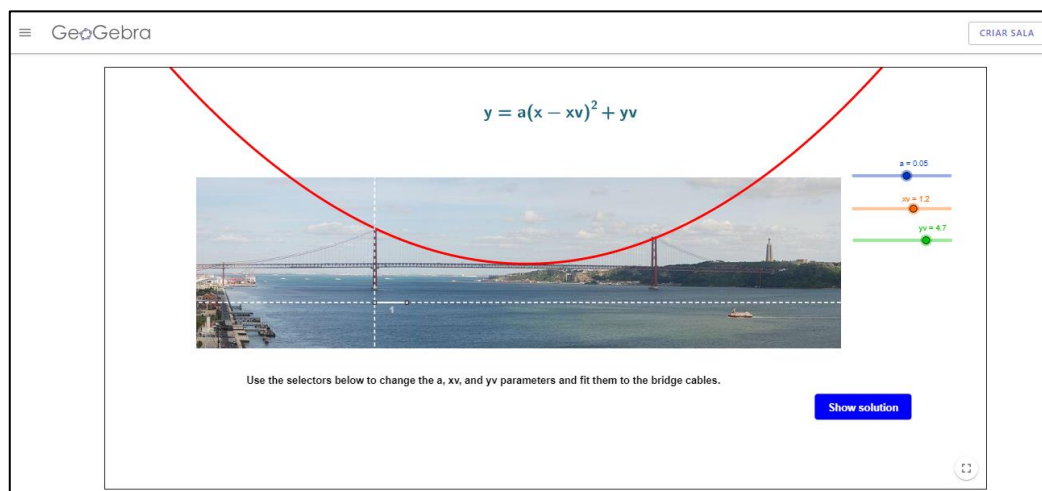


Figure 7. Solution to be Found by the Student

In this way, the aim is for the student to reach the solution found in Figure 7, with the parameter values being, respectively, $a = 0,05$, $x_v = 1,2$ and $y_v = 4,7$.

Institutionalization dialectic: At this stage the teacher must intervene, making a parallel with everything that was proposed as a solution by the students, discarding the incorrect premises with a formal justification and making a synthesis of the correct assertions to, from there, present them with due rigor mathematical the theoretical concept of parabola. An explanation that can be added by the professor is that in some suspension bridges, the chain connected to the vertical rods that support the platform is shaped like a parabola.

In this way, the teacher can seek an institutionalization that relates the subject of quadratic functions in its canonical form with the equation of the parabola within the topic of Analytical Geometry. A suggested model of institutionalization for a teacher can be described in subsequent passages. The equation presented in this example describes the parabola from the coordinates of its vertex $V(x_v, y_v)$ and the parameter a is the concavity of the parabola. The canonical form is viable in solving situations where the vertex coordinates are known. Parabolas, in addition to geometrically representing quadratic (or 2nd degree) functions, are also the result of studies of conics in Analytical Geometry.

According to Lima [29], parabolas appear as graphs of quadratic functions, with the quadratic function of a variable represented by $f(x) = y = ax^2 + bx + c$, with a, b and c constants, $a \neq 0$. The graph f of the function is the set of points $P(x, y) \in \mathbb{R}^2$ where $y = ax^2 + bx + c$. In this case, to show that the curve described by this function is, in fact, a parabola, we can enter new coordinates (s, t) and translate the axes so that $x = s + h, y = t + k$, where h and k have been selected conveniently. From the new coordinates, the point $P = (x, y) = (s + h, t + k)$ belongs to the parabola if, and only if (6) and (7):

$$t + k = a(s + h)^2 + b(s + h) + c = as^2 + (2ah + b)s + (ah^2 + bh + c) \quad (6)$$

Thus, when taking $h = -b/2a$ and $k = ah^2 + bh + c$, the equality above reduces to $t = as^2$, that is, regarding the new coordinates, the point (s, t) belongs to the graph of the parabola G if and only if $t = as^2$. Therefore, Lima [29] points out, in terms of the original x, y coordinates, the focus of the parabola $y = ax^2 + bx + c$ is the point (7):

$$\left(-\frac{b}{2a}, \frac{4ac - b^2 + 1}{4a}\right) \quad (7)$$

and the directrix (8) is the horizontal line:

$$y = \frac{4ac - b^2 - 1}{4a} \quad (8)$$

That said, the teacher can make an analogy between the parameters of the quadratic function and the equation of the parabola, showing its focus and directrix as a function of the elements given in the construction, exploring the algebraic/analytical and geometric view, seeking to clarify this relationship for the student. Another possibility for the teacher to work with GeoGebra within the theme addressed in this work would be to reproduce this construction with the class using 3D tools, as illustrated in Figures 8, 9 and 10:

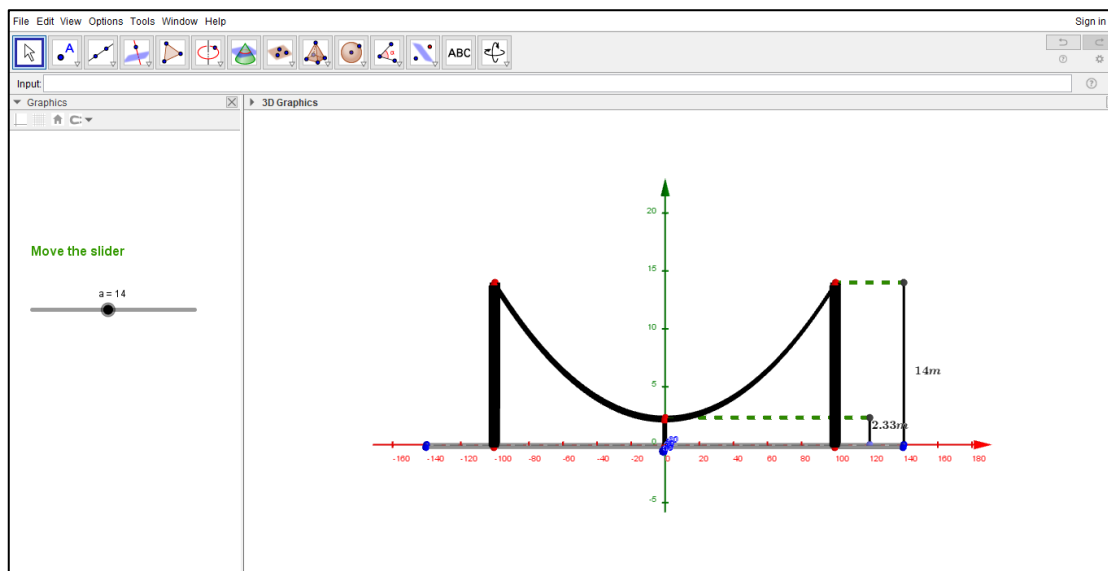


Figure 8. Front View of a Suspension Bridge in GeoGebra 3D

It is also interesting to use the graphic resources and the different colors, as a way to highlight the different geometric elements that can be explored and apprehended. The use of colors in representations of the most diverse types, in addition to the aesthetic character, can have a pedagogical value as it allows a clear distinction to be made in the identification of the different elements involved in a geometric construction. According to Lieban et al. [30] in dynamic geometry, especially with GeoGebra software, it is common for

developers of files aimed at teaching mathematics to explore the resource of coloring existing objects, both for its visual appeal and its didactic potential.

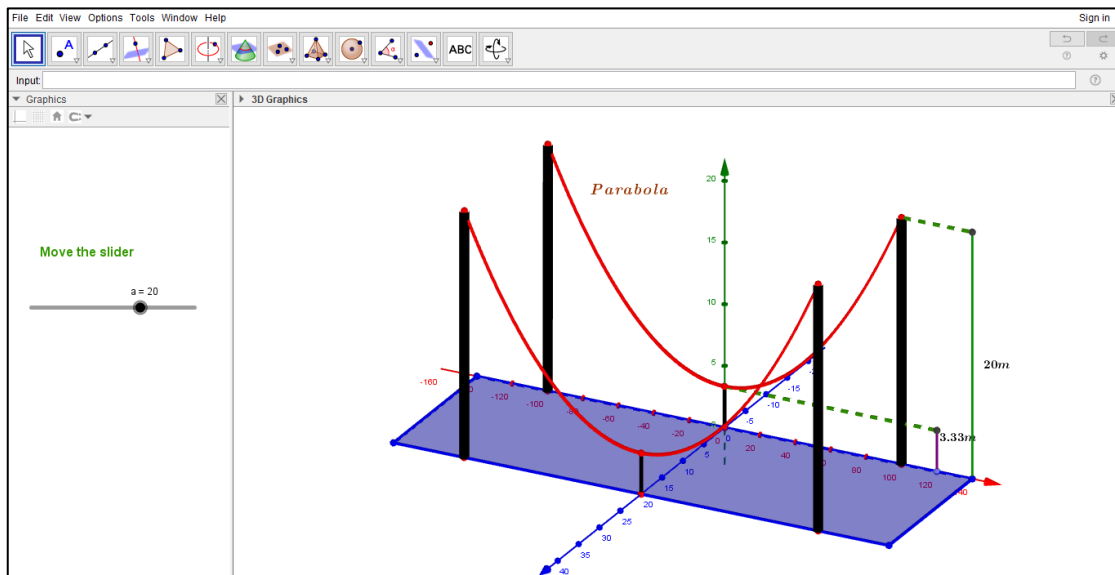


Figure 9. Left Side View of a Suspension Bridge in GeoGebra 3D

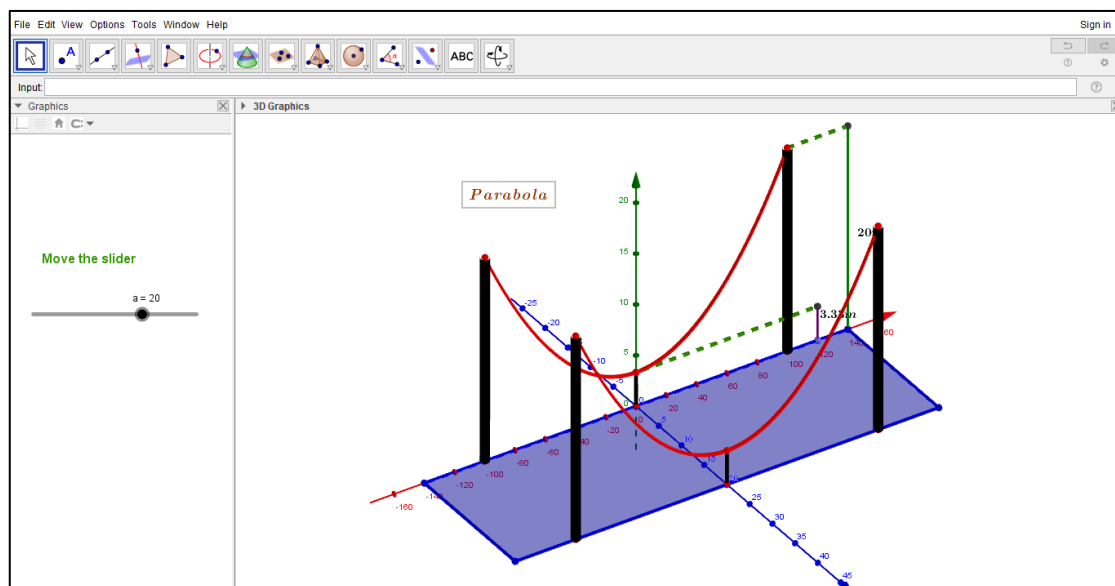


Figure 10. Right Side View of a Suspension Bridge in GeoGebra 3D

In this case, the teacher has the possibility to explore the elements of the parabola, both in the context of functions and in Analytical Geometry, using 3D visualization, which can expand the student's ability to build geometric concepts from a three-dimensional visualization, according to Sousa et al. [31].

As the main difficulties of this study, we have the lack of a more in-depth discussion about the concept of quadratic function and the existing relationship with the parabola in Analytical Geometry within the literature and textbooks used in Brazilian schools in the classroom and the attempt to explore this learning object made in GeoGebra using the cell phone, because we are afraid that the development of this LO in the classroom using the cell phone will not have a satisfactory effect. For a possible a posteriori analysis and validation when implementing this didactic situation, it is important that the teacher has the mastery of observing/analyzing whether the students, in fact, are achieving an

evolution of algebraic, analytical, and geometric reasoning, from the demonstration or manifesting conjectures that are understandable and relevant to their learning; otherwise, measures must be taken in view of the difficulties presented. In solving the proposed activity, students can analyze the construction in pairs or small groups, in order to facilitate the discussion of observations, facilitating the teaching and learning process.

Parabolas, in addition to representing the quadratic (or 2nd degree) functions geometrically, are also the result of the study of conics in Analytical Geometry. Cerqueira [2], Macedo [3], Halberstadt [12], Alves and Pereira [16], and Siqueira [19] point out in their works that this subject is approached in a laconic way, in an analytical/algebraic view, not exploring the characteristics and geometric possibilities of the parabola, nor relating the elements focus, directrix, vertex and axis of symmetry to the study of quadratic functions, as a way of making a parallel between these two subjects. With the help of the works of Venturi [13], Lima et al. [14], and Lima [29] it was possible to rewrite this relationship seeking to make it more explicit to the reader (student or professor).

Starting from this premise, we seek to explore in this work the parabola and its characteristics, aiming at an understanding that relates the context of quadratic functions to Analytical Geometry. In addition, we also brought the use of technology in the exploration of its elements using the GeoGebra dynamic geometry software, from some possibilities of geometric construction of the parabola, in order to subsidize the work of the mathematics teacher.

4. CONCLUSION

After a long search, there was a limited amount of work on the triad Parabolas – Learning Objects – Didactic Engineering, which reinforces the importance of this investigative study, as a form of methodological support for Mathematics teachers who face difficulties to work on this subject in a dynamic way in the classroom. This work carried out a study on the parabola, seeking to relate the analytical view and the geometric view through a Learning Object (LO), being structured based on Didactic Engineering (DE) and on the Theory of Didactic Situations (TDS). Thus, we used the first two phases of DE in this research, given its theoretical character.

In the preliminary analysis presented, a scarce number of works point to classroom activities that bring the teaching of parabola in Analytical Geometry with the use of technology and that make a parallel between the quadratic function and the equation of the parabola, relating them to the form canonical. Thus, in the a priori analysis, we present an LO supported by the GeoGebra software, which explores the visualization of the canonical form of the quadratic function, in which there is the possibility for the teacher to extend his institutionalization within the field of Analytical Geometry, presenting the existing relationship between a quadratic function and the equation of a parabola. TDS enters into this work as a way to organize the teaching session based on the proposed didactic situation.

Based on the above, we understand that there is great value in the didactic proposal presented in this work, which brings a methodological alternative to the teacher to address this theme in the classroom, given the scarcity of bridges between these two themes in Brazilian basic education textbooks, which creates a gap for the student when entering higher education. Finally, as a future perspective, we intend to develop this LO in the classroom and perform data collection, seeking a correction of routes and improvements in LO, showing the relevance of this research for the teaching of parabolas, being a contribution to the Mathematics Education field.

ACKNOWLEDGMENTS

We would like to thank the National Council for Scientific and Technological Development - CNPq for the financial support for the development of this research in Brazil.

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